Announcement

Homework #3 has been posted in Blackboard and class website.

Due on Tuesday, September 20th before class starts.

On the Midterm Exam

- Tuesday, 10/11 in class
- Closed book and closed notes
- One page cheat sheet is allowed
- A calculator is allowed for +-*/
- Covers the topics until the class on Thursday, 10/6

Proposal of Final Project

Due: 11:59 pm, October 4th.

Late submission penalty applies.

Include

- Title and names of the team member
- Topic: a research project or a survey
- Brief introduction on the background
- Timeline and project management for a teamwork

At most one page

Each team only needs one abstract

On the Paper Reading (Section 001)

For students in Section 001 :

Each student has 10 minutes (9 minutes for presentation and 1 minute for questions) to present the paper chosen by yourself.

Presentation days:

- Tuesday, Oct. 18
- Thursday, Oct. 20
- Tuesday, Oct. 25

On the Paper Reading (Section 006)

For students in Section 006 :

Each student has 9 minutes to present the paper chosen by yourself.

Submit a link to your prerecorded video to Blackboard by **11:59pm, Oct. 25**

Uploading a video to Blackboard – YouTube

Detailed instructions of preparing prerecorded videos will be sent by email

On the Paper Reading (Both Sections)

Send me an email (tongy@cec.sc.edu) by 11:59pm of Sep. 29, which includes:

- The paper you are going to present
  - Title, authors, where and when it was published, pages
  - Example: Sing Bing Kang, Ashish Kapoor, Dani Lischinski , "Personalization of Image Enhancement ", in *Proceedings of IEEE Conference on computer vision and Pattern Recognition (CVPR)*, 2010
- **Section 001 only:** Your name and preference of these three days in a decreasing order. Earlier email has higher priority in choosing the day

I will provide feedback (approve/suggest to change) to your selected paper

Where to Find the Paper

The paper you choose must be published in an official journal or conference!

A journal paper is preferred!

You can find papers from journals

- IEEE Transactions on Pattern Analysis and Machine Intelligence
- http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?reload=true&punumber=34
- IEEE Transactions on Image Processing
- http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=83

- Other premier conferences or journals, CVPR, ICCV, ECCV, IEEE Trans. Medical Imaging …

Deadline for your email: 11:59pm, Sep. 29

Today’s Agenda

- **Fourier Transform**
  - FT of simple functions

Fourier Series

***f*(*t*) is a continuous function with period *T*, we have**

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{j2\pi nt}{T}}$$

Discrete frequency

Coefficient

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-\frac{j2\pi nt}{T}}\, dt, \quad n = 0, \pm 1, \pm 2, \ldots$$

https://en.wikipedia.org/wiki/Fourier_transform#/media/File:Fourier_transform_time_and_frequency_domains_(small).gif

Fourier Transform in 1D

$f(t)$ **is an arbitrary non-periodic function and can be represented by**

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Coefficient

Continuous frequency

where

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Fourier Transform in 1D

$f(t)$ is an arbitrary non-periodic function and can be represented by

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

Coefficient

Continuous frequency

where

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Fourier series

Discrete frequency

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{\frac{j2\pi nt}{T}}$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-\frac{j2\pi nt}{T}} dt$$

Fourier Transform in 1D

Spatial domain → Frequency domain

$$F(\mu)=\int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}dt$$ Forward transform

Frequency domain → Spatial domain

$$f(t)=\int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t}d\mu$$ Inverse transform

Fourier transform pair

Basic Properties of FT

Linearity $h(t) = af(t) + bg(t) \leftrightarrow H(\mu) = aF(\mu) + bG(\mu)$

Translation $h(t) = f(t - t_0) \leftrightarrow H(\mu) = e^{-j2\pi t_0 \mu} F(\mu)$

Translation in spatial domain → Rotation in frequency domain

Modulation $h(t) = e^{j2\pi\mu_0 t} f(t) \leftrightarrow H(\mu) = F(\mu - \mu_0)$

