Today’s Agenda

- Image Degradation and Restoration

Degradation VS Restoration

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Note: a linear, position invariant degradation system with additive noise can be modeled as the convolution of the degradation function with the image plus the additive noise.

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha, y-\beta)\,d\alpha\, d\beta + \eta(x,y)$$

Degradation (convolution)

$f(x,y)$ → $g(x,y)$

Restoration (deconvolution)

Estimate the Degradation Function

- Observation
- Experimentation
- Mathematical modeling

Estimation by Modeling – Motion Blur

Constant velocity along x and y direction:

$$x_0(t) = \frac{at}{T} \qquad y_0(t) = \frac{bt}{T}$$

a b

FIGURE 5.26
(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Estimation by Modeling – Cont.

An example of motion blur

$$g(x,y)=\int_0^T f[x-x_0(t),\, y-y_0(t)]dt$$

Motion in both x and y direction during acquisition

Estimation by Modeling – Cont.

An example of motion blur

$$G(u,v) = F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

$$\Rightarrow \quad H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

Estimation by Modeling – Example

Constant velocity along x and y direction:

$$x_0(t) = at/T \qquad y_0(t) = bt/T$$

What is $H(u,v)$?

$$H(u,v) = T\,\frac{\sin[\pi(ua+vb)]}{\pi(ua+vb)}\,e^{-j\pi(ua+vb)}$$

Image Restoration

Given the degradation system **H** and the input image **G**, recover the original image **F**

- Inverse filtering
- Wiener filtering

Inverse Filtering

Ideally:

$$G(u,v) = H(u,v)F(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

In practice:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Limiting the analysis to frequencies near the origin (0,0)

Low pass filtering

An Example of Inverse Filtering

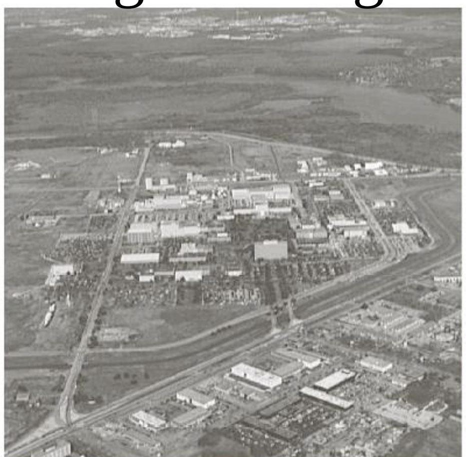
Original image

Degraded image

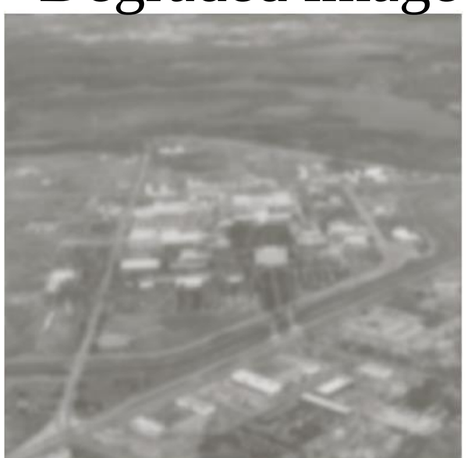
$$\frac{G(u,v)}{H(u,v)}$$

An Example of Inverse Filtering (Cont.)

Original image



Degraded image



$$\frac{G(u,v)}{H(u,v)}$$

Full filter

Cutoff radius = 40

Cutoff radius = 70

Cutoff radius = 85

Minimum Mean Square Error (Wiener) Filtering

Assumptions:

- Noise and image are uncorrelated
- Noise has zero mean

Original

Inverse filtering with cutoff = 70

Wiener filtering

The Formulation

Minimize mean squared error: $e^2 = E\{(f - \hat{f})^2\}$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + |N(u,v)|^2 / |F(u,v)|^2} \right] G(u,v)$$

Least square error filter

$|N(u,v)|^2$ is the power spectrum of noise

$|F(u,v)|^2$ is the power spectrum of undegraded image

The Formulation (Cont.)

Signal to noise ratio: the metric to evaluate the restoration performance

Frequency domain: $$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left|F(u,v)\right|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left|N(u,v)\right|^2}$$

Spatial domain: $$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\hat{f}(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\hat{f}(x,y) - f(x,y)]^2}$$

The Formulation

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + |N(u,v)|^2 / |F(u,v)|^2} \right] G(u,v)$$

⬇

An approximation

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] G(u,v)$$

Example 2 – Motion Blur + Additive Noise

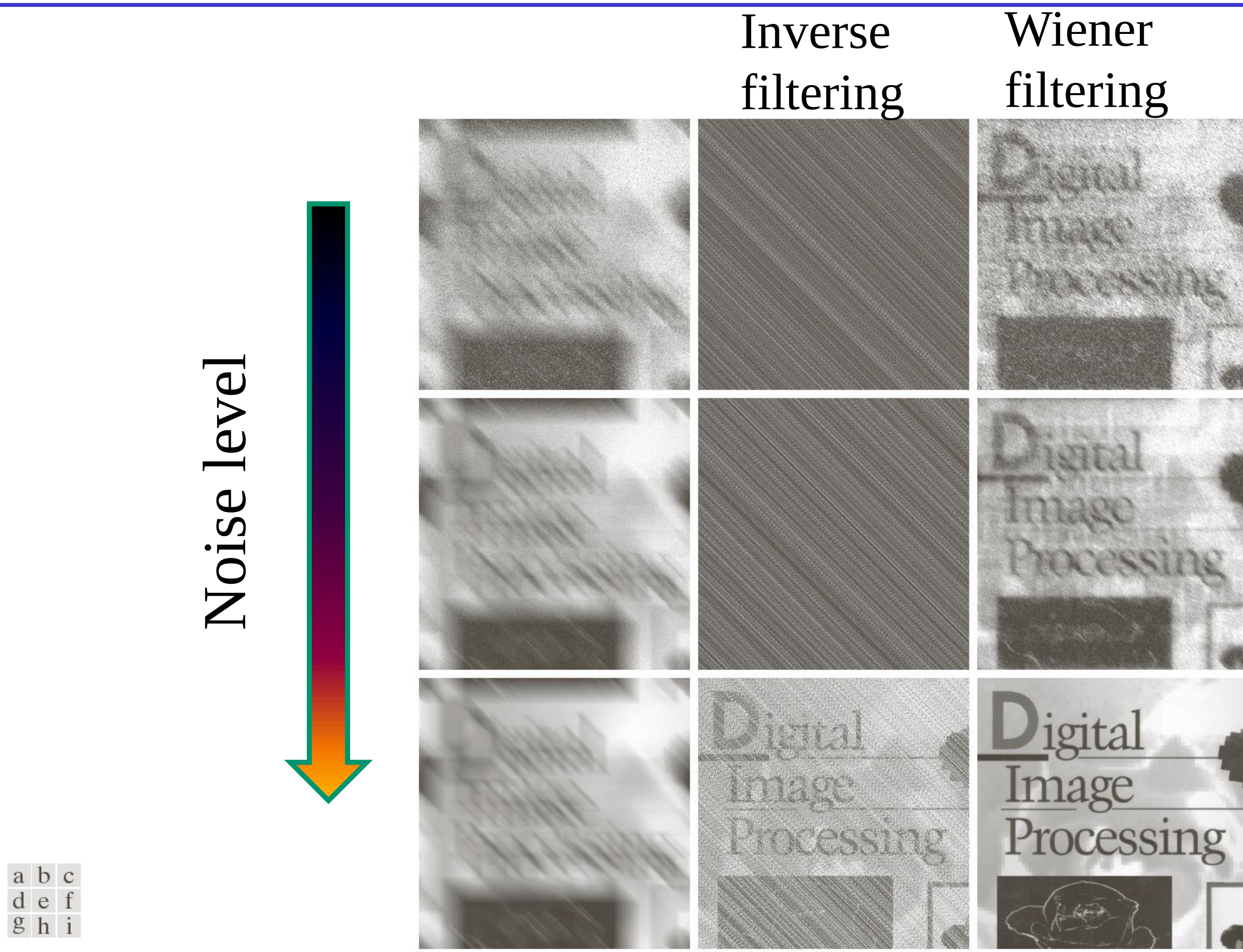


FIGURE 5.29 (a) 8-bit image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

Constrained Least Square Filtering

Assumption: the mean and variance of the noise is known

Matrix-vector representation: $\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\eta}$

Alleviate the noise sensitivity by minimizing

$$C = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1}[\nabla^2 f(x,y)]^2$$

The image is smooth

subject to

$$\left\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\right\|^2 = \left\|\boldsymbol{\eta}\right\|^2$$

Constrained Least Square Filtering

Frequency domain solution:

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2} \right] G(u,v)$$

Fourier transform of Laplacian $p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

Parameter to satisfy the constraint

Example

CLSF

Wiener

Optimize γ

Residual: $$\mathbf{r} = \mathbf{g} - \mathbf{H}\hat{\mathbf{f}}$$

$$\phi(\gamma) = \|\mathbf{r}\|^2$$ ➡ A function monotonically increasing of γ

Adjust γ to satisfy the constraint:

$$\|\mathbf{r}\|^2 = \|\eta\|^2 \pm a$$ → Accuracy factor

Optimize γ

1. Specify an initial γ
2. Compute $\phi(\gamma) = \|\mathbf{r}\|^2$
3. If $\|\mathbf{r}\|^2 = \|\eta\|^2 \pm a$, stop or adjust γ accordingly

How to Calculate $\|\eta\|^2$

$$\|\eta\|^2 = MN\left({\sigma_\eta}^2 + {m_\eta}^2\right)$$

Example

Iteratively search for optimal parameter γ

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

Reading Assignments

Chapter 5.10 – 5.11

Color Image Processing

- **The world is colorful**
- **Color feature is one of the natural cue human used for object detection/recognition**
  - Thousands of color shades vs dozens of gray levels
  - Various applications
- **Challenges**
  - Illumination
  - Variations

http://okanaganokanogan.com/2015/10/

https://johnhowie.wordpress.com/2009/12/22/445/

http://www.tutorialized.com/tutorial/Grasslands-in-3ds-Max/57927

Fundamentals of Color Image Processing

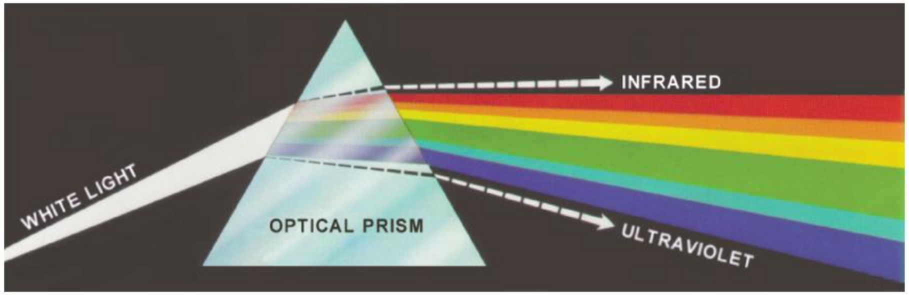


FIGURE 6.1 Color spectrum seen by passing white light through a prism. (Courtesy of the General Electric Co., Lamp Business Division.)

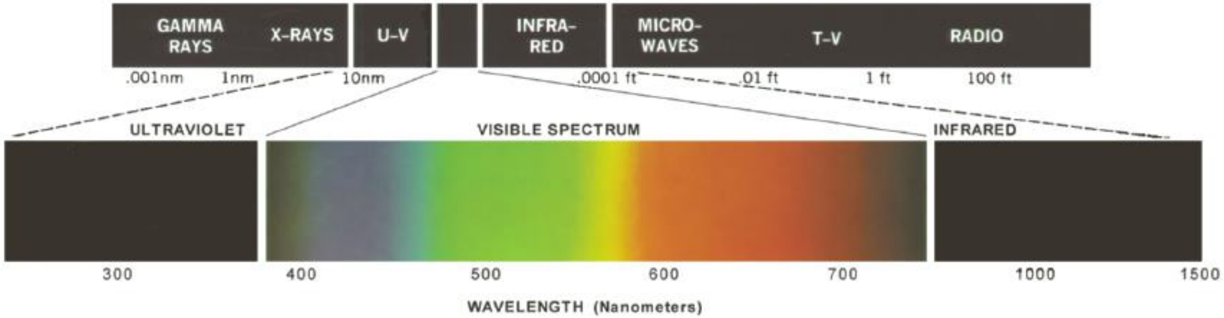


FIGURE 6.2 Wavelengths comprising the visible range of the electromagnetic spectrum. (Courtesy of the General Electric Co., Lamp Business Division.)

Color Representations

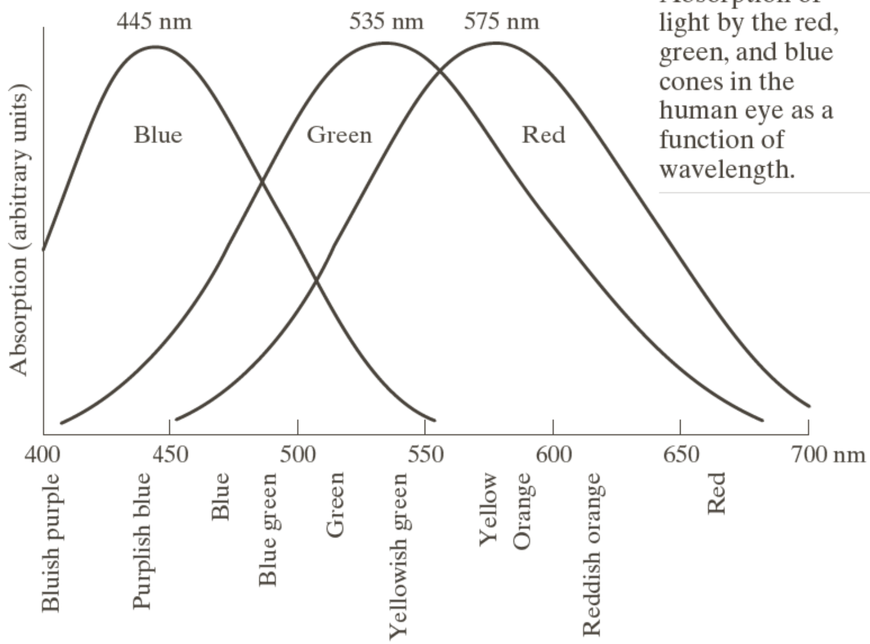
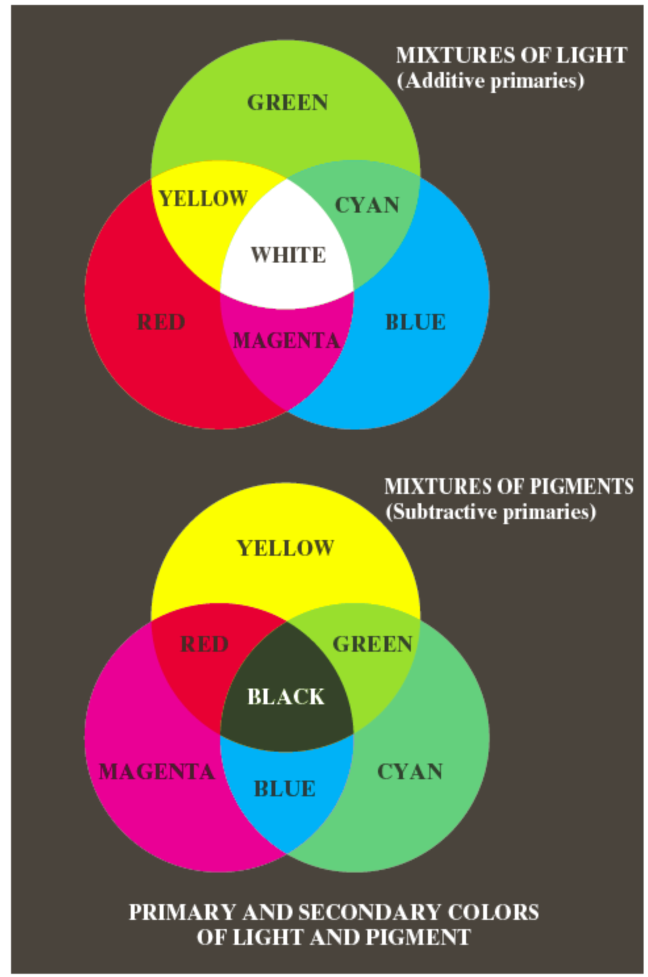


FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.



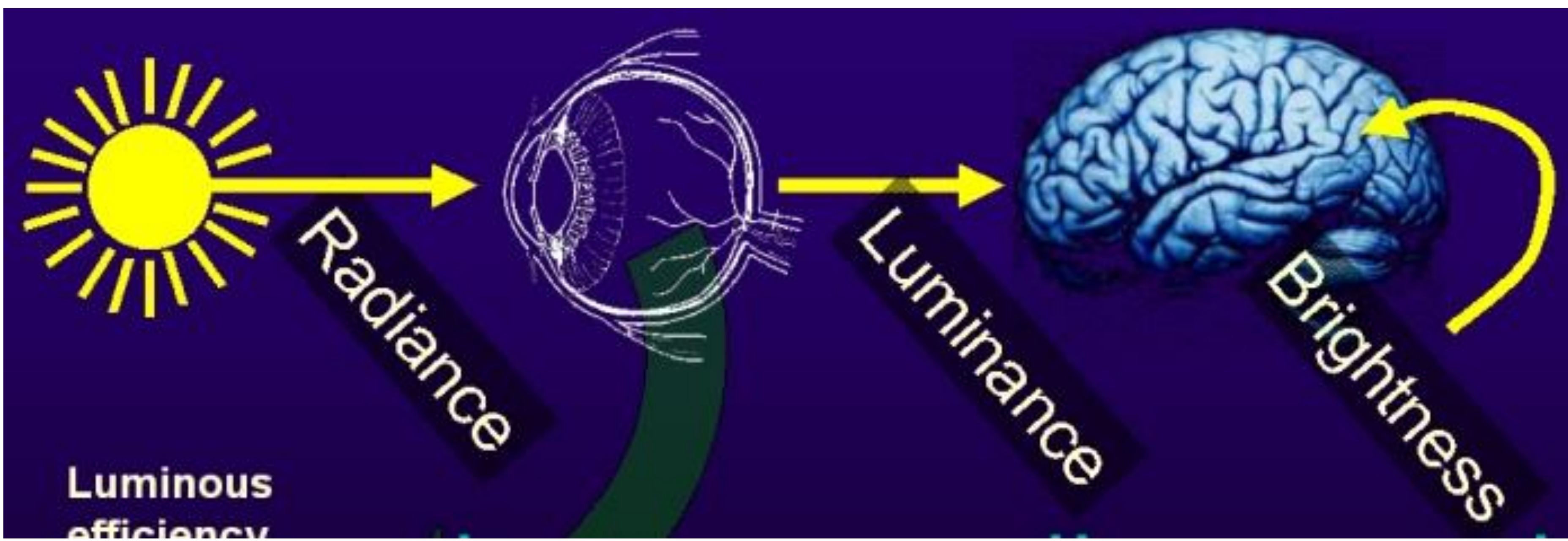
a
b

FIGURE 6.4 Primary and secondary colors of light and pigments. (Courtesy of the General Electric Co., Lamp Business Division.)

- primary/secondary colors
- primary/secondary pigments
- all visible colors

Characteristics of Light

- Radiance
- Luminance
- Brightness



Picture was adapted from Dr. Gordon Kindlmann's talk "**Face-based Luminance Matching for Perceptual Colormap Generation**"
http://www.cs.utah.edu/~gk/papers/vis02/talk/

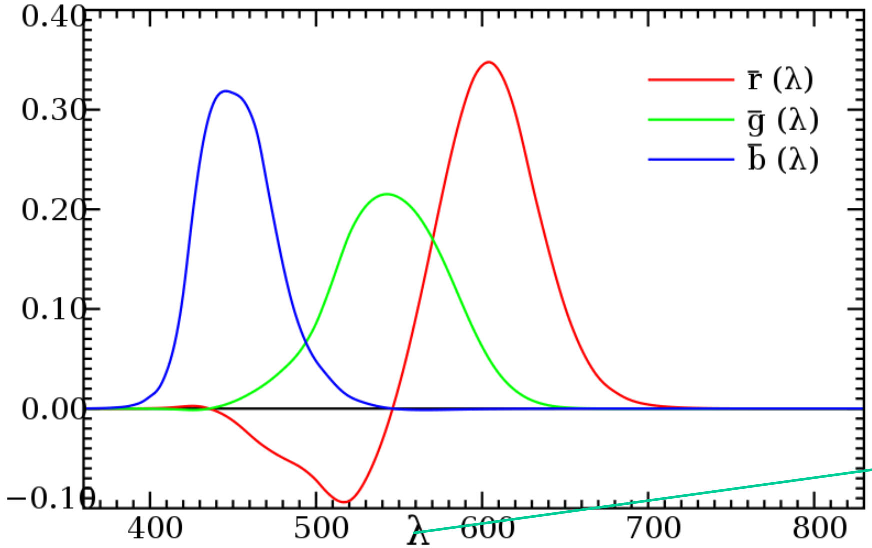
Characteristics of Color Light

- **Radiance**
- **Luminance**
- **Brightness**
- **Chromaticity**
  - Hue – dominant color/wavelength
  - Saturation – color purity

White and grey has the same chromaticity, while different brightness

Chromaticity

