Announcement

A take-home quiz #2 will be posted at 4:00 pm
Submit your solution to dropbox by 11:59:59pm, April 16.
Final Exam Schedule

Final exam has been scheduled
12:30 pm – 3:00 pm, May 7
Location: INNOVA 1400
It will cover all the topics discussed in class

One page double-sided cheat sheet is allowed
A calculator is allowed for +-*/
On the Final-Project Presentation

Final project presentation:

- 13 minutes presentation + 2 minutes Q&A
- You will be timed, don’t go over

<table>
<thead>
<tr>
<th>April 23</th>
<th>April 25</th>
<th>April 30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brandon</td>
<td>Hao &amp; Yuhang</td>
<td>Bharat &amp; Brendan</td>
</tr>
<tr>
<td>Andrew</td>
<td>Yuxiang</td>
<td>Kevin</td>
</tr>
<tr>
<td>Konstantin</td>
<td>Yang</td>
<td>Joshua &amp; James</td>
</tr>
<tr>
<td></td>
<td>Noah and Betsy</td>
<td>Xiaoyi</td>
</tr>
<tr>
<td></td>
<td>Xiangyu &amp; Zhiyuan</td>
<td>Jeremy &amp; Noel</td>
</tr>
</tbody>
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In the Final-Project Presentation

An introduction of the background
Methodologies/algorithms
Comparisons and discussions (for a survey project)
Experiments (for a research project)
  • Experiment setup
  • Experimental results if available
Conclusion
## Final presentation criteria

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of Problem</td>
<td>Did you make clear the nature of the problem you were trying to solve?</td>
<td>10</td>
</tr>
<tr>
<td>Description of Methodology</td>
<td>Did you present the methodology clearly? Did you provide sufficient (key) information to the audience?</td>
<td>25</td>
</tr>
<tr>
<td>Description of Experiment (Research project)</td>
<td>Could your experiment design – dataset, evaluation metrics, baseline approaches effectively demonstrate your proposed method?</td>
<td>25</td>
</tr>
<tr>
<td>Discussion and Comparison (Survey project)</td>
<td>Did your discussion clearly show your understanding of the methods?</td>
<td>25</td>
</tr>
<tr>
<td>Visual Aids</td>
<td>Were visual aids used effectively? Were slides clear and easy to read by the audience?</td>
<td>10</td>
</tr>
<tr>
<td>Clarity &amp; Organization</td>
<td>Was the presentation easy to understand; did it have a logical flow and organization?</td>
<td>10</td>
</tr>
<tr>
<td>Timing</td>
<td>Was the presentation well-paced? Did it fit within the time allotted?</td>
<td>10</td>
</tr>
<tr>
<td>Question</td>
<td>How well did you respond to questions?</td>
<td>10</td>
</tr>
</tbody>
</table>
On the Final Project Report

Written report due time: 11:59:59pm. May 4\textsuperscript{th}

\begin{itemize}
  \item Report format: the same as a conference paper
    \begin{itemize}
      \item For example, you can use a template for ICIP 2018
        \url{https://2018.ieeeicip.org/Papers/PaperKit.html#Templates}
      \item Length: around 4 pages double-column
    \end{itemize}
  \item Executable code must be submitted with clear comments
    (Research project only)
\end{itemize}

Academic integrity (avoiding plagiarism)

\begin{itemize}
  \item don’t copy other person’s work
  \item describe using your own words
  \item complete citation and acknowledgement whenever you use any other work (either published or online)
\end{itemize}
Requirement for Final Project

In form of a complete research project
• Introduction (problem formulation/definition)
• literature review
• the proposed method and analysis
• experiment
• conclusion
• reference

A special case: survey research
• A well-defined problem or topic
• a complete list of previous (typical) work on this problem
• clearly and briefly describe it
• analyze/discuss these methods and compare them
• give the conclusion and list of references
## Project Report Grading Criteria

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of Problem</td>
<td>Did you make clear the nature of the problem you were trying to solve?</td>
<td>10</td>
</tr>
<tr>
<td>Description of Methodology</td>
<td>Did you present the methodology clearly? Did you provide sufficient (key) information to the audience?</td>
<td>35</td>
</tr>
<tr>
<td>Description of Experiment (Research project)</td>
<td>Could your experiments effectively demonstrate your proposed method?</td>
<td>30</td>
</tr>
<tr>
<td>Discussion and Comparison (Survey project)</td>
<td>Was the problem comprehensively reviewed? Did your discussion clearly show your understanding of the methods?</td>
<td>30</td>
</tr>
<tr>
<td>Writing Clarity</td>
<td>Does the report read well? Is it easy to understand?</td>
<td>10</td>
</tr>
<tr>
<td>Organization &amp; Length</td>
<td>Is the report well-organized? Does it have a logical flow?</td>
<td>10</td>
</tr>
<tr>
<td>Reference</td>
<td>Is the reference section complete and in consistent format; are the citations in the text in consistent format?</td>
<td>5</td>
</tr>
</tbody>
</table>
Basic Concepts