Rotation in spatial domain → Translation in frequency domain

Basic Properties of FT

Scaling

$$h(t) = f(at) \leftrightarrow H(\mu) = \frac{1}{|a|} F(\frac{\mu}{a})$$

Conjugation

$$h(t) = f^*(t) \leftrightarrow H(\mu) = F^*(-\mu)$$

Symmetry

$$f(t) \leftrightarrow F(\mu) \Rightarrow F(t) \leftrightarrow f(-\mu)$$

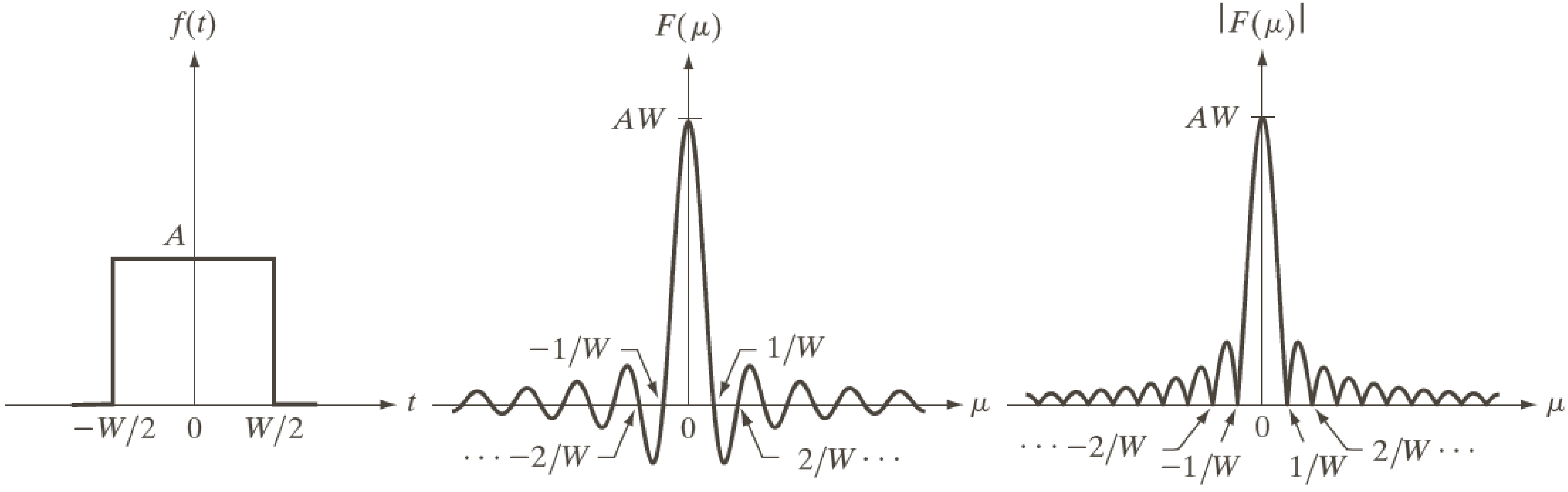
FT of Simple Functions

$$f(t) = \begin{cases} A & -\frac{w}{2} \leq t \leq \frac{w}{2} \\ 0 & otherwise \end{cases}$$

$$F(\mu) = \frac{A}{\pi\mu} \sin \pi w \mu = Aw \frac{\sin \pi w \mu}{\pi w \mu} = Aw\ sinc(\pi w \mu)$$

FT of a Rectangle Function

Rectangle function → Sinc function



a b c

FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Continuous Impulses and Sifting Property

Unit impulse

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Sifting property

$$\left.\begin{aligned} \int_{-\infty}^{\infty} \delta(t) g(t) dt &= g(0) \\ \int_{-\infty}^{\infty} \delta(t - t_0) g(t) dt &= g(t_0) \end{aligned}\right\} \begin{array}{l}\text{The value of function at} \\ \text{the impulse location}\end{array}$$

