Today's Calendar

Intensity Transformation

Histogram processing

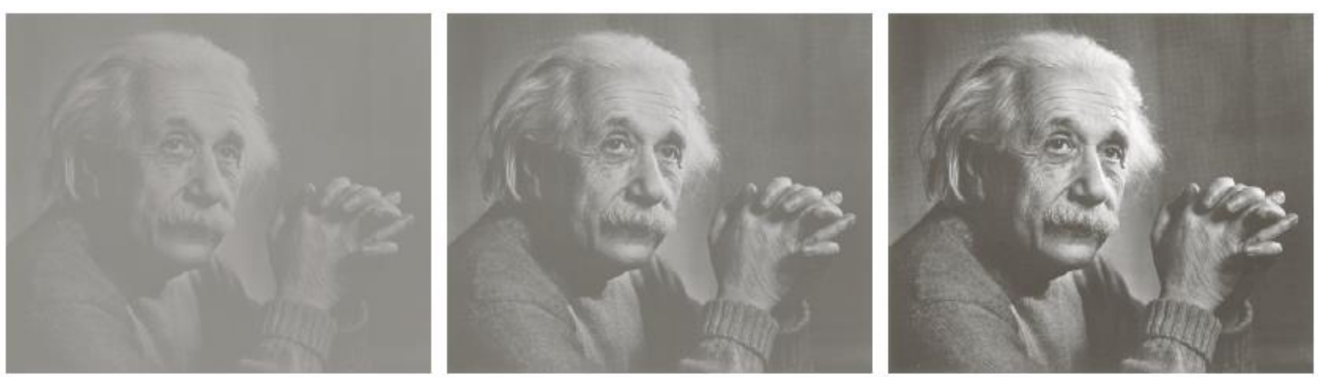
Probability Methods

$$m = \sum_{k=1}^{L-1} z_k\, p(z_k), \quad \sigma^2 = \sum_{k=1}^{L-1} (z_k - m)^2\, p(z_k)$$

What do they mean?

$$\mu_n(z) = \sum_{k=1}^{L-1} (z_k - m)^n\, p(z_k)$$

n^{th} moment of z



a b c

FIGURE 2.41 Images exhibiting (a) low contrast, (b) medium contrast, and (c) high contrast.

Std=14.3 Std=31.6 Std=49.2

Stochastic Image-Sequence Processing

Using probability and random-process tools

Each pixel is a random event → each image frame is a random event, related to time

Probability plays a central role in modern image processing and computer vision

Summary

In this course, we will discuss all the concepts in details.

Now,

Intensity Transformation and Spatial Filtering

Reading: Chapter 3.

Spatial Domain

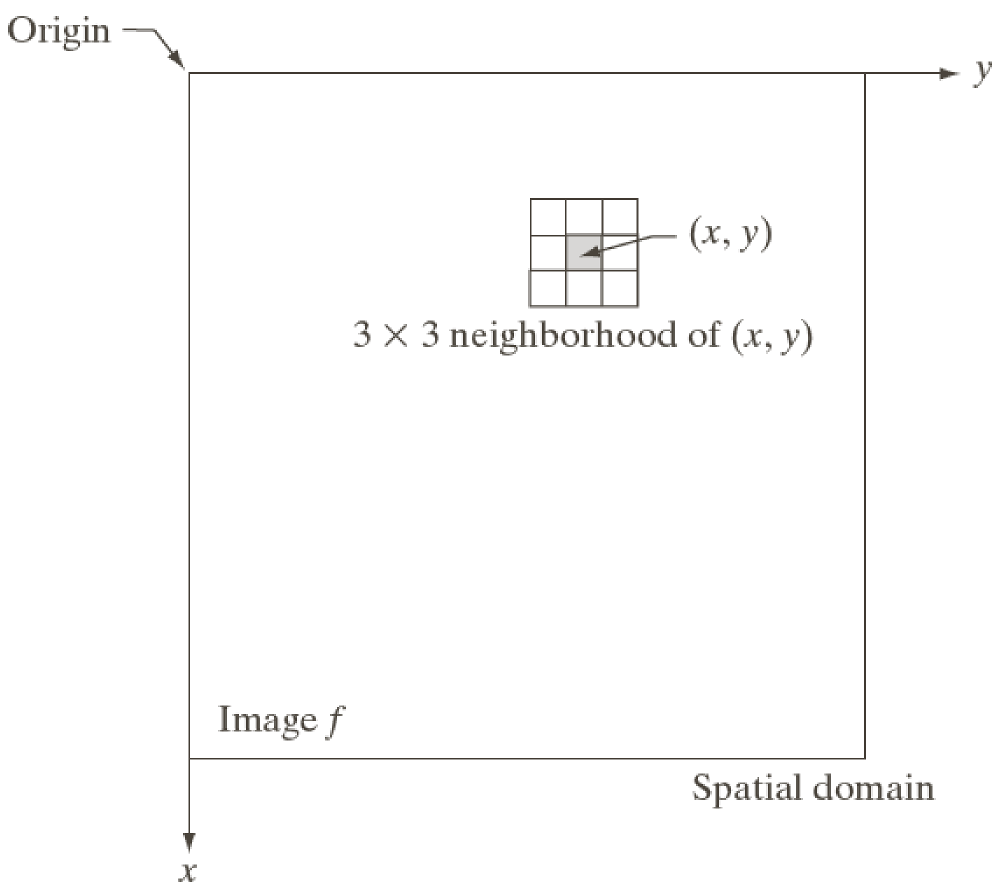


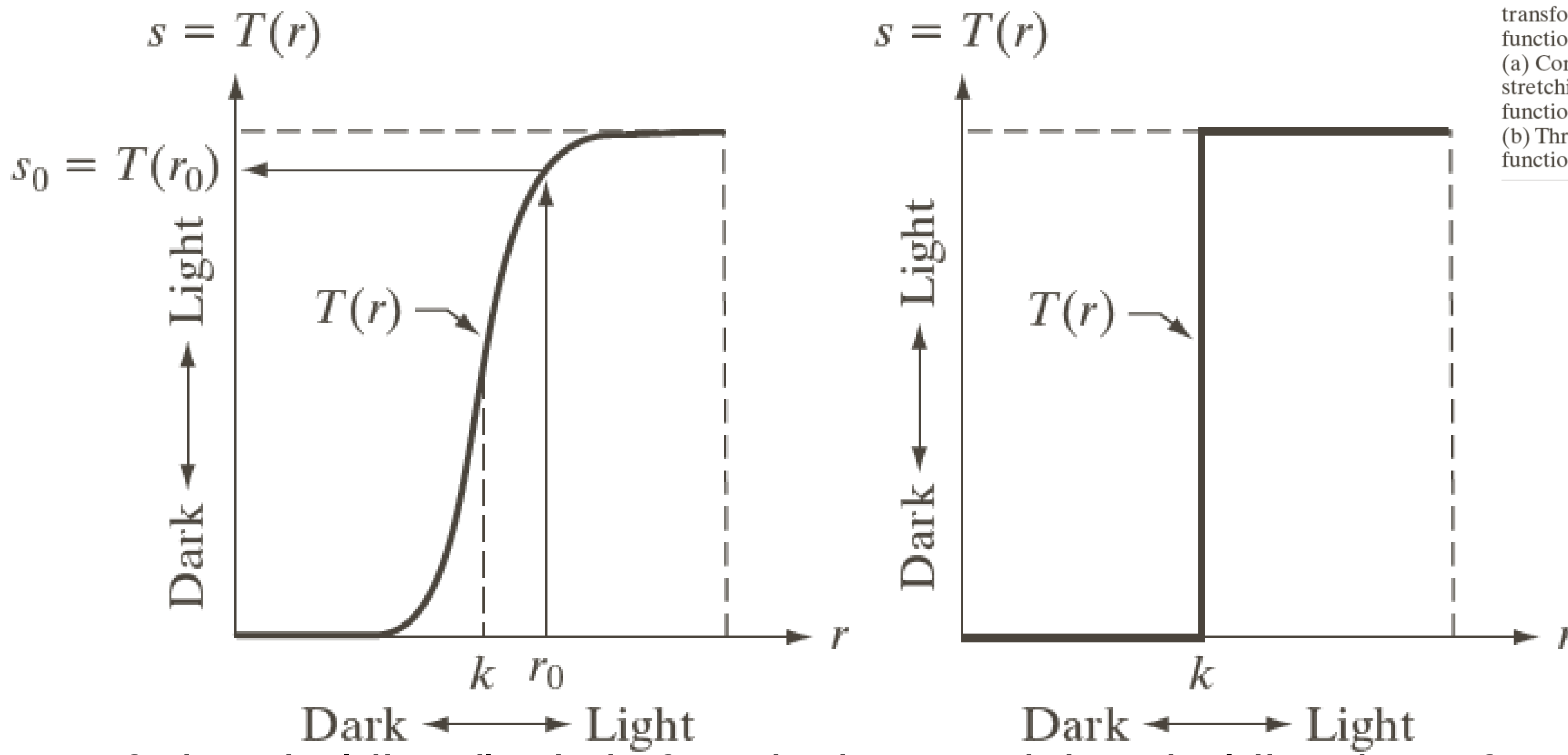
FIGURE 3.1 A 3 × 3 neighborhood about a point (x, y) in an image in the spatial domain. The neighborhood is moved from pixel to pixel in the image to generate an output image.

