Announcement

We will have Quiz #2 in Blackboard.

Open book and open notes

Starting from 3:35pm, Monday, April 15.

Due: 11:59pm, Monday, April 15.

Course Evaluation

The course evaluation is available in Blackboard.

Please complete online course evaluation at Blackboard, which closes in April 23

Your feedback is crucial to improving the course

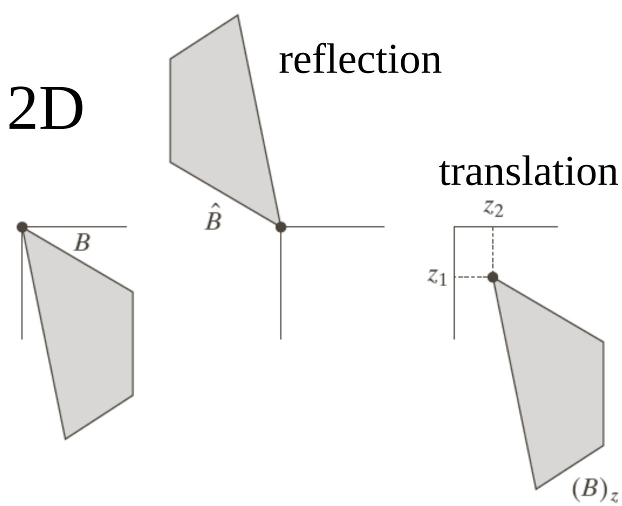
Today’s Agenda

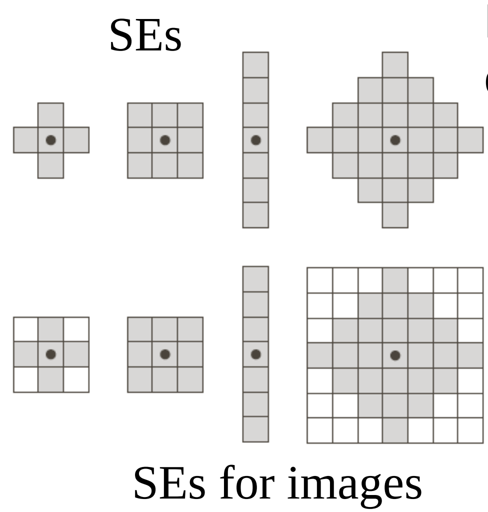
Morphological image processing techniques

Final project presentations

Review: Basic Concepts

- **2D Integer space Z^2**
- **Union, intersection, complement, difference**
- **Set reflection** $\hat{B} = \{ w \mid w = -b, b \in B \}$ B is a set of 2D points (x, y)
- **Set translation** $(B)_z = \{ \mathbf{c} \mid \mathbf{c} = \mathbf{b} + \mathbf{z}, \mathbf{b} \in B \}$ -- move the center/origin of ***B*** by *z* pixels
- **Structure elements (SEs): small sets/subimages used in morphology**





Black dots are centers/origins of SE

Common Morphological Operations

Two basic operations

- Erosion
- Dilation

Other operations

- Opening/closing
- Hit-or-Miss transform
- Thinning/thickening
- Hole filling

Hit-or-Miss Transform

Shape detection: find D

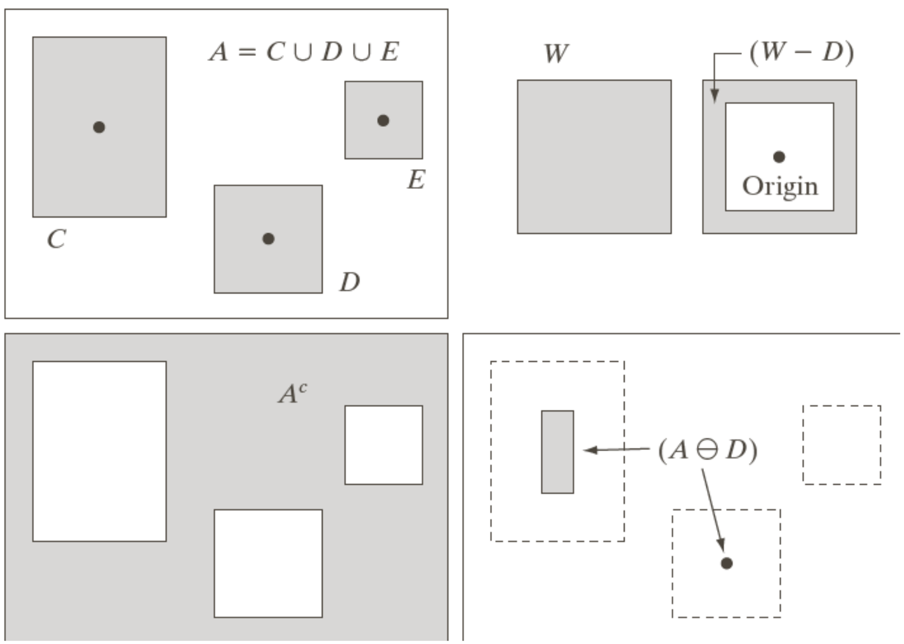
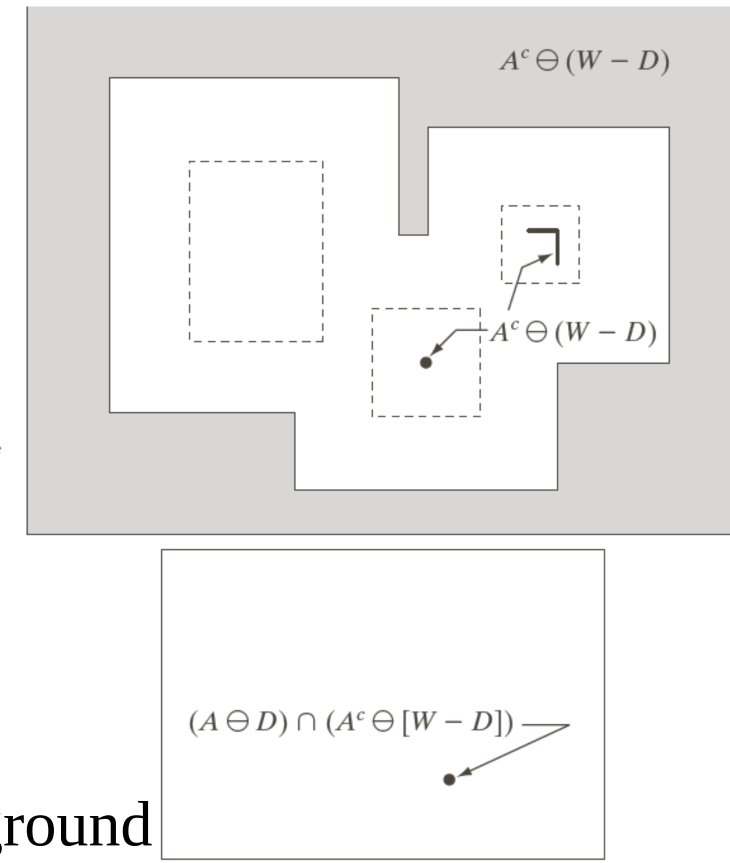


FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

a b e
c d f



Define a set B consisting of D and its background

$$B = (D, W - D) \Rightarrow A \circledast B = (A \ominus D) \cap [A^c \ominus (W - D)]$$

Other Basic Morphological Algorithms

Thinning $A \otimes B = A - (A \circledast B)$

$$B = \{B^1, B^2, B^3, \cdots, B^n\} \Rightarrow A \otimes B = \left(\left(\left((A \otimes B^1) \otimes B^2 \right) \cdots \right) \otimes B^n \right)$$

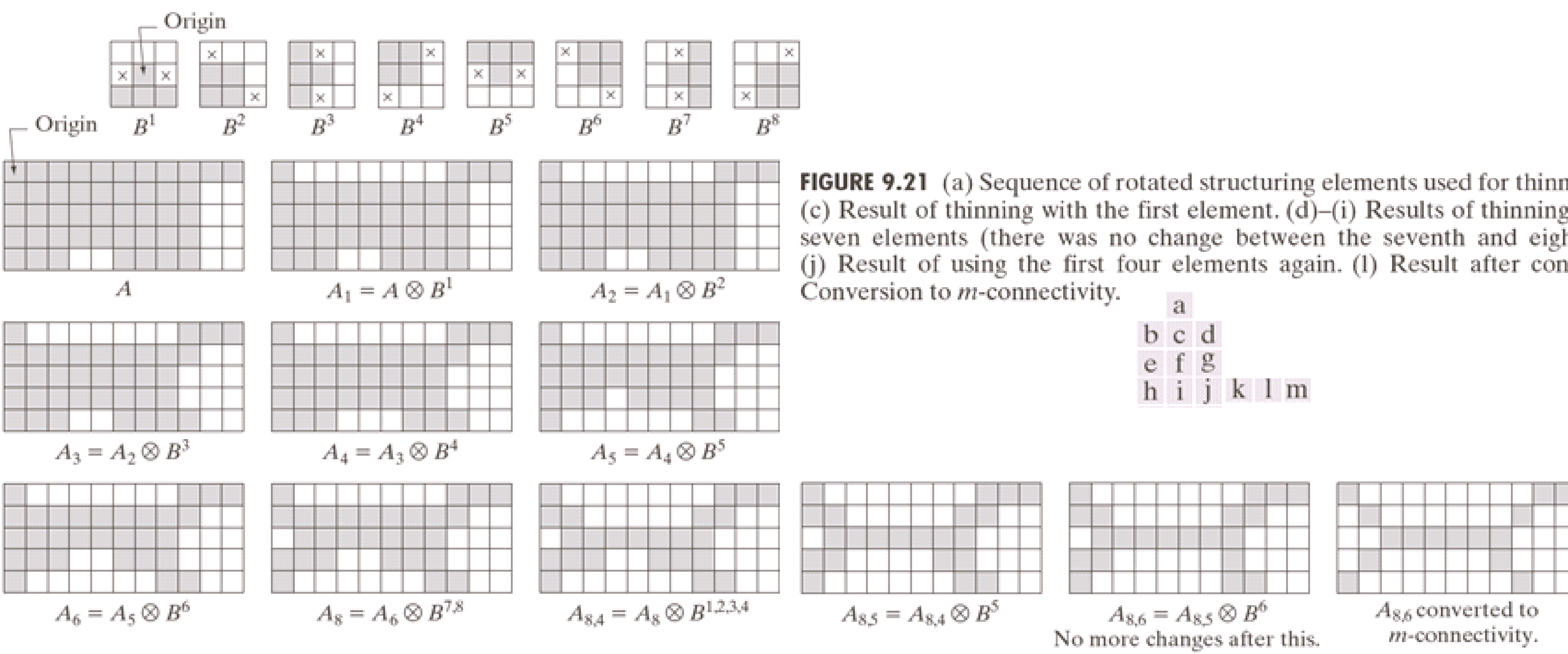


FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to m-connectivity.