FT of an Impulse

$$\delta(t) \leftrightarrow ?$$

$$\delta(t - t_0) \leftrightarrow ?$$

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

Proof with

- sifting property

$$\int_{-\infty}^{\infty} \delta(t - t_0) g(t) dt = g(t_0)$$

- translation property

$$h(t) = f(t - t_0) \leftrightarrow H(\mu) = e^{-j2\pi t_0 \mu} F(\mu)$$

FT of an Impulse

$$\delta(t) \leftrightarrow F(\mu) = 1$$

$$\delta(t - t_0) \leftrightarrow F(\mu) = e^{-j2\pi\mu t_0}$$

FT of an Impulse

$$e^{j2\pi t_0 t} \leftrightarrow ?$$

$$F\left(e^{j2\pi t_0 t}\right) = \delta(\mu - t_0)$$

Symmetry property

$$f(t) \leftrightarrow F(\mu) \Rightarrow F(t) \leftrightarrow f(-\mu)$$

$$\delta(t - t_0) \leftrightarrow F(\mu) = e^{-j2\pi u t_0}$$

⇒

$$F\left(e^{-j2\pi t_0 t}\right) = \delta(-\mu - t_0)$$
$$= \delta(\mu + t_0)$$

⇓

$$F\left(e^{j2\pi t_0 t}\right) = \delta(\mu - t_0)$$

Discrete Impulses and Sifting Property

Unit impulse

$$\delta(x)=\begin{cases}1 & \text{if } x=0\\ 0 & \text{if } x\neq 0\end{cases} \quad \text{and} \quad \sum_{x=-\infty}^{+\infty}\delta(x)=1$$

Sifting property

$$\sum_{x=-\infty}^{\infty}\delta(x)g(x)=g(0)$$

$$\sum_{x=-\infty}^{\infty}\delta(x-x_0)g(x)=g(x_0)$$

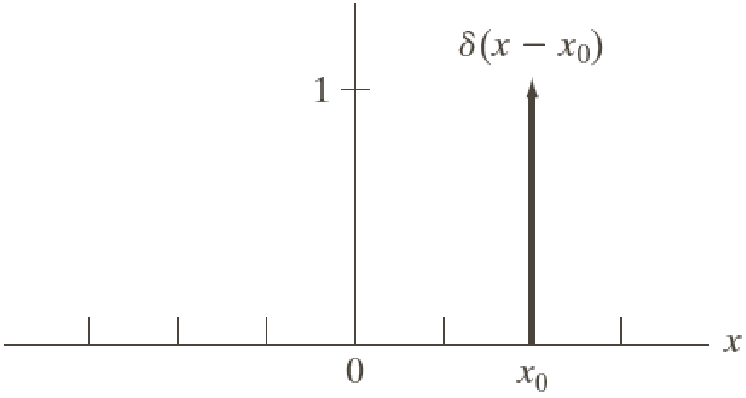


FIGURE 4.2 A unit discrete impulse located at $x = x_0$. Variable x is discrete, and δ is 0 everywhere except at $x = x_0$.