- **Union, intersection, complement, difference**
- **Set reflection** \( \hat{B} = \{ w | w = -b, b \in B \} \)
- **Set translation** \( (B)_z = \{ c | c = b + z, b \in B \} \) -- move the center of \( B \) by \( z \) pixels
- **Structure elements (SEs):** small sets/subimages used in morphology

Black dots are centers/origins of SE

SEs for images
Common Morphological Operations

Two basic operations

- Erosion
- Dilation

Other operations

- Opening/closing
- Hit-or-Miss transform
- Thinning/thickening
- Hole filling
**Erosion**

\[ A \ominus B = \{ z \mid (B)_z \subseteq A \} \quad \text{or} \quad A \ominus B = \{ z \mid (B)_z \cap A^c = 0 \} \]

Shrink or thin objects and remove the details smaller than the SE
Dilation

\[ A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq 0 \right\} \]

or \[ A \oplus B = \left\{ z \mid \left[ (\hat{B})_z \cap A \right] \subseteq A \right\} \]

Grows or thickens objects and remove the gaps smaller than the SE
Properties of Erosion and Dilation

• Dilation is commutative \[ A \oplus B = B \oplus A \]

• Dilation is associative \[ A \oplus B \oplus C = A \oplus (B \oplus C) \]

• Dilation \[ A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C) \]
Properties of Erosion and Dilation

• Erosion $A \ominus B \ominus C = A \ominus (B \oplus C)$

• Erosion and dilation are duals of each other

$$A \oplus B = (A^c \ominus \hat{B})^c$$

• $A \subseteq (C \ominus B)$ if and only if $(A \oplus B) \subseteq C$

• If $A \subseteq C$, $A \oplus B \subseteq C \oplus B$ and $A \ominus B \subseteq C \ominus B$
Opening

- Smooth the contour of an object
- Break narrow bridges
- Eliminate thin protrusions

\[ A \circ B = \bigcup \{ (B)_z \mid (B)_z \subseteq A \} \]

The SE rolls within the boundary of \( A \).

\[ A \circ B = (A \ominus B) \oplus B \]
\[ (A \circ B) \circ B = A \circ B \]
\[ (A \circ B) \subseteq A \]

if \( A \subseteq C, A \circ B \subseteq C \circ B \)
Opening (Cont’d)

Thin isthmus

Isthmus removed

Smoothed outer corners

$A \circ B = (A \Theta B) \oplus B$
Closing

- Smooth the contour of an object
- Fill narrow breaks and gaps
- Eliminate long and thin gulfs
- Eliminate small holes

Opening and closing are duals of each other

\[ A \cdot B = (A \oplus B) \ominus B \]
\[ (A \cdot B) \cdot B = A \cdot B \]
\[ A \subseteq (A \cdot B) \]
\[ if \ A \subseteq C, A \cdot B \subseteq C \cdot B \]

The SE rolls outside the boundary of \( A \).

**FIGURE 9.9** (a) Structuring element \( B \) “rolling” on the outer boundary of set \( A \). (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade \( A \) in (a) for clarity.
Closing (Cont’d)

\[ A \ominus B \]

Smoothed inner corners

\[ A \cdot B = (A \oplus B) \ominus B \]

Eliminated thin gulf
Opening & Closing

\[ A \ominus B \subseteq A \odot B \subseteq A \subseteq A \ominus (A \ominus B) \ominus B \]

\[ A \odot B = (A \ominus B) \ominus B \]

\[ A \odot B \subseteq A \ominus (A \ominus B) \ominus B \subseteq A \ominus B \subseteq A \ominus B \]

\[ A \ominus B \subseteq A \odot B \subseteq A \subseteq A \ominus (A \ominus B) \ominus B \subseteq A \ominus B \subseteq A \ominus B \]

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\[ A \ominus B \subseteq A \odot B \subseteq A \subseteq A \ominus (A \ominus B) \ominus B \subseteq A \ominus B \subseteq A \ominus B \]

\[ A \ominus B \subseteq A \odot B \subseteq A \subseteq A \ominus (A \ominus B) \ominus B \subseteq A \ominus B \subseteq A \ominus B \]
An Example of Opening & Closing