a
b c d
e f g
h i j k l m

Other Basic Morphological Algorithms

Thickening $A \odot B = A \cup (A \circledast B)$ ➡ In practice,

$$A \odot B = A^c \otimes B$$

$$B = \{B^1, B^2, B^3, \cdots, B^n\}$$

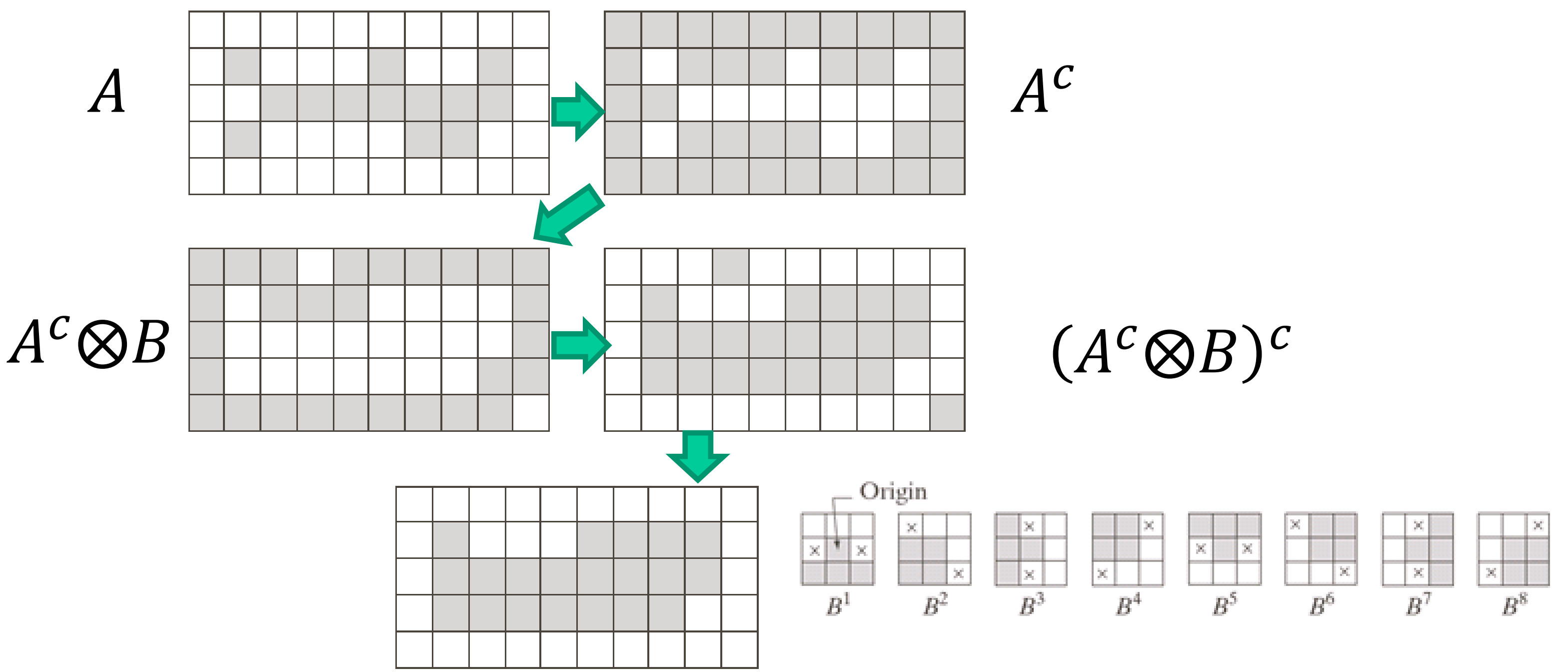


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Applications of Morphological Operations

- **Boundary extraction**
- **Hole filing**
- **Connected component analysis**
- **Convex hull extraction**
- **Skeleton analysis**

Basic Morphological Algorithms

Boundary extraction $$\beta(A) = A - (A \ominus B)$$

a b

FIGURE 9.14 (a) A simple binary image, with 1s represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Hole Filling

Hole: a background region surrounded by a connected foreground pixels.

Objective: given a point in a hole, fill the hole with foreground pixels.

X_0 is a set of all 0s except the selected background point
A is the set of foreground boundary

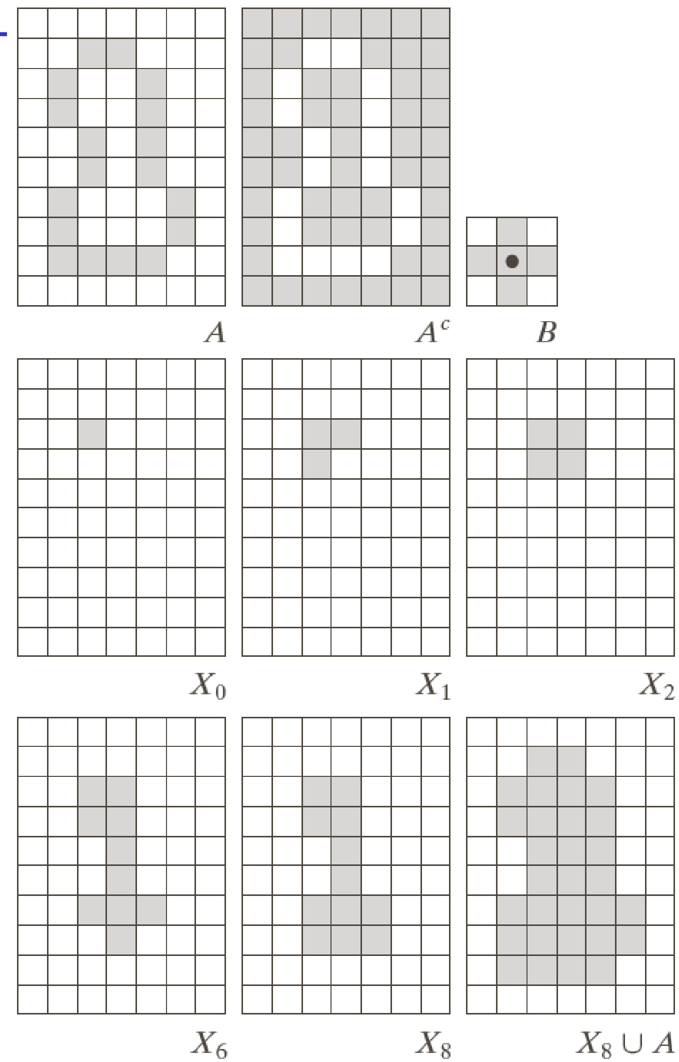
Repeat:

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad \text{k=1,2,3,...}$$

Until

$$X_k = X_{k-1}$$

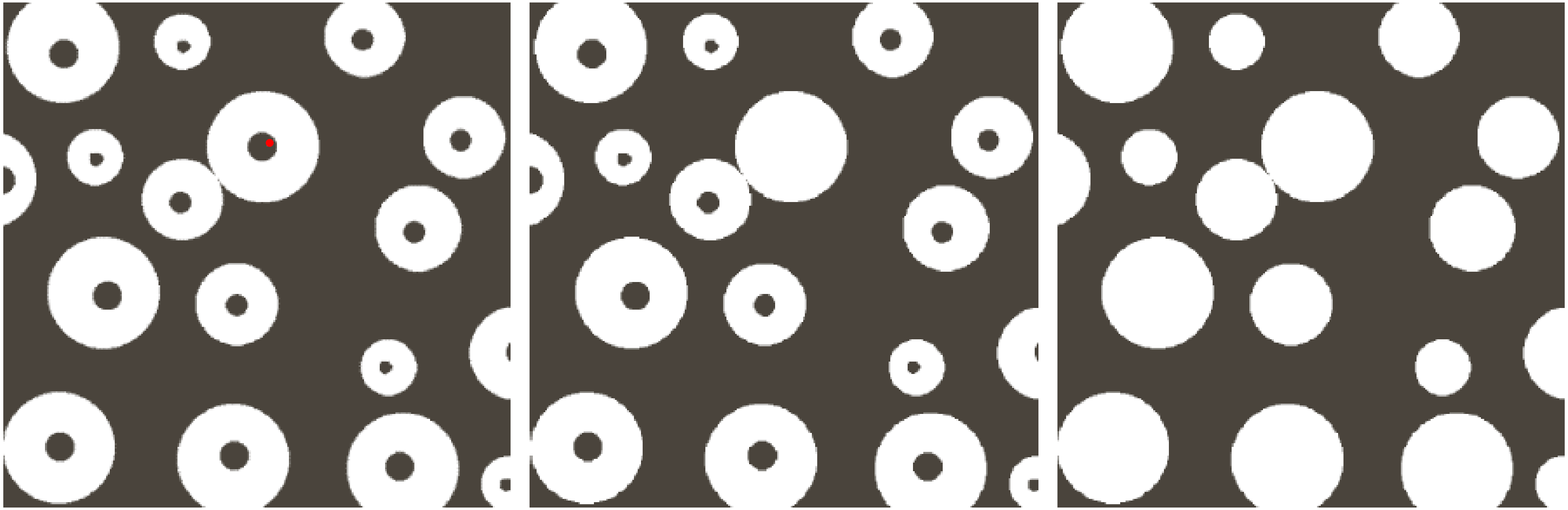
Conditional dilation



a	b	c
d	e	f
g	h	i

FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example

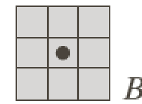


a b c

FIGURE 9.16 (a) Binary image (the red dot inside one of the regions is the starting point for the hole-filling algorithm). (b) Result of filling that region. (c) Result of filling all holes.

Connected Component Analysis

Blob extraction or region labeling



How to select B?

Objective: find connected components in a binary image.

Applications: finding candidates of target object for recognition

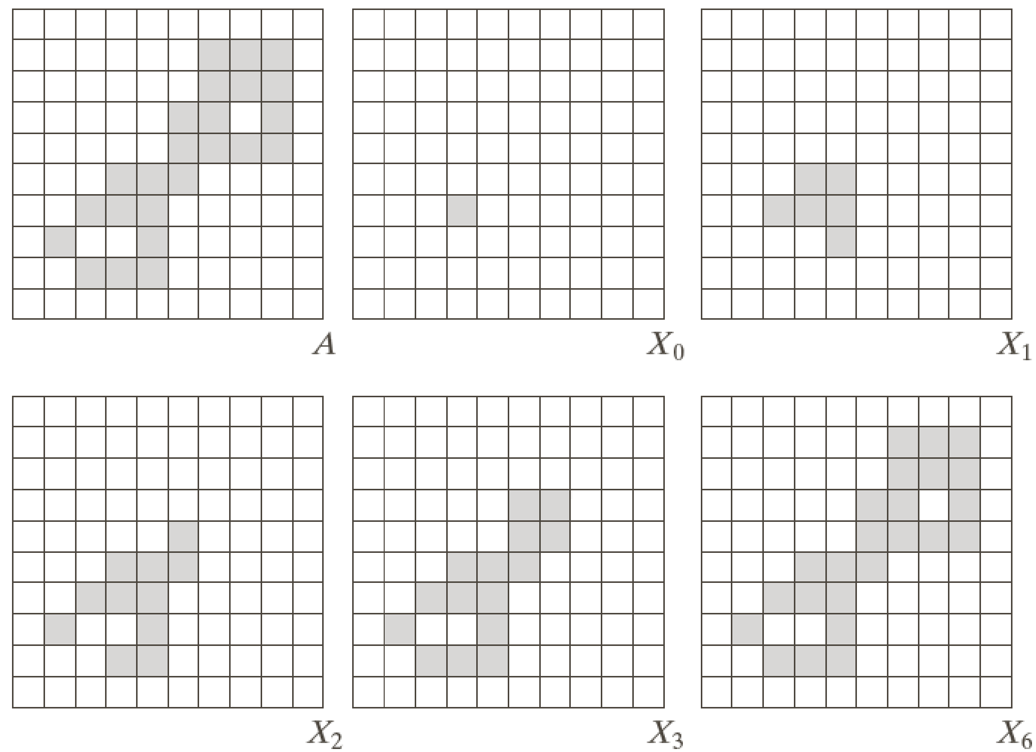
X_0 is a set of all 0s except the selected point belong to the connected component

A is the set containing one or more connected components

Repeat:

$X_k = (X_{k-1} \oplus B) \cap A$ k=1,2,3,…

Until $X_k = X_{k-1}$



Example

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex Hull

Convex: a set ***A*** is convex if the line segment connecting any two points in ***A*** is entirely belong to ***A***

Examples: rectangle, triangle, circle **are** convex

Ring, hand, many other objects with dents or hollows **are not** convex

Convex hull of a set *S*: the minimal convex set containing ***S***

Applications of finding convex hull: an abstract representation for high level image understanding

Extract Convex Hull

A is the target object and $X_0^i = A$

For each structure element $B^i, i = 1,2,3,4$

Repeat:

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$$

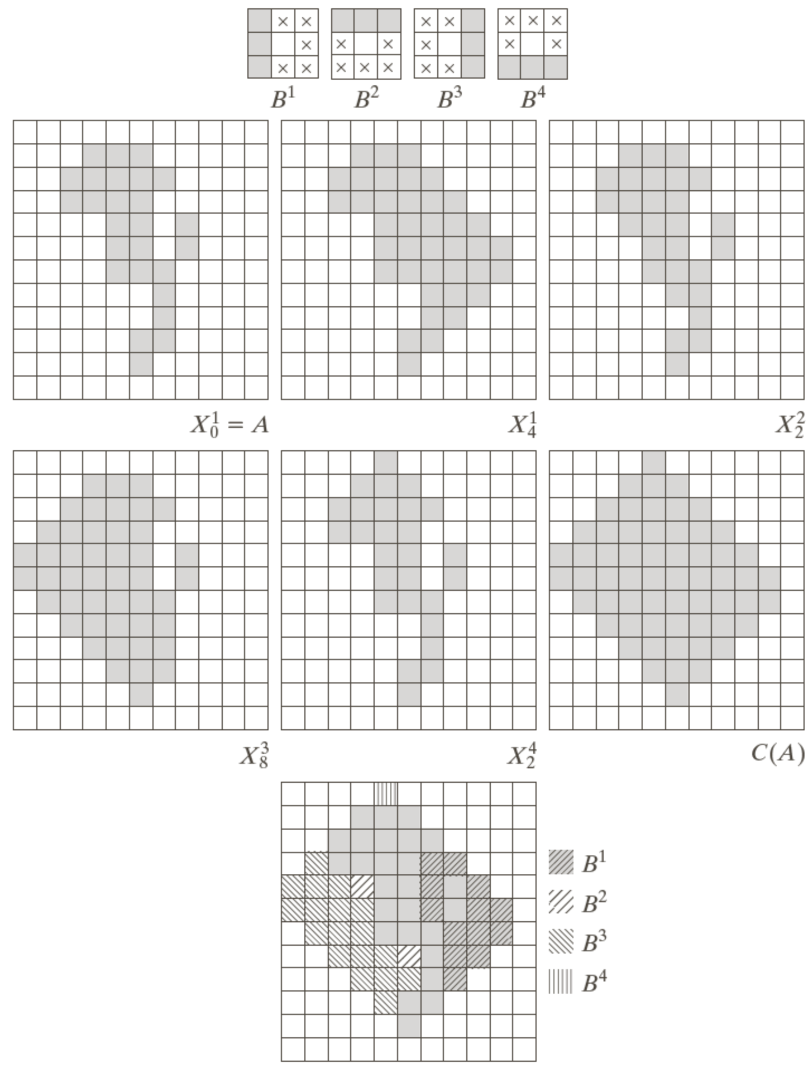
Until

$$X_k^i = X_{k-1}^i$$

Hit-or-miss

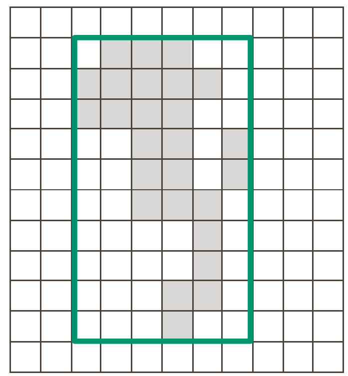
The convex hull of A is

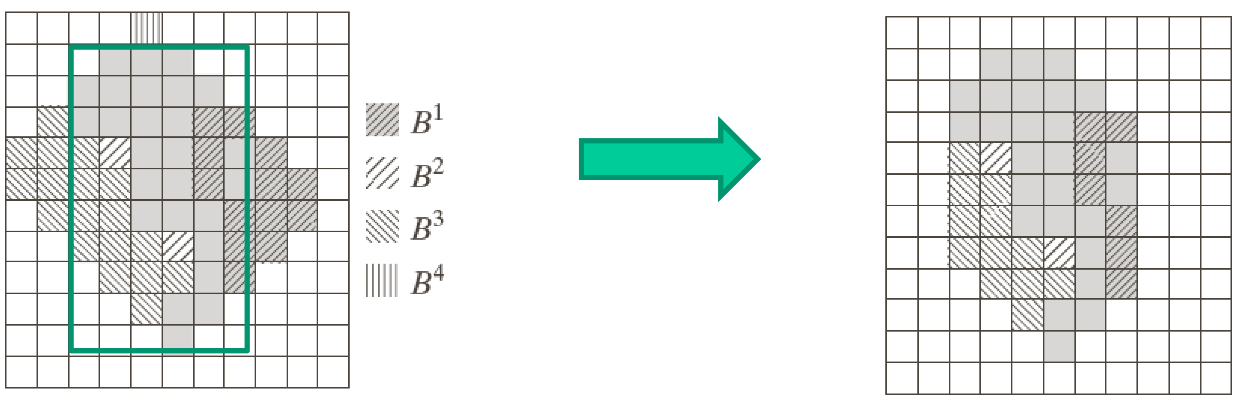
$$C(A) = \bigcup_{i=1}^{4} X_k^i$$



Extract Convex Hull (Cont'd)

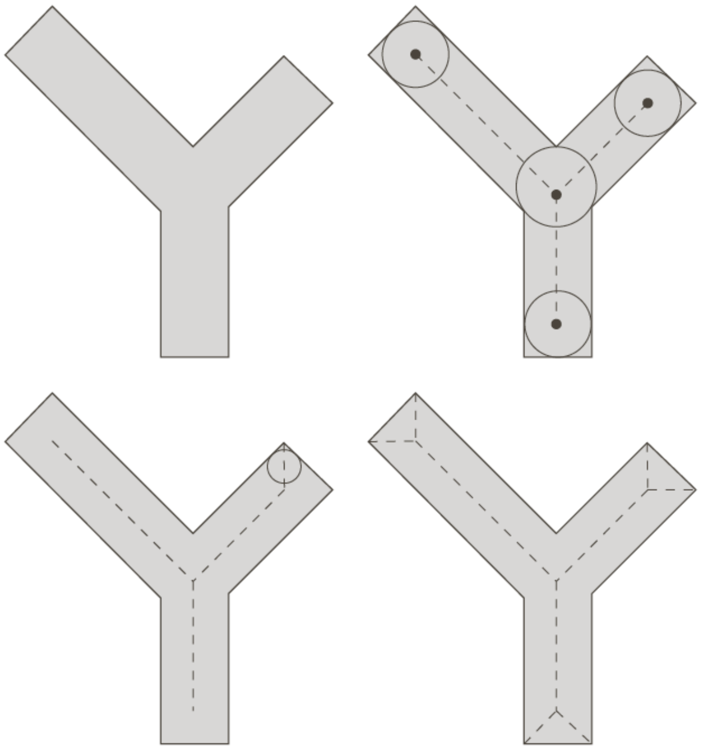
Potential issue:





Morphological Skeleton

Skeleton (Defined by centers of maximal disks):

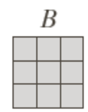


If a point ***Z*** belongs to the skeleton of ***A***, we can find a maximal disk that entirely lies in ***A*** and touches the boundary of ***A*** at no less than two positions.

Applications: an abstract shape representation for high level image understanding, e.g. Optical Character Recognition (OCR)

Morphological Skeleton

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$



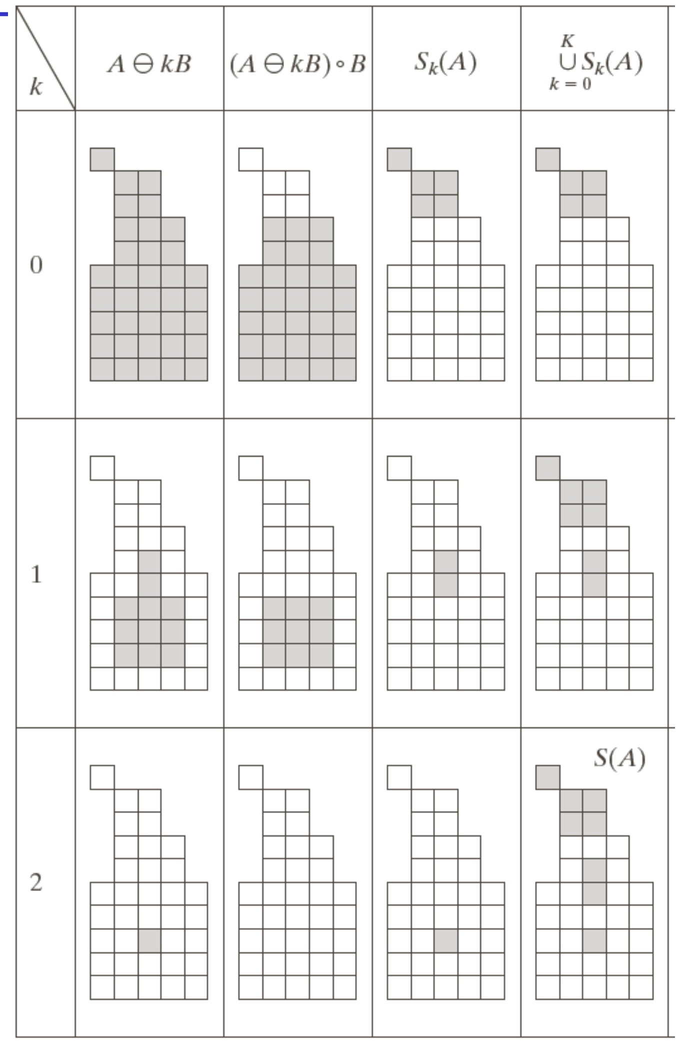
The k^th erosion

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

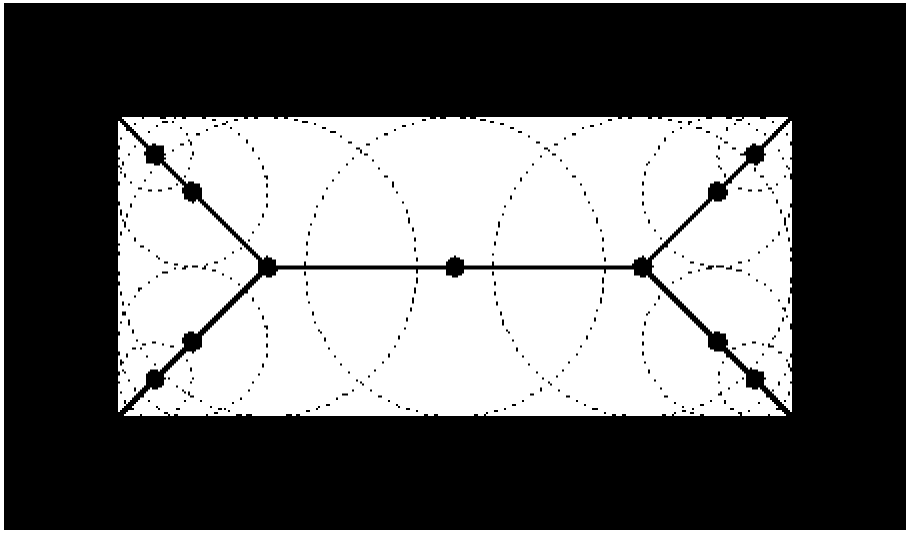
$$K = max\{k | (A \ominus kB) \neq \emptyset\}$$

Reconstruct ***A*** from its skeleton

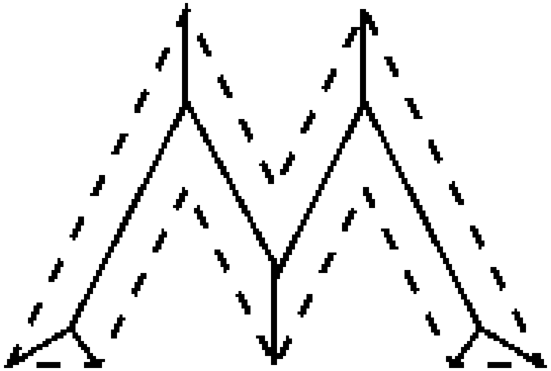
$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$



Examples

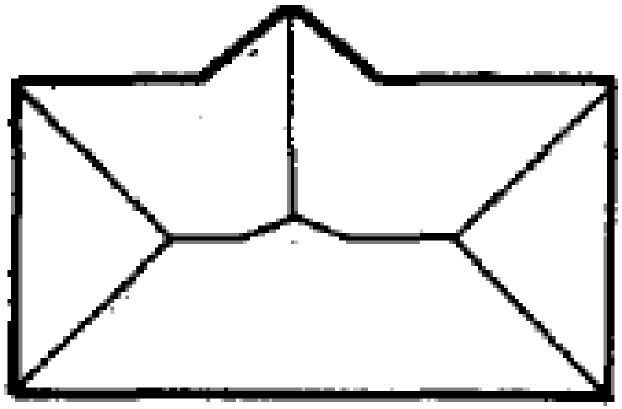


http://homepages.inf.ed.ac.uk/rbf/HIPR2/skeleton.htm



Potential issues with skeleton?

Sensitive to noise



Maragos and Schafer, "Morphological Skeleton Representation and Coding of Binary Images", IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. 34, No. 5, 1986.

Review of Chapter 8, 9, and 10

Chapter 8

- Fundamentals of image compression
- Image compression methods

Chapter 9

- Basic morphological operations

Chapter 10

- Edge-based image segmentation
- Region-based image segmentation

Fundamentals of Image Compression

Compression ratio $$C = \frac{b}{b'}$$

Data redundancy $$R = 1 - \frac{1}{C}$$

Three types of data redundancy and their examples

- Coding redundancy
- Spatial/temporal redundancy
- Irrelevant information

Entropy and average number of bits

$$H = -\sum_{k=0}^{L-1} p_r(r_k)\log_2 p_r(r_k)$$

$$L_{\text{avg}} = \sum_{k=0}^{L-1} l(r_k)\,p_r(r_k)$$

Objective fidelity criteria

$$e_{\text{rms}} = \left[\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y) - f(x,y)\right]^2\right]^{\frac{1}{2}}$$

$$SNR_{\text{ms}} = \frac{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\hat{f}(x,y)^2}{\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\hat{f}(x,y) - f(x,y)\right]^2}$$

Image Compression Methods

Huffman coding

Arithmetic coding

Run-length coding

Requirement: encode/decode with these methods, working conditions for these methods; develop a compression method with a combination of these methods for a specific task

Edge-based Image Segmentation

Connectivity

Edge detectors

- First-order derivative: Roberts (2x2), Prewitt (3x3), Sobel (3x3)
- Second-order derivative: Laplacian
- Complex edge detectors: LoG, DoG, and canny

Requirement: characteristics of these edge detector, perform simple edge detectors (1^{st} and 2^{nd} order) on an image

Edge linking

- Hough transform

Requirement: find a basic shape (e.g. a line) using Hough transform

Brief Review on Simple Edge Detectors

-First-order derivative

- Roberts (2x2)
- Prewitt (3x3)
- Sobel (3x3, smooth + difference)
- Thicker edge
- One operator for one edge direction

-Second-order derivative

- Laplacian (3x3)
- Double edge
- Zero-crossing

-Common issues:

- Sensitive to image noise
- Cannot deal with the scale change of the image

Brief Review on Advanced Edge Detectors

Laplacian of Gaussian (LoG) $\nabla^2 G(x,y)$

- Gaussian smoothing + Laplacian
- Localize the edge position based on zero-crossings
- Spaghetti effect of zero-crossings
- Multiple filtering processing for scale variations

Difference of Gaussian (DoG)

- Approximation of LoG
- Two different sigmas handle the scale variations

Canny edge detector

- Gaussian smoothing + 1st order derivative
- Nonmaxima suppression to get single edge
- Double thresholding and connectivity analysis to detect and link edges

Thresholding

- Key factors affect thresholding
  - Separation between peaks
  - Noise level
  - Relative size of objects and background
  - Illumination and reflectance

Region-based Image Segmentation

Region growing

Region-splitting and merging

Requirement: develop a region-based image segmentation method for a specific image

Basics of Mathematical Morphology

Set reflection and translation $\hat{B} = \{w \mid w = -b, b \in B\}$ $(B)_z = \{c \mid c = b + z, b \in B\}$

Erosion and dilation

Opening: $A \circ B = (A \ominus B) \oplus B$

- Smooth the contour of an object: smoothing outer corners
- Break narrow isthmuses
- Eliminate thin protrusions

Closing: $A \bullet B = (A \oplus B) \ominus B$

- Fill narrow breaks and gaps
- Eliminate long and thin gulfs
- Eliminate small holes

Hit-or-miss transform

Other operations

- Thinning and thickening
- Hole filling
- Connected component analysis
- Convex hull
- Skeleton

Requirement: the applications of these operations; perform erosion, dilation, opening, and closing operation on an image

Properties of Erosion and Dilation

- Dilation is commutative $A \oplus B = B \oplus A$
- Dilation is associative $A \oplus B \oplus C = A \oplus \left(B \oplus C\right)$
- Dilation $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
- Erosion $(A \cap B) \ominus C = (A \ominus C) \cap (B \ominus C)$
- Erosion and dilation are duals of each other

$$A \oplus B = \left(A^c \ominus \hat{B}\right)^c$$

- $A \subseteq (C \ominus B)$ if and only if $(A \oplus B) \subseteq C$
- If $A \subseteq C, \quad A \oplus B \subseteq C \oplus B$ and $A \ominus B \subseteq C \ominus B$

Properties of Opening and Closing

Opening:

$$(A \circ B) \circ B = A \circ B$$

$$(A \circ B) \subseteq A$$

$$if\ A \subseteq C, A \circ B \subseteq C \circ B$$

Closing

$$(A \bullet B) \bullet B = A \bullet B$$

$$A \subseteq (A \bullet B)$$

$$if\ A \subseteq C, A \bullet B \subseteq C \bullet B$$