Basic Concepts in Digital Image Processing
Announcement

Homework #1 was posted in dropbox and on class website.

Due time: Wednesday, Jan 31, before class starts.
Today’s Agenda

• Basic Relationships between Pixels
Basic Set and Logical Operations

- A is a set: \[ A = \{.\} \quad \text{e.g. } A = \{1, \ldots, 255\} \] or \[ A = \{w | w = 1, \ldots, 255\} \]
  \[ A = \emptyset \quad \text{for empty set} \]
- a is an element of A (a \in A) or a isn’t an element of A (a \notin A)
- A is a subset of B if every element in A also is in B \( A \subseteq B \)
- C is the union of two sets A and B \( C = A \cup B \)
- C is the intersection of A and B \( C = A \cap B \)
- Disjoint or mutual exclusive sets \( A \cap B = \emptyset \)
- Set universe is the set of all elements in an application
- Set difference \( A - B = \{w | w \in A, w \notin B\} \)
Set Operations Based on Coordinates

A region in an image is represented by a set of coordinates within the region.

**FIGURE 2.31**
(a) Two sets of coordinates, $A$ and $B$, in 2-D space. (b) The union of $A$ and $B$. (c) The intersection of $A$ and $B$. (d) The complement of $A$. (e) The difference between $A$ and $B$. In (b)–(e) the shaded areas represent the member of the set operation indicated.
Some Basic Relationships between Pixels

Neighbors of a pixel

\[ N_4(p) \]

\[ N_8(p) \]

\[ N_D(p) \]
Adjacency

Adjacency is the relationship between two pixels $p$ and $q$. $V$ is a set of intensity values used to define adjacency:

- **Binary image**: $V = \{1\}$ or $V = \{0\}$
- **Gray level image**: $V \subseteq \{0, 1, \ldots, 255\}$

\[ f(p) \in V \quad \text{and} \quad f(q) \in V \quad \text{Intensity constraints} \]

Three types of adjacency:

- **4-adjacency**
- **8-adjacency**
- **m-adjacency**

\[
\begin{align*}
p & \quad q \in N_4(p) \\
0 & 1 1 \\
0 & 0 0 \\
1 & 0 1 \\
1 & 1 1
\end{align*}
\]

\[
\begin{align*}
p & \quad q \in N_8(p) \\
0 & 1 1 \\
0 & 1 0 \\
0 & 0 0 \\
0 & 0 1
\end{align*}
\]

\[
\begin{align*}
p & \quad q \in N_D(p) \quad \text{and} \quad N_4(q) \cap N_4(p) = \emptyset \\
0 & 1 1 \\
1 & 0 0 \\
1 & 1 0 \\
0 & 0 1
\end{align*}
\]

or $q \in N_4(p)$
Connectivity

- Path from p to q: a sequence of distinct and adjacent pixels with coordinates 

  \[(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\]

- Closed path: if the starting point is the same as the ending point
- p and q are connected: if there is a path from p to q in S
- Connected component: all the pixels in S connected to p
- Connected set: S has only one connected component

Are they connected sets?

\[
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{array}
\]
Regions

- $R$ is a region if $R$ is a connected set
- $R_i$ and $R_j$ are adjacent if $R_i \cup R_j$ is a connected set
Boundaries

- **Inner boundary (boundary) -- the set of pixels each of which has at least one background neighbor**
- **Outer boundary – the boundary pixels in the background**
Distance Measures

For pixels p, q, and z, with coordinates (x,y), (s,t) and (v,w), D is a distance function or metric if

\( (a) \ D(p, q) \geq 0 \quad D(p, q) = 0 \iff p = q \)

\( (b) \ D(p, q) = D(q, p), \quad \text{and} \)

\( (c) \ D(p, z) \leq D(p, q) + D(q, z) \)
Distance Measures

Euclidean distance
\[ D_e(p, q) = \sqrt{(x-s)^2 + (y-t)^2} \]

City-block (D4) distance
\[ D_4(p, q) = |x-s| + |y-t| \]

Chessboard (D8) distance (Chebyshev distance)
\[ D_8(p, q) = \max(|x-s|, |y-t|) \]
Distance: Sample Problem

D4 distance

6

D8 distance

5

Euclidean distance

\[ \sqrt{1 + 5^2} \]

Distance vs length of a path?
Mathematic Tools

**Array versus Matrix operations**

**Array Multiplications**

\[
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \cdot
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix} =
\begin{bmatrix}
a_{11}b_{11} & a_{12}b_{12} \\
a_{21}b_{21} & a_{22}b_{22}
\end{bmatrix}
\]

**Matrix Multiplications**

\[
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix} \times
\begin{bmatrix}
b_{11} & b_{12} \\
b_{21} & b_{22}
\end{bmatrix} =
\begin{bmatrix}
a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\
a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22}
\end{bmatrix}
\]
Mathematic Tools

Linear/nonlinear operations

Linearity: \[ H[a_if_i(x, y) + a_jf_j(x, y)] = a_iH[f_i(x, y)] + a_jH[f_j(x, y)] \]

Arithmetic Operations – single pixel operations
- Image averaging, image subtraction, image multiplication

Set and logic operations

Spatial operations
- Single pixel operations and neighborhood operations

Image transformation

Probabilistic methods
**Image Averaging – Noise Reduction**

\[
g(x, y) = f(x, y) + \eta(x, y)
\]

\[
\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^{K} g_i(x, y)
\]

\[
E\{\bar{g}(x, y)\} = f(x, y)
\]

\[
\sigma_{\bar{g}(x,y)}^2 = \frac{1}{K} \sigma_{\eta(x,y)}^2
\]

**Assumption:** the noise is uncorrelated in image and has zero mean.
**Image Subtraction – Enhance Difference**

![Image Subtraction Examples](image)

**FIGURE 2.27** (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.
Image Subtraction

The images used in averaging & subtraction must be registered!
**Image Multiplication (Division)**

\[ g(x,y) = f(x,y) / h(x,y) \]

**FIGURE 2.30** (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

**FIGURE 2.29** Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)
Notes on Arithmetic Operations

The images used in averaging & subtraction must be registered!

Output images should be normalized to the range of [0,255]

\[ f_m = f - \min(f) \]
\[ f_s = K \left[ \frac{f_m}{\max(f_m)} \right] \]
Set Operations Based on Intensities

Complement – negative image

\[ A^c = \{(x, y, K - z) | (x, y, z) \in A\} \]

Thresholding

\[ A \cup B = \{(x, y, \max(z_a, z_b)) | (x, y, z_a) \in A, (x, y, z_b) \in B\} \]

**FIGURE 2.32** Set operations involving gray-scale images.
(a) Original image.
(b) Image negative obtained using set complementation.
(c) The union of (a) and a constant image.
(Original image courtesy of G.E. Medical Systems.)
Logic Operations for Binary Image

Foreground/background

• Binary image: 0/1
• Fuzzy set: [0,1]

FIGURE 2.33
Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.