Announcement

A take-home quiz #2 will be posted at 4:00 pm

Submit your solution to dropbox by 11:59:59pm, April 16.
Final Exam Schedule

Final exam has been scheduled
12:30 pm – 3:00 pm, May 7

Location: INNOVA 1400

It will cover all the topics discussed in class

One page double-sided cheat sheet is allowed

A calculator is allowed for +-*
In the Final-Project Presentation

An introduction of the background
Methodologies/algorithms
Comparisons and discussions (for a survey project)
Experiments (for a research project)
  • Experiment setup
  • Experimental results if available
Conclusion
# Final presentation criteria

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
<th>Worth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description of Problem</td>
<td>Did you make clear the nature of the problem you were trying to solve?</td>
<td>10</td>
</tr>
<tr>
<td>Description of Methodology</td>
<td>Did you present the methodology clearly? Did you provide sufficient (key) information to the audience?</td>
<td>25</td>
</tr>
<tr>
<td>Description of Experiment (Research project)</td>
<td>Could your experiment design – dataset, evaluation metrics, baseline approaches effectively demonstrate your proposed method?</td>
<td>25</td>
</tr>
<tr>
<td>Discussion and Comparison (Survey project)</td>
<td>Did your discussion clearly show your understanding of the methods?</td>
<td>25</td>
</tr>
<tr>
<td>Visual Aids</td>
<td>Were visual aids used effectively? Were slides clear and easy to read by the audience?</td>
<td>10</td>
</tr>
<tr>
<td>Clarity &amp; Organization</td>
<td>Was the presentation easy to understand; did it have a logical flow and organization?</td>
<td>10</td>
</tr>
<tr>
<td>Timing</td>
<td>Was the presentation well-paced? Did it fit within the time allotted?</td>
<td>10</td>
</tr>
<tr>
<td>Question</td>
<td>How well did you respond to questions?</td>
<td>10</td>
</tr>
</tbody>
</table>
On the Final Project Report

Written report due time: 11:59:59pm. May 4th

• Report format: the same as a conference paper
  – For example, you can use a template for ICIP 2018
    https://2018.ieeeicip.org/Papers/PaperKit.html#Templates
  – Length: around 4 pages double-column

• Executable code must be submitted with clear comments
  (Research project only)

Academic integrity (avoiding plagiarism)

• don’t copy other person’s work
• describe using your own words
• complete citation and acknowledgement whenever you use any other work (either published or online)
Requirement for Final Project

In form of a complete research project
- Introduction (problem formulation/definition)
- literature review
- the proposed method and analysis
- experiment
- conclusion
- reference

A special case: survey research
- A well-defined problem or topic
- a complete list of previous (typical) work on this problem
- clearly and briefly describe it
- analyze/discuss these methods and compare them
- give the conclusion and list of references
# Project Report Grading Criteria

<table>
<thead>
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</tr>
</thead>
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<td>Did you make clear the nature of the problem you were trying to solve?</td>
<td>10</td>
</tr>
<tr>
<td>Description of Methodology</td>
<td>Did you present the methodology clearly? Did you provide sufficient (key) information to the audience?</td>
<td>35</td>
</tr>
<tr>
<td>Description of Experiment (Research project)</td>
<td>Could your experiments effectively demonstrate your proposed method?</td>
<td>30</td>
</tr>
<tr>
<td>Discussion and Comparison (Survey project)</td>
<td>Was the problem comprehensively reviewed? Did your discussion clearly show your understanding of the methods?</td>
<td>30</td>
</tr>
<tr>
<td>Writing Clarity</td>
<td>Does the report read well? Is it easy to understand?</td>
<td>10</td>
</tr>
<tr>
<td>Organization &amp; Length</td>
<td>Is the report well-organized? Does it have a logical flow?</td>
<td>10</td>
</tr>
<tr>
<td>Reference</td>
<td>Is the reference section complete and in consistent format; are the citations in the text in consistent format?</td>
<td>5</td>
</tr>
</tbody>
</table>
Basic Concepts

- Union, intersection, complement, difference
- Set reflection \( \hat{B} = \{ w \mid w = -b, b \in B \} \)
- Set translation \( (B)_z = \{ c \mid c = b + z, b \in B \} \) -- move the center of \( B \) by \( z \) pixels
- Structure elements (SEs): small sets/subimages used in morphology