• An opening removes all noise
  • removing the white noise by erosion
  • removing the black noise by dilation
• An additional closing fills the gaps

FIGURE 9.11
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of $A$.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)
Hit-or-Miss Transform

Shape detection: find D

Define a set $B$ consisting of $D$ and its background

$$B = (D, W - D)$$

$$A \circ B = (A \ominus D) \cap [A^c \ominus (W - D)]$$
Basic Morphological Algorithms

**Thinning**  
\[ A \bigotimes B = A - (A \bigodot B) \]

\[ B = \{B^1, B^2, B^3, \ldots, B^n\} \rightarrow A \bigotimes B = \left( ( (A \bigotimes B^1) \bigotimes B^2 ) \ldots \right) \bigotimes B^n \]

FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to \( m \)-connectivity.
Basic Morphological Algorithms

**Thickening** \( A \ominus B = A \cup (A \odot B) \)  
\[ B = \{B^1, B^2, B^3, \ldots, B^n\} \]

In practice, \( A \ominus B = A^c \ominus B \)

**FIGURE 9.22** (a) Set \( A \). (b) Complement of \( A \). (c) Result of thinning the complement of \( A \). (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.
Applications of Morphological Operations

• Boundary extraction
• Hole filing
• Connected component analysis
• Convex hull extraction
• Skeleton analysis
Basic Morphological Algorithms

Boundary extraction

\[ \beta(A) = A - (A \ominus B) \]
Hole Filling

**Hole:** a background region surrounded by a connected foreground pixels.

**Objective:** given a point in a hole, fill the hole with foreground pixels.

\( X_0 \) is a set of all 0s except the selected background point

A is the set of foreground boundary

**Repeat:**

\[ X_k = (X_{k-1} \oplus B) \cap A^c \quad k=1,2,3,\ldots \]

**Until**

\[ X_k = X_{k-1} \]

Conditional dilation

**Figure 9.15** Hole filling. (a) Set \( A \) (shown shaded). (b) Complement of \( A \). (c) Structuring element \( B \). (d) Initial point inside the boundary. (e)-(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].
Example

**FIGURE 9.16** (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.
**Objective:** find connected components in a binary image.

**Applications:** finding candidates of target object for recognition

\( X_0 \) is a set of all 0s except the selected point belong to the connected component

\( A \) is the set containing one or more connected components

Repeat:\n\[ X_k = (X_{k-1} \oplus B) \cap A \quad k=1,2,3,\ldots \]

Until \( X_k = X_{k-1} \)

Other algorithms (two-pass and one-pass) are based on the connectivity analysis directly
Figure 9.18
(a) X-ray image of chicken fillet with bone fragments.
(b) Thresholded image of (a).
(c) Image (b) eroded with a $5 \times 5$ structuring element.
(b) Number of connected components of (c).
(d) Image (b) eroded with a $5 \times 5$ structuring element.
(e) Image (b) eroded with a $5 \times 5$ structuring element.
(f) Image (b) eroded with a $5 \times 5$ structuring element.
(g) Image (b) eroded with a $5 \times 5$ structuring element.
(h) Image (b) eroded with a $5 \times 5$ structuring element.
Convex Hull

**Convex**: a set $A$ is convex if the line segment connecting any two points in $A$ is entirely belong to $A$

**Examples**: rectangle, triangle, circle are convex

Ring, hand, many other objects with dents or hollows are not convex

**Convex hull of a set $S$**: the minimal convex set containing $S$

**Applications of finding convex hull**: an abstract representation for high level image understanding
Extract Convex Hull

A is the target object and $X_0^i = A$
For each structure element $B^i$, $i = 1, 2, 3, 4$
Repeat:

\[ X_k^i = (X_{k-1}^i \odot B^i) \cup A \]

Until

\[ X_k^i = X_{k-1}^i \]

Hit-or-miss

The convex hull of A is

\[ C(A) = \bigcup_{i=1}^{4} X_i \]
Extract Convex Hull (Cont’d)

Potential issue:
Morphological Skeleton

Skeleton (Defined by centers of maximal disks):

If a point $Z$ belongs to the skeleton of $A$, we can find a maximal disk that entirely lies in $A$ and touches the boundary of $A$ at no less than two positions.

Applications: an abstract shape representation for high level image understanding, e.g. Optical Character Recognition (OCR)
Morphological Skeleton

\[
S(A) = \bigcup_{k=0}^{K} S_k(A)
\]

The kth erosion

\[
S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B
\]

\[K = \max \{k | (A \ominus kB) \neq \emptyset \}\]

Reconstruct \( A \) from its skeleton

\[
A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)
\]
Potential issues with skeleton?

Sensitive to noise