Impulse Train

$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

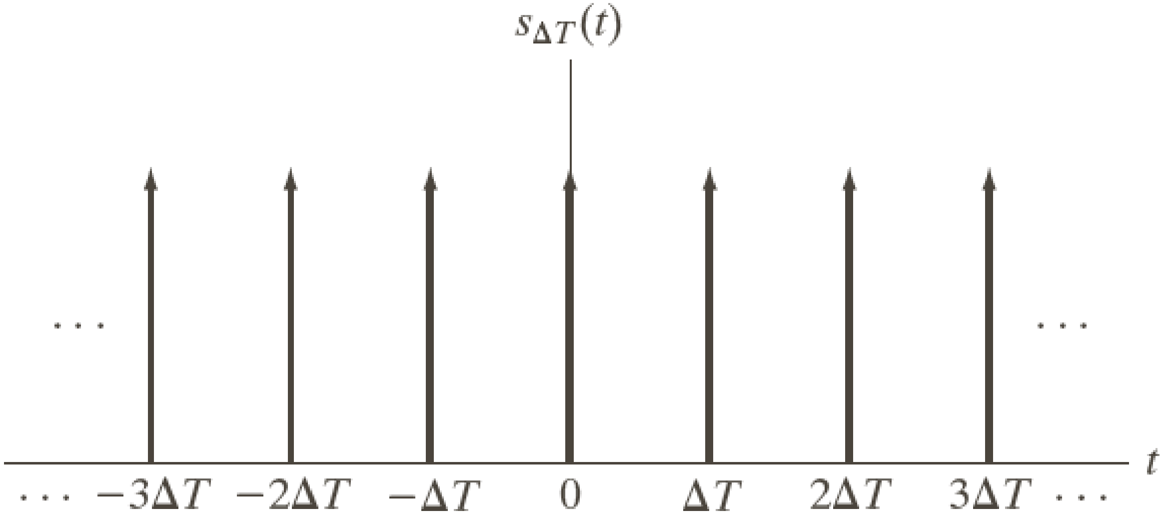


FIGURE 4.3 An impulse train.

Sampling in Spatial Domain

$$\tilde{f}(t) = f(t)s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t)\delta(t - n\Delta T)$$

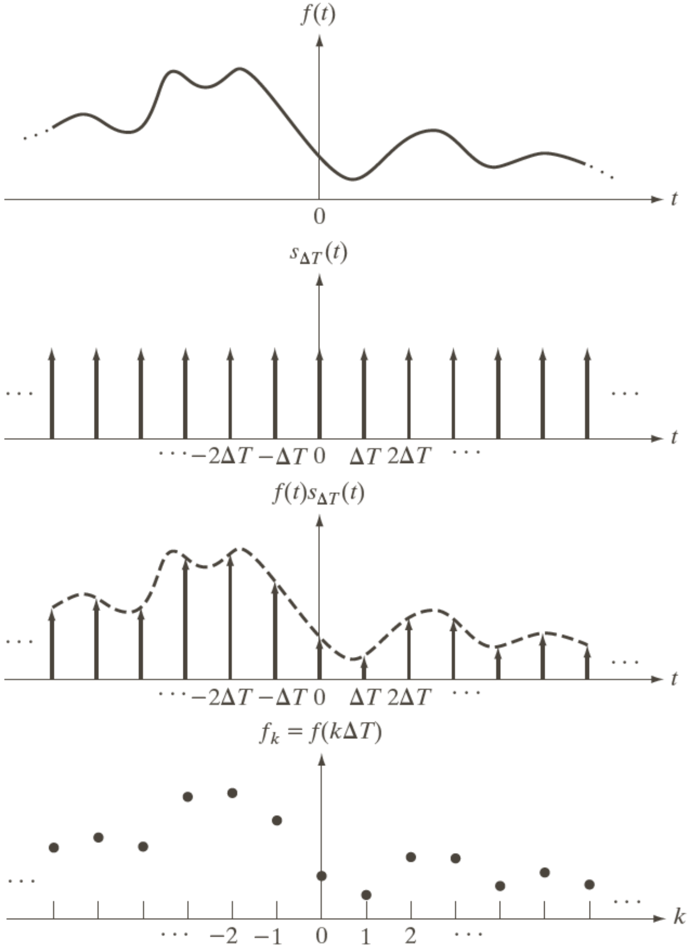


FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

FT of an Impulse Train

$$s_{\Delta T}(t) \leftrightarrow \; S(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta(\mu - \frac{n}{\Delta T})$$

Fourier series of an impulse train $s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{\Delta T}t}$

where $c_n = \frac{1}{\Delta T}\int_{-\frac{\Delta T}{2}}^{\frac{\Delta T}{2}} s_{\Delta T}(t) e^{-\frac{j2\pi nt}{\Delta T}} dt$ ⇨ $c_n = \frac{1}{\Delta T}$

$$S(\mu) = \frac{1}{\Delta T}\sum_{n=-\infty}^{\infty}\delta(\mu - \frac{n}{\Delta T})$$

⇨ FT of an impulse train is an impulse train in frequency domain

FT of an Impulse and Impulse Train

$$\delta(t) \leftrightarrow F(\mu) = 1$$

$$\delta(t - t_0) \leftrightarrow F(\mu) = e^{-j2\pi\mu t_0}$$

$$e^{j2\pi t_0 t} \leftrightarrow \delta(\mu - t_0)$$

$$s_{\Delta T}(t) \leftrightarrow \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\mu - \frac{n}{\Delta T} \right)$$