2D reflection

2D translation

Black dots are centers/origins of SE

SEs

SEs for images
Common Morphological Operations

Two basic operations
- Erosion
- Dilation

Other operations
- Opening/closing
- Hit-or-Miss transform
- Thinning/thickening
- Hole filling
Erosion

\[ A \ominus B = \{ z \mid (B)_z \subseteq A \} \quad \text{or} \quad A \ominus B = \{ z \mid (B)_z \cap A^c = 0 \} \]

Shrink or thin objects and remove the details smaller than the SE
Dilation

\[ A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq 0 \right\} \]

or \[ A \oplus B = \left\{ z \mid [(\hat{B})_z \cap A] \subseteq A \right\} \]

Grows or thickens objects and remove the gaps smaller than the SE
Properties of Erosion and Dilation

- **Dilation is commutative**  
  \[ A \bigoplus B = B \bigoplus A \]

- **Dilation is associative**  
  \[ A \bigoplus B \bigoplus C = A \bigoplus (B \bigoplus C) \]

- **Dilation**  
  \[ A \bigoplus (B \cup C) = (A \bigoplus B) \cup (A \bigoplus C) \]
Properties of Erosion and Dilation

• Erosion $A \ominus B \ominus C = A \ominus (B \oplus C)$

• Erosion and dilation are duals of each other
  $$A \oplus B = (A^c \ominus \hat{B})^c$$

• $A \subseteq (C \ominus B)$ if and only if $(A \oplus B) \subseteq C$

• If $A \subseteq C$, $A \oplus B \subseteq C \oplus B$ and $A \ominus B \subseteq C \ominus B$
Opening

- Smooth the contour of an object
- Break narrow bridges
- Eliminate thin protrusions

\[ A \circ B = \bigcup \{(B)_z | (B)_z \subseteq A \} \]

\[ A \circ B = (A \ominus B) \oplus B \]
\[ (A \circ B) \circ B = A \circ B \]
\[ (A \circ B) \subseteq A \]

if \( A \subseteq C, A \circ B \subseteq C \circ B \)

The SE rolls within the boundary of \( A \).

**FIGURE 9.8** (a) Structuring element \( B \) “rolling” along the inner boundary of \( A \) (the dot indicates the origin of \( B \)). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade \( A \) in (a) for clarity.
Opening (Cont’d)

- Thin isthmus
- Isthmus removed
- Smoothed outer corners
- $A \circ B = (A \ominus B) \oplus B$
Closing

- Smooth the contour of an object
- Fill narrow breaks and gaps
- Eliminate long and thin gulfs
- Eliminate small holes

Opening and closing are duals of each other

\[
A \subseteq (A \bullet B) \ominus B
\]

\[(A \bullet B) \bullet B = A \bullet B\]

\[
(A \cdot B) \circ B = A \cdot B
\]

if \( A \subseteq C, A \cdot B \subseteq C \circ B \)

\[
A \cdot B = (A^c \circ \hat{B})^c
\]

The SE rolls outside the boundary of \( A \).
Closing (Cont’d)

Smoothed inner corners

Eliminated thin gulf

\[ A \ominus B \]

\[ A \cdot B = (A \oplus B) \ominus B \]
Opening & Closing

\[ A \ominus B \subseteq A \circ B = (A \ominus B) \oplus B \]

\[ A \ominus B \subseteq A \circ B \subseteq A \subseteq A \bullet B \subseteq A \oplus B \]
An Example of Opening & Closing

- An opening removes all noise
  - removing the white noise by erosion
  - removing the black noise by dilation
- An additional closing fills the gaps

**FIGURE 9.11**
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of A.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)
Shape detection: find D

Define a set B consisting of D and its background

\[ B = (D, W - D) \]

\[ A \odot B = (A \ominus D) \cap [A^c \ominus (W - D)] \]
Basic Morphological Algorithms

**Thinning**  
\[ A \otimes B = A - (A \odot B) \]

\[ B = \{B^1, B^2, B^3, \ldots, B^n\} \quad \rightarrow \quad A \otimes B = \left( \left( (A \otimes B^1) \otimes B^2 \right) \ldots \right) \otimes B^n \]

**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set \( A \). (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to \( m \)-connectivity.  