$$g(x,y)=T\,[f(x,y)]$$

→ spatial filter

1x1 Neighborhood → Intensity Transformation → Image Enhancement

Contrast stretch



a b

FIGURE 3.2 Intensity transformation functions. (a) Contrast-stretching function. (b) Thresholding function.

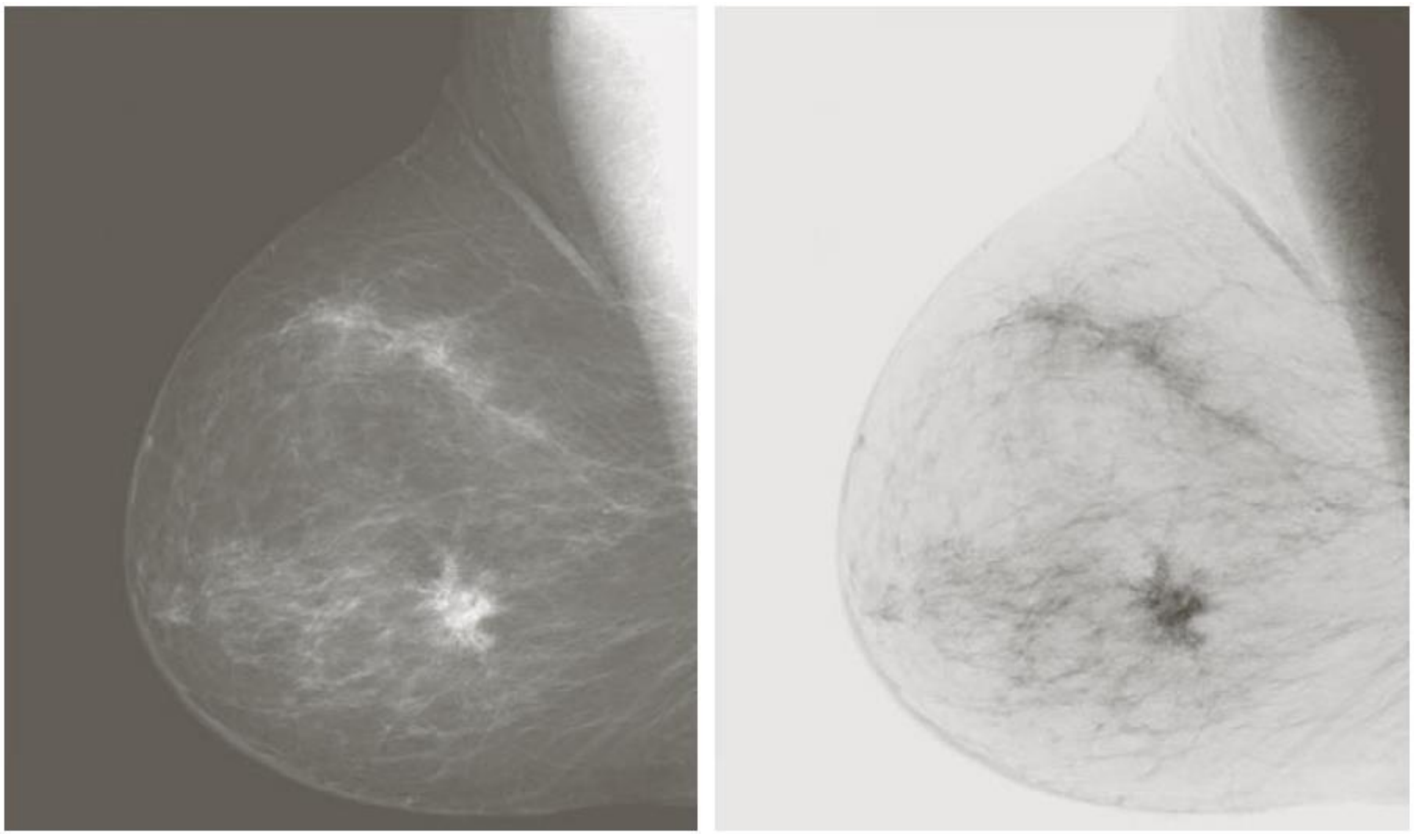
Soft thresholding (logistic function)   Hard thresholding (step function)

$$\sim s = \frac{1}{1 + e^{r}}$$

Some Basic Intensity Transformation Functions

- Thresholding – Logistic function
- Log transformation
- Power-law (Gamma correction)
- Piecewise-linear transformation
- Histogram processing

Some Basic Intensity Transformation Functions



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

Image Negative: $s = L - 1 - r$

Basic Intensity Transformation Functions

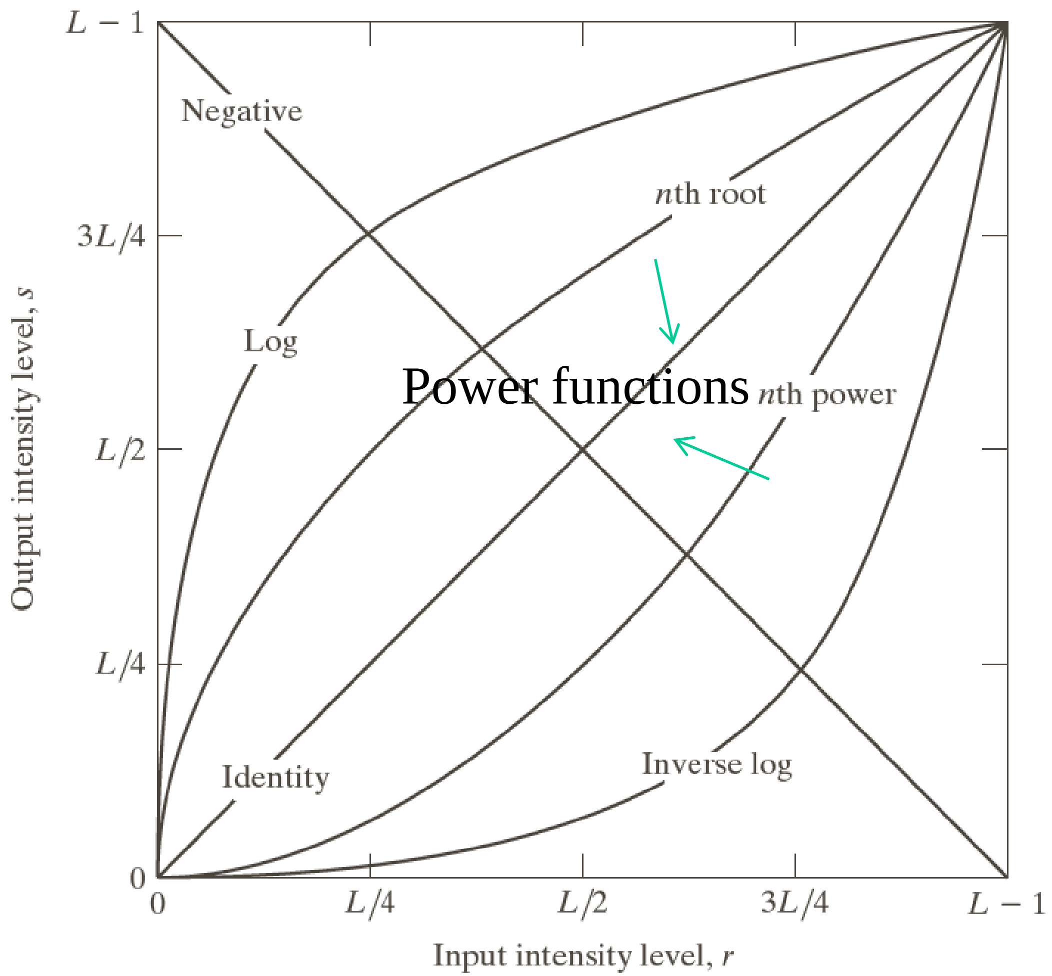


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Log/Inverse Log Transformation Functions