\[ a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \quad i \quad j \quad k \quad l \quad m \]

\[ A_6 = A_5 \otimes B^6 \]
\[ A_8 = A_6 \otimes B^7 \]
\[ A_{8,4} = A_8 \otimes B^{1,2,3,4} \]
\[ A_{8,5} = A_8 \otimes B^5 \]
\[ A_{8,6} = A_{8,5} \otimes B^6 \]
No more changes after this.  

\[ A_{8,6} \text{ converted to } m\text{-connectivity.} \]
Basic Morphological Algorithms

Thickening \( A \ominus B = A \cup (A \odot B) \) → In practice, \( A \ominus B = A^c \odot B \)

\[
B = \{B^1, B^2, B^3, \ldots, B^n\}
\]

**FIGURE 9.22** (a) Set \( A \). (b) Complement of \( A \). (c) Result of thinning the complement of \( A \). (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.
Applications of Morphological Operations

- Boundary extraction
- Hole filing
- Connected component analysis
- Convex hull extraction
- Skeleton analysis
**Basic Morphological Algorithms**

**Boundary extraction** 
\[ \beta(A) = A - (A \ominus B) \]
**Hole Filling**

**Hole:** a background region surrounded by a connected foreground pixels.

**Objective:** given a point in a hole, fill the hole with foreground pixels.

$X_0$ is a set of all 0s except the selected background point

$A$ is the set of foreground boundary

**Repeat:**

$X_k = (X_{k-1} \oplus B) \cap A^c$ for $k=1,2,3,...$

**Until**

$X_k = X_{k-1}$

Conditional dilation

---

**Figure 9.15** Hole filling. (a) Set $A$ (shown shaded).
(b) Complement of $A$.
(c) Structuring element $B$.
(d) Initial point inside the boundary.
(e)-(h) Various steps of Eq. (9.5.2).
(i) Final result [union of (a) and (h)].
Example

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.
Connected Component Analysis

Blob extraction or region labeling

Objective: find connected components in a binary image.

Applications: finding candidates of target object for recognition

$X_0$ is a set of all 0s except the selected point belong to the connected component

$A$ is the set containing one or more connected components

Repeat:

$X_k = (X_{k-1} \oplus B) \cap A$ \(k=1,2,3,\ldots\)

Until \(X_k = X_{k-1}\)

Other algorithms (two-pass and one-pass) are based on the connectivity analysis directly.
FIGURE 9.18
(a) X-ray image of chicken fillet with bone fragments.
(b) Thresholded image. (c) Image eroded with a $5 \times 5$ structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

<table>
<thead>
<tr>
<th>Connected component</th>
<th>No. of pixels in connected comp</th>
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<tbody>
<tr>
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<tr>
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<tr>
<td>15</td>
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</tr>
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</table>
Convex Hull

Convex: a set $A$ is convex if the line segment connecting any two points in $A$ is entirely belong to $A$

Examples: rectangle, triangle, circle are convex

Ring, hand, many other objects with dents or hollows are not convex

Convex hull of a set $S$: the minimal convex set containing $S$

Applications of finding convex hull: an abstract representation for high level image understanding
Extract Convex Hull

A is the target object and $X_0^i = A$

For each structure element $B^i, i = 1,2,3,4$

Repeat:

$$X_k^i = (X_{k-1}^i \ast B^i) \cup A$$

Until

$$X_k^i = X_{k-1}^i$$  \hspace{1cm} \text{Hit-or-miss}$$

The convex hull of $A$ is

$$C(A) = \bigcup_{i=1}^{4} X^i$$
Potential issue:
Morphological Skeleton

Skeleton (Defined by centers of maximal disks):

If a point $Z$ belongs to the skeleton of $A$, we can find a maximal disk that entirely lies in $A$ and touches the boundary of $A$ at no less than two positions.

Applications: an abstract shape representation for high level image understanding, e.g. Optical Character Recognition (OCR)
Morphological Skeleton

\[ S(A) = \bigcup_{k=0}^{K} S_k(A) \]

The \( k \)th erosion

\[ S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B \]

\[ K = \max\{k \mid (A \ominus kB) \neq \emptyset \} \]

Reconstruct \( A \) from its skeleton

\[ A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB) \]
Potential issues with skeleton?

Sensitive to noise