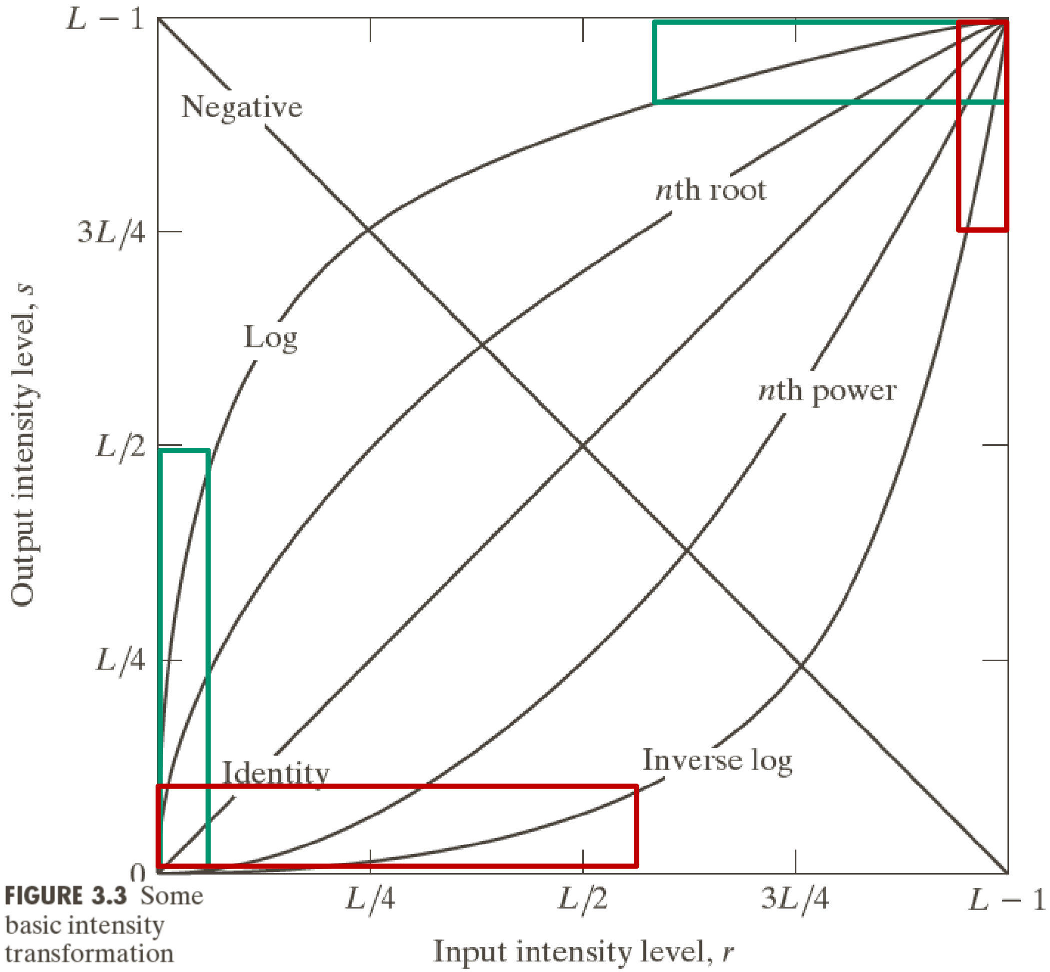


FIGURE 3.3 Some basic intensity transformation functions. All curves were scaled to fit in the range shown.

Log function:

$$s = c\log(1+r) \quad r \geq 0$$

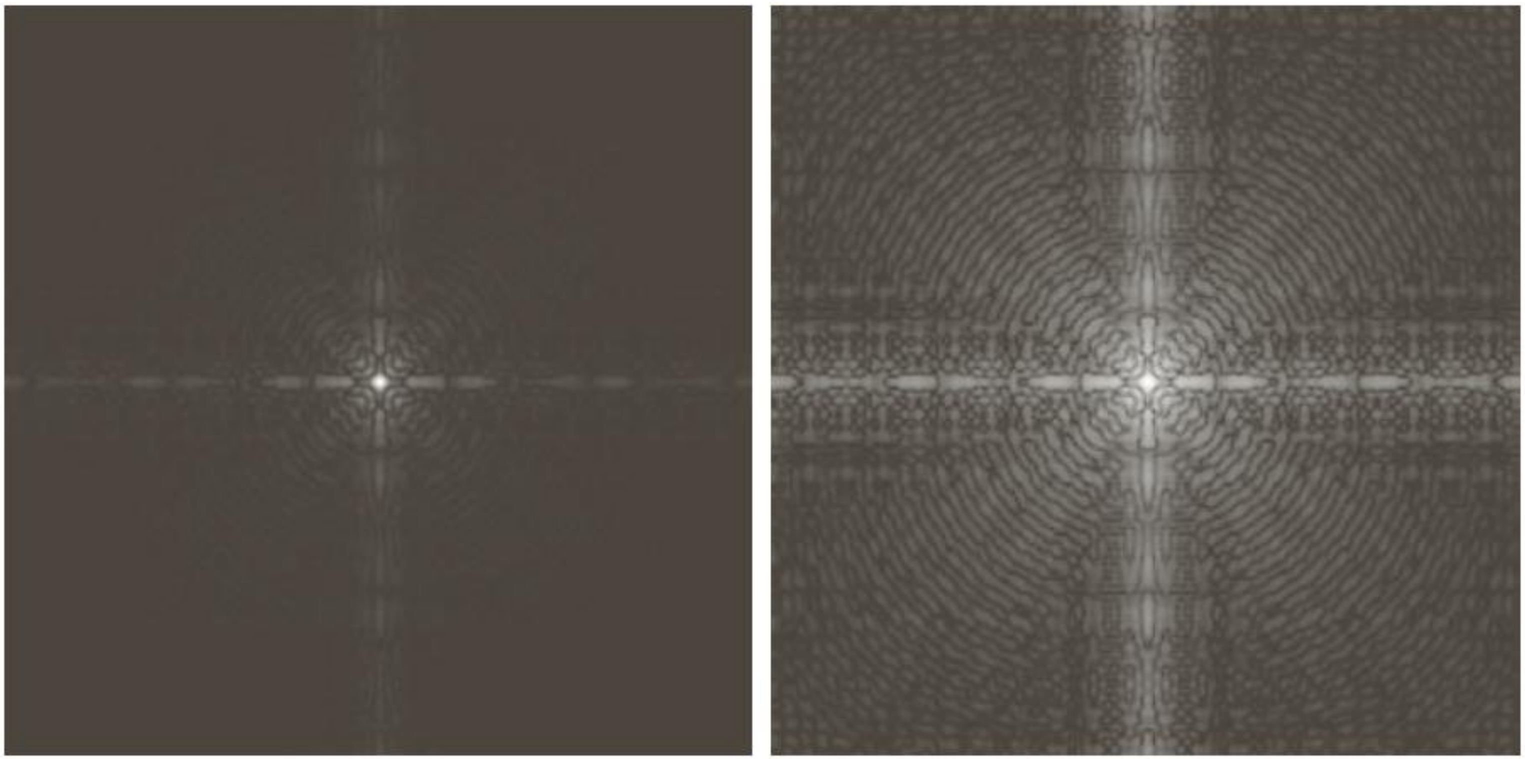
Stretch low intensity levels
Compress high intensity levels

Inverse log function:

$$s = c\log^{-1}(r)$$

Stretch high intensity levels
Compress low intensity levels

Log Transformations: $s = c \log(1+r)$



a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

Power-Law (Gamma) Transformations

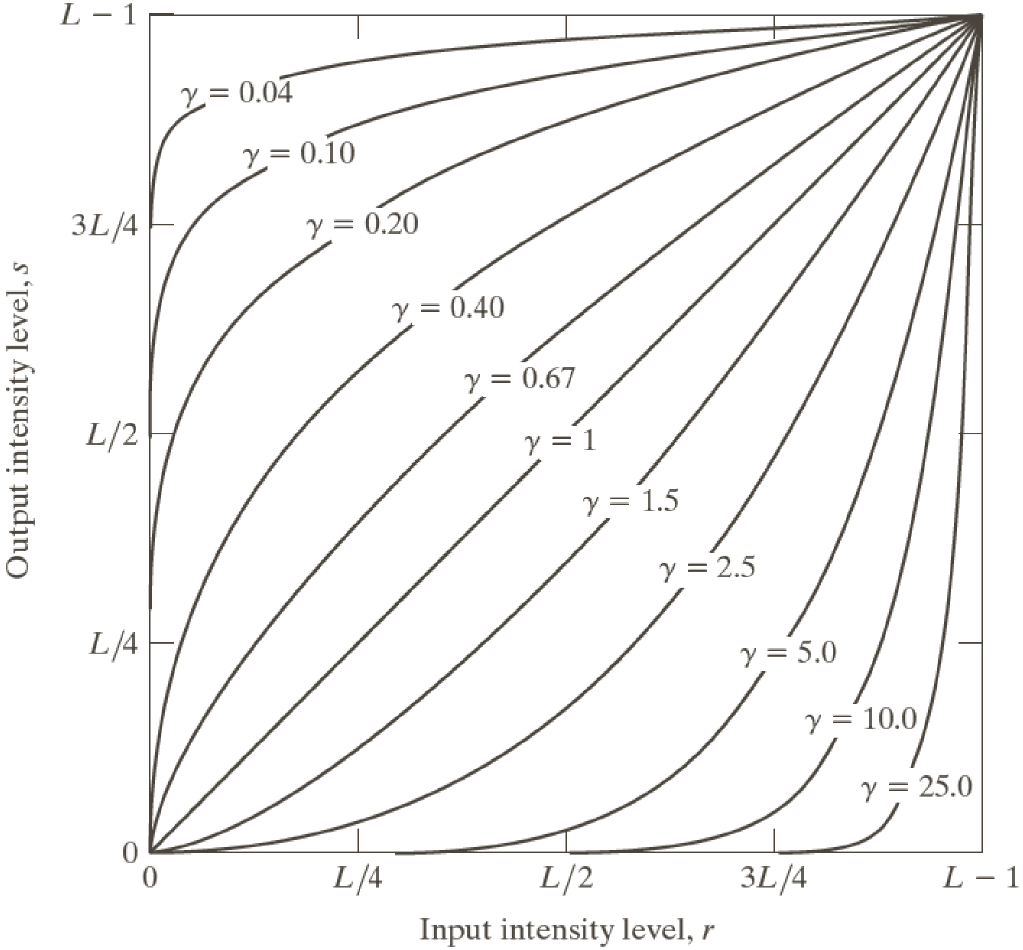
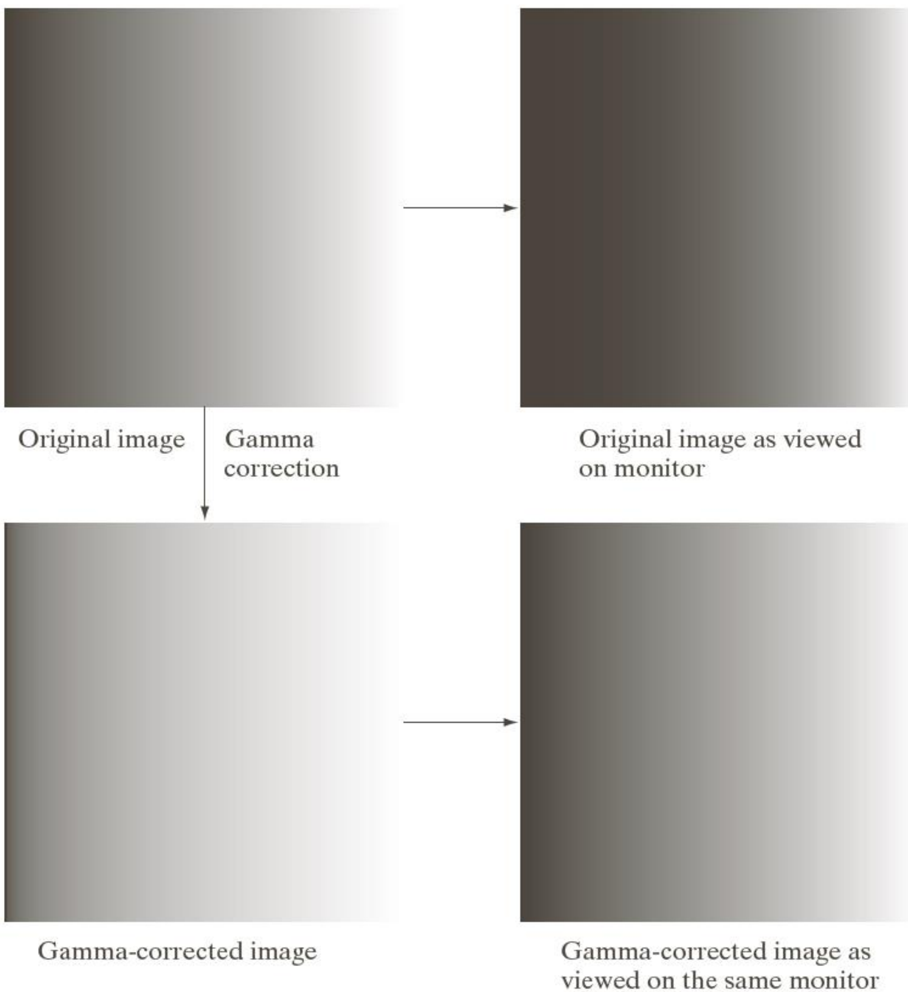


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases). All curves were scaled to fit in the range shown.

$$s = cr^{\gamma}$$

- More versatile than log transformation
- Performed by a lookup table

Power-Law (Gamma) Transformations



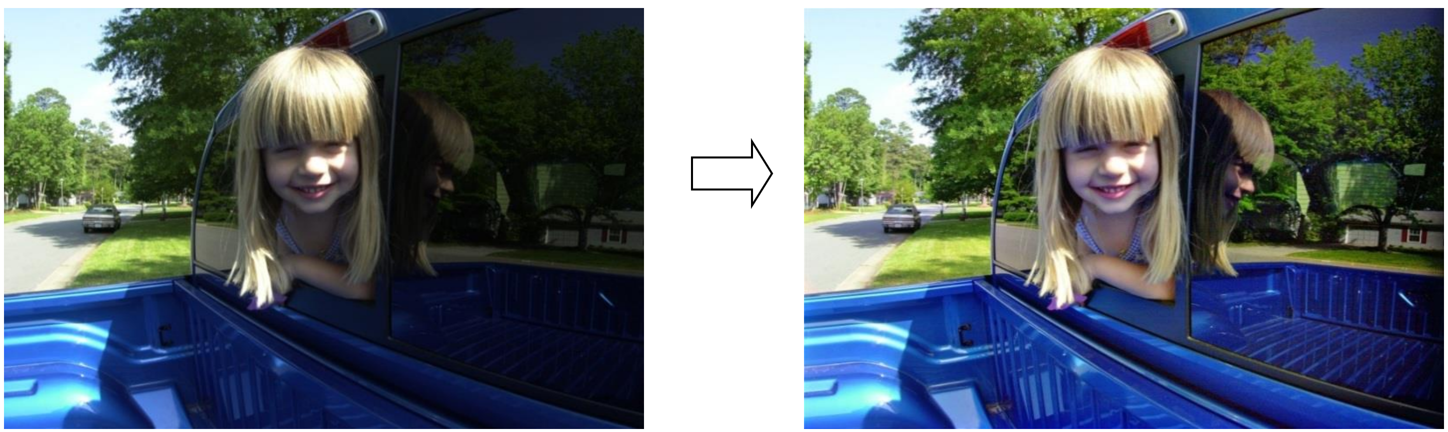
Monitors have an intensity-to-voltage response with a power function

$$s = r^{1/2.5}$$

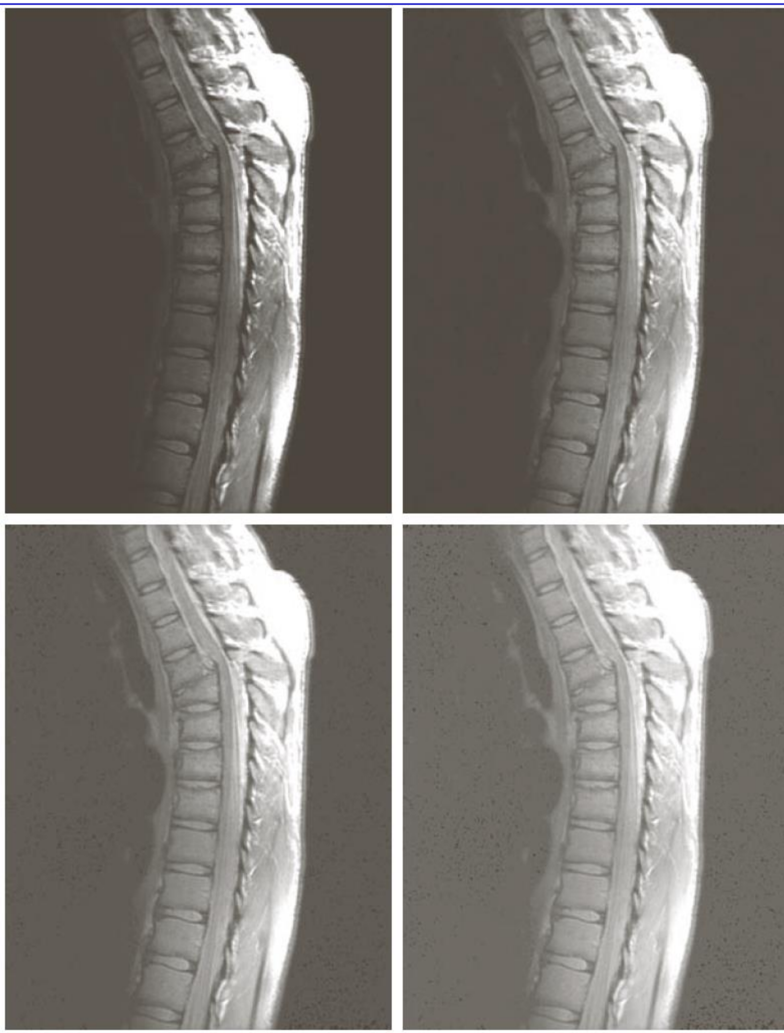
a	b
c	d

FIGURE 3.7
(a) Intensity ramp image. (b) Image as viewed on a simulated monitor with a gamma of 2.5. (c) Gamma-corrected image. (d) Corrected image as viewed on the same monitor. Compare (d) and (a).

Image Enhancement Using Gamma Correction



Power-Law (Gamma) Transformations for Contrast Manipulation



a b
c d

FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Washed-out appearance caused by a small gamma value

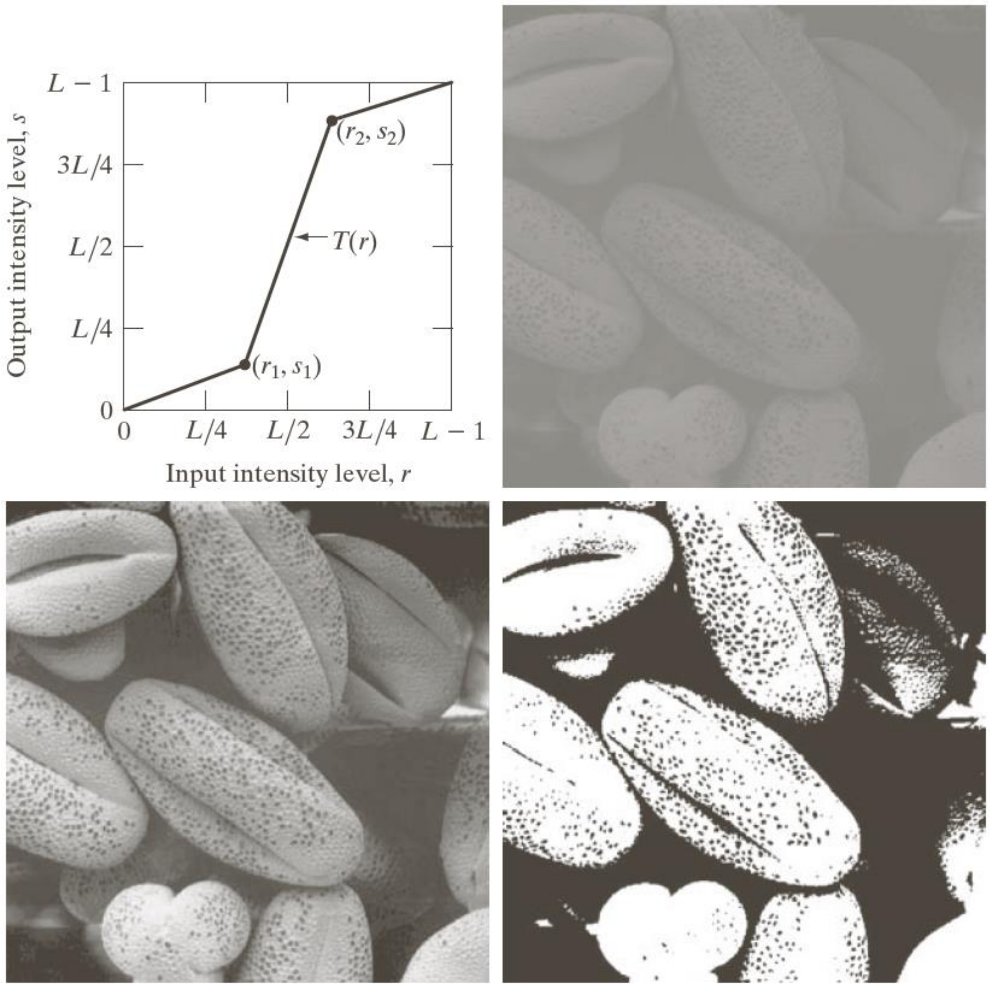
Power-Law (Gamma) Transformations for Contrast Manipulation

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and 5.0, respectively. (Original image for this example courtesy of NASA.)

Washed-out appearance was reduced by a large gamma value

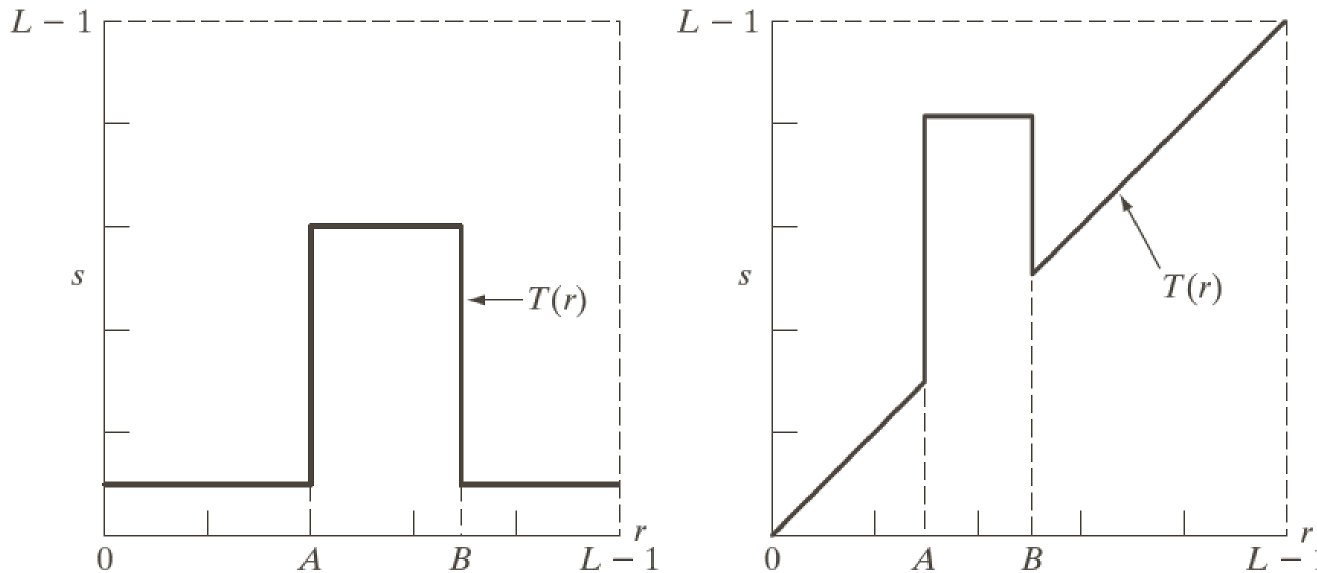
Piecewise-Linear Transformation Functions: Contrast Stretching



a b
c d

FIGURE 3.10 Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)

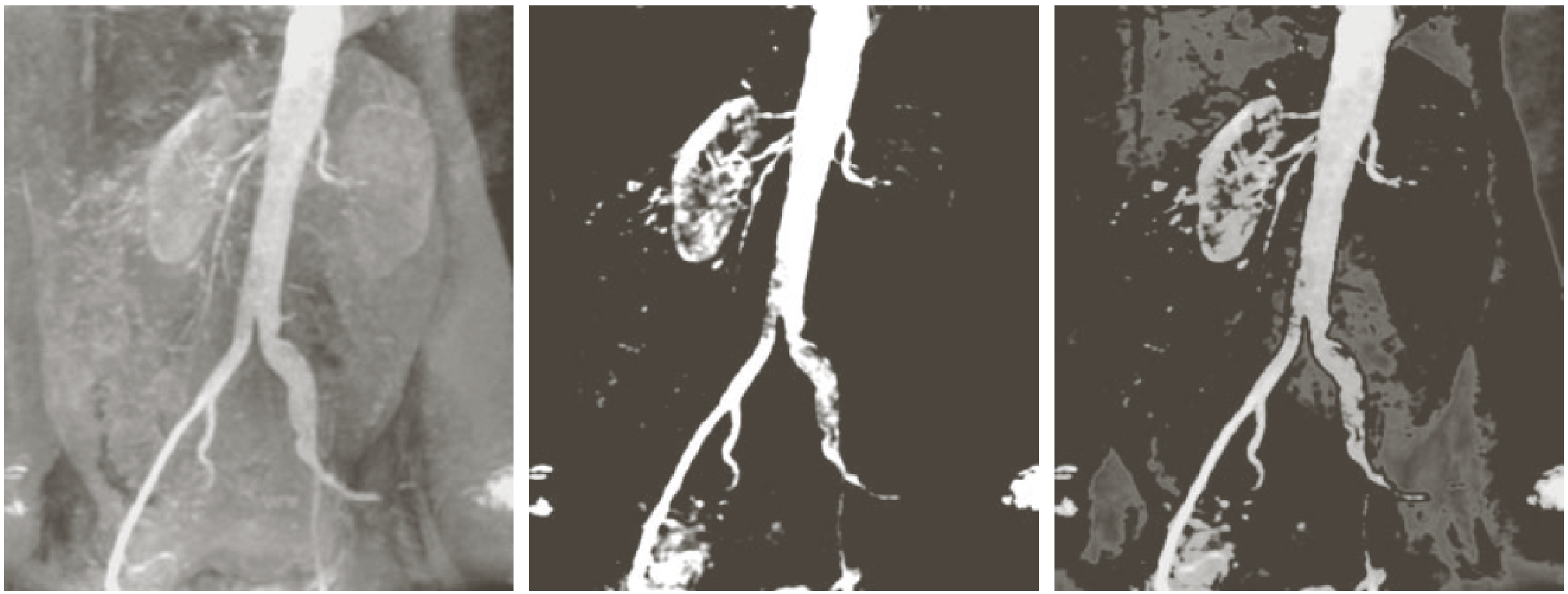
Piecewise-Linear Transformation Functions: Intensity-Level Slicing



a b

FIGURE 3.11 (a) This transformation highlights intensity range [A, B] and reduces all other intensities to a lower level. (b) This transformation highlights range [A, B] and preserves all other intensity levels.

An Example of Intensity-Level Slicing



a b c

FIGURE 3.12 (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

Piecewise-Linear Transformation Functions: Bit-Plane Slicing

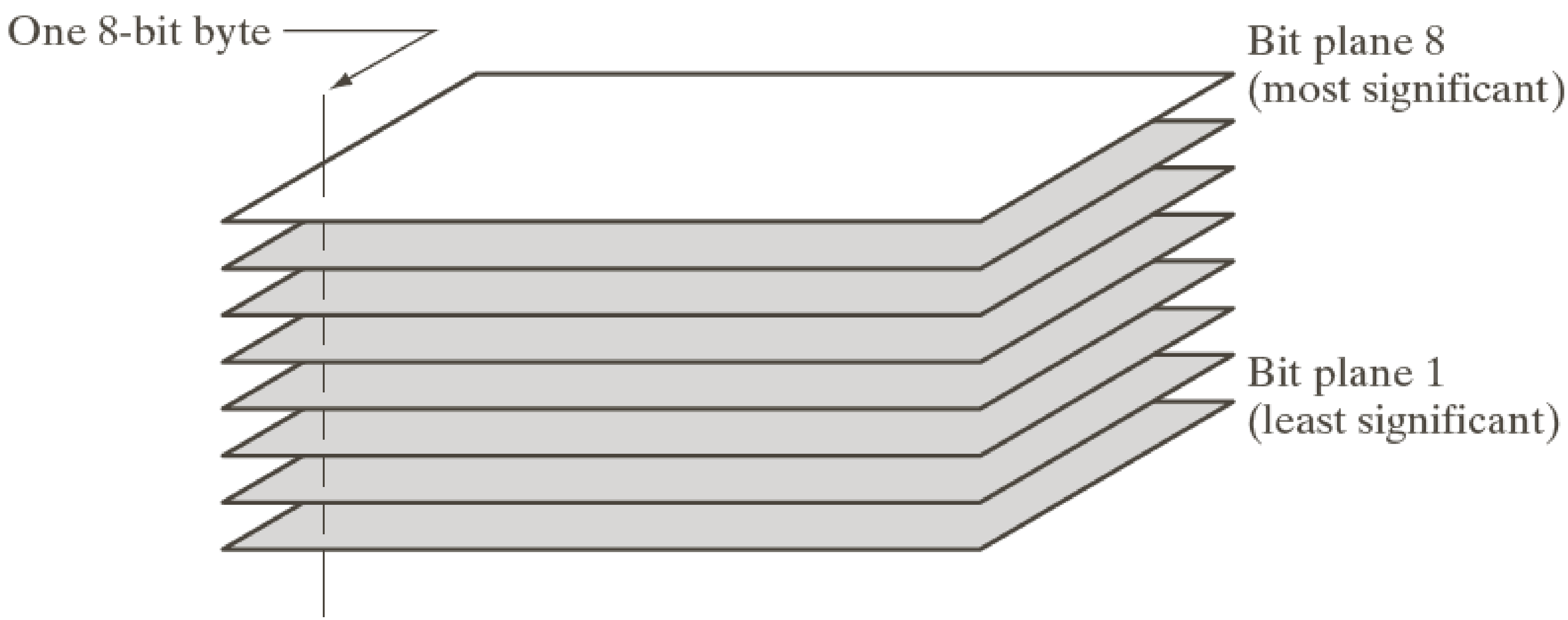


FIGURE 3.13
Bit-plane representation of an 8-bit image.

An Example

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Use for Image Compression

original

a b c

FIGURE 3.15 Images reconstructed using (a) bit planes 8 and 7; (b) bit planes 8, 7, and 6; and (c) bit planes 8, 7, 6, and 5. Compare (c) with Fig. 3.14(a).

Less bit planes are sufficient to obtain an acceptable details, while require half of the storage

Histogram Processing

Dark:

Light:

Low contrast:

High contrast:

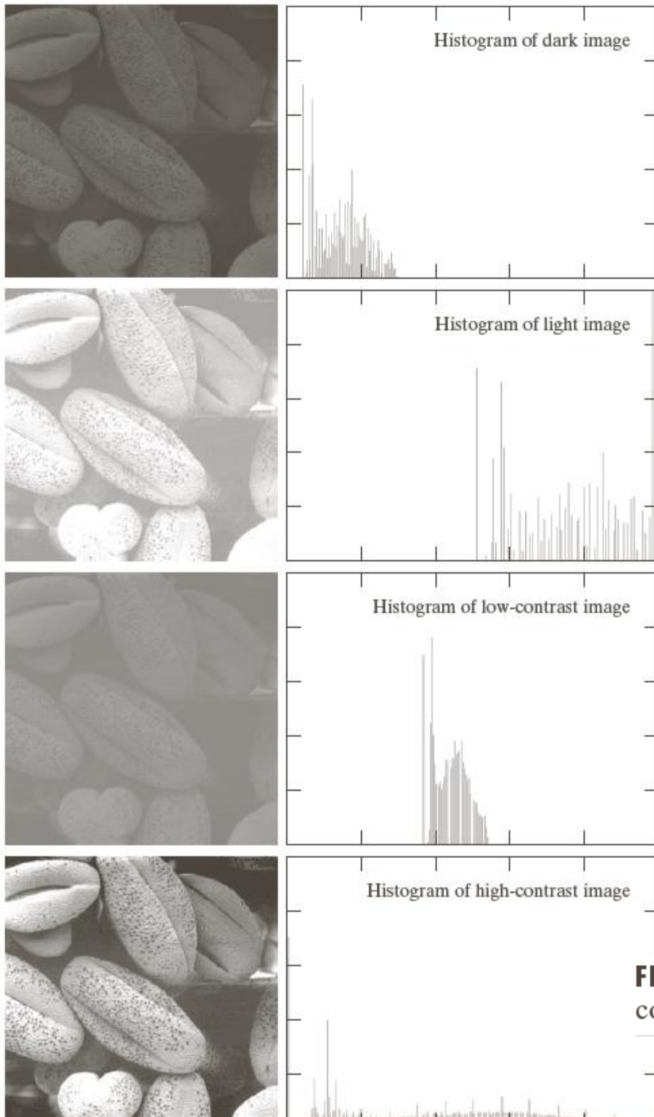


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram

$$h(r_k) = n_k$$

Normalized histogram

$$p(r_k) = n_k / MN$$

$$\sum_{k=0}^{255} p(r_k) = 1$$

Transformation Function

$$s = T(r) \quad 0 \le r \le L-1$$

A valid transformation function must satisfy two conditions:

(a) $T(r)$ is monotonically increasing, i.e., $T(r_1) \ge T(r_2)$ if $r_1 > r_2$

(b) $0 \le T(r) \le L-1$ The same range as input

(a') $T(r)$ is strictly monotonic : one-to-one mapping $r = T'(s)$

Intensity Transformation Function

$$s = T(r) \quad 0 \le r \le L-1$$

A valid transformation function must satisfy two conditions:

$(a)\ T(r)$ is monotonically increasing, i.e., $\mathrm{T}(\mathrm{r}_1) \ge \mathrm{T}(\mathrm{r}_2)$ if $\mathrm{r}_1 > \mathrm{r}_2$

$(b)\ 0 \le T(r) \le L-1$

$(a')\ T(r)$ is strictly monotonic : one - to - one mapping $\mathrm{r} = \mathrm{T}'(\mathrm{s})$

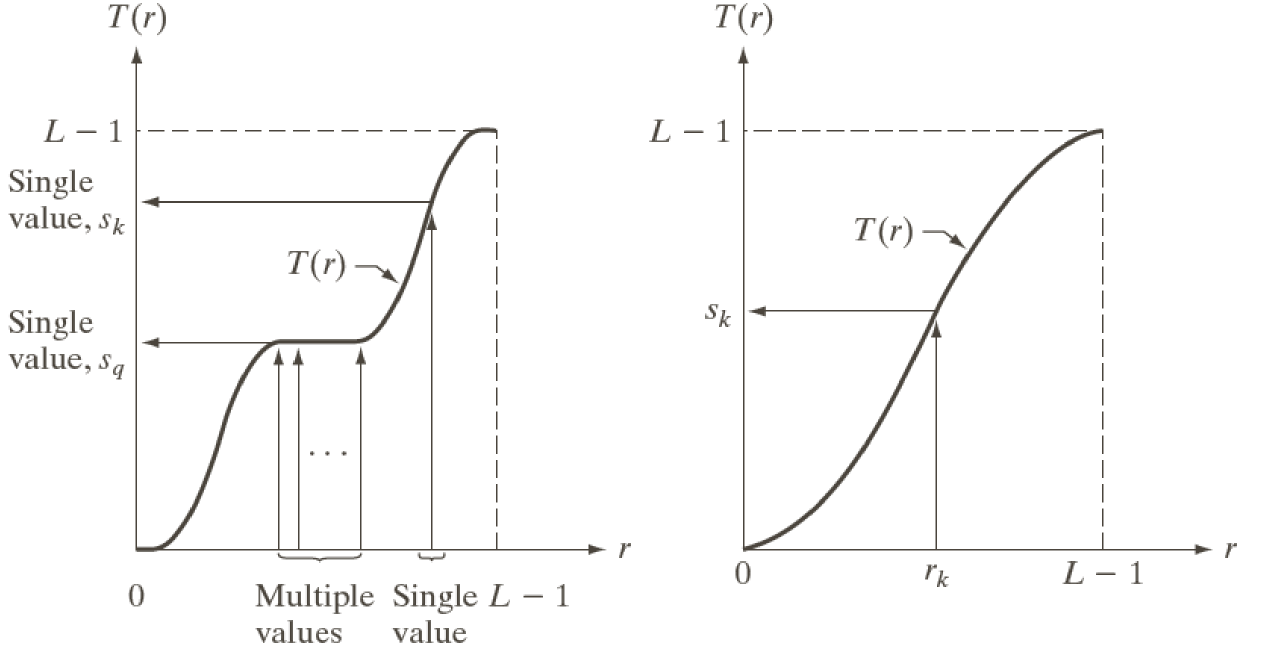


FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Processing

If $T(r)$ is continuous and differentiable over the range of *r*, then

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

Probability density function of intensity value

Histogram Equalization

A special transformation function

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Cumulative distribution function of r

Is it a valid transformation function?

Yes.

(a) $T(r)$ is monotonically increasing, i.e., $T(r_1) \geq T(r_2)$ if $r_1 > r_2$

(b) $0 \leq T(r) \leq L-1$

Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\Rightarrow \quad p_s(s) = \frac{1}{L-1}$$

How to prove it?

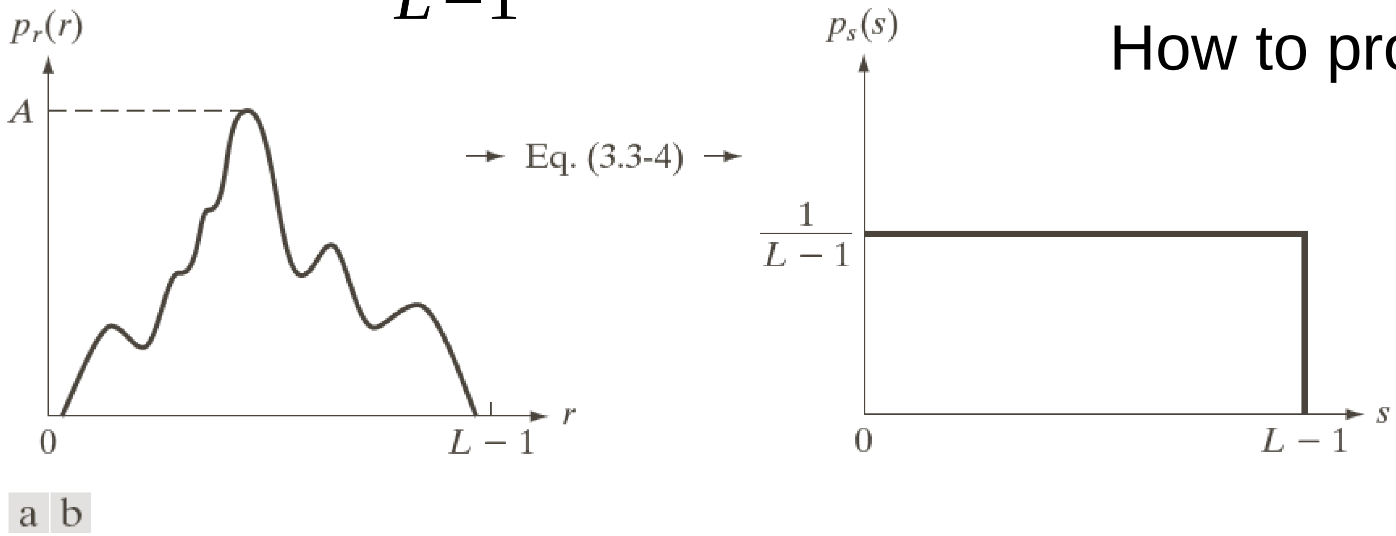


FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|, \quad s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\Rightarrow \quad p_s(s) = \frac{1}{L-1}$$

An Example

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & 0 \le r \le (L-1) \\ 0 & otherwise \end{cases}$$

$$s = (L-1)\int_0^r p_r(w)dw = \cdots = \frac{r^2}{L-1}$$

Histogram Equalization – Discrete Case

$$p_r(r_k) = n_k / MN, k = 0,1,2,..., L-1$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1
Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

$$s_0 = T(r_0) = (L-1)\sum_{j=0}^{0} p_r(r_j) = (8-1) * 0.19 = 1.33 \rightarrow 1$$

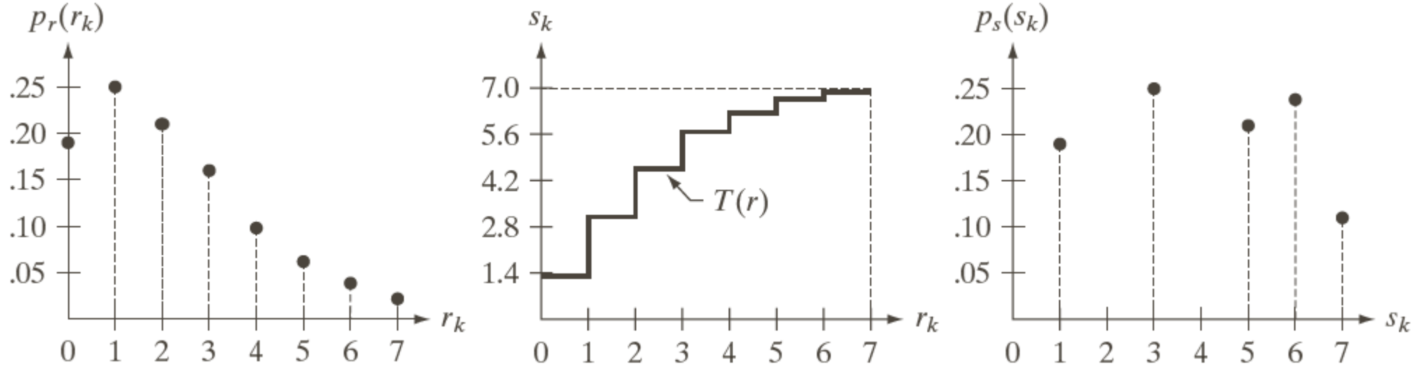
$$\vdots$$

$$s_7 = T(r_7) = (L-1)\sum_{j=0}^{7} p_r(r_j) = (8-1) * 1 = 7$$

Histogram Equalization – Discrete Case

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64 × 64 digital image.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Histogram equalization is not guaranteed to result in a uniform histogram.

Examples

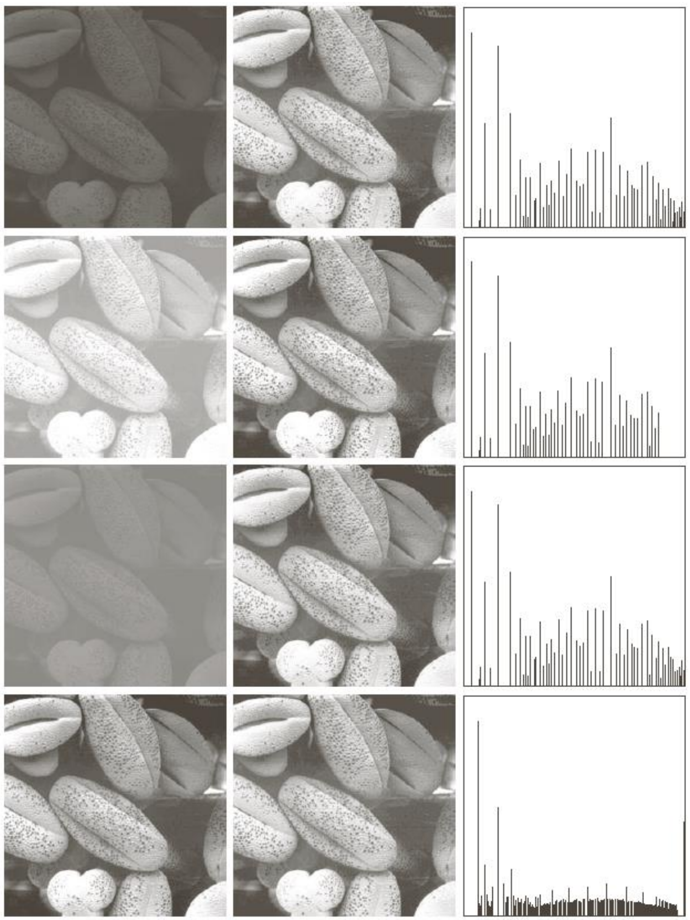


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

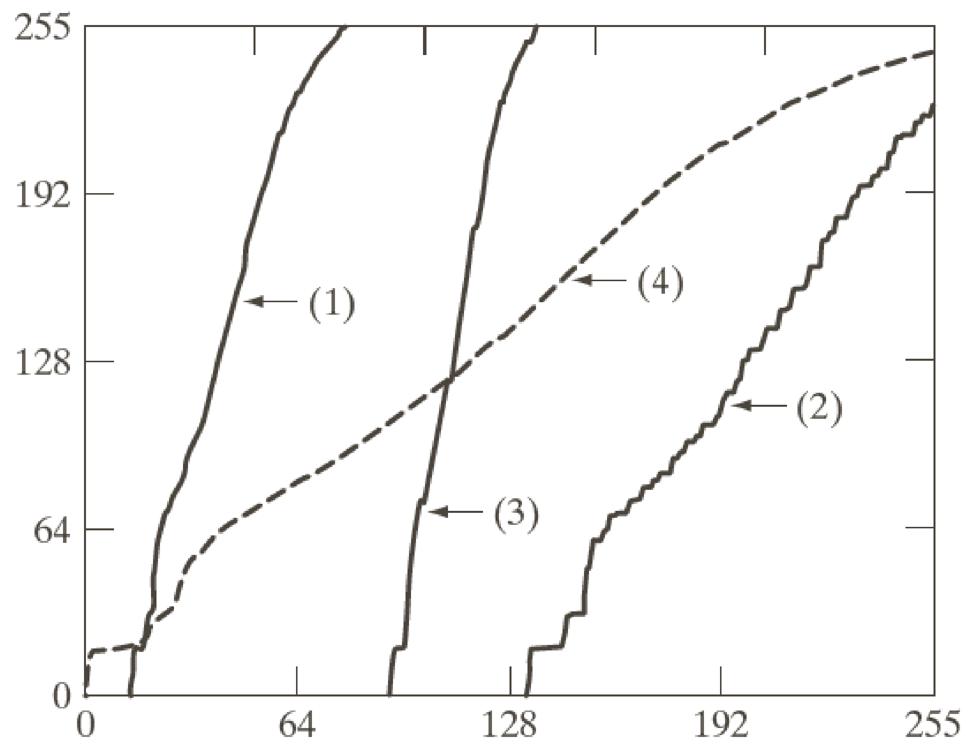


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

A Continuous Example

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & 0 \le r \le (L-1) \\ 0 & otherwise \end{cases}$$

$$p_z(z) = \begin{cases} \dfrac{3z^2}{(L-1)^3} & 0 \le z \le (L-1) \\ 0 & otherwise \end{cases}$$

Compute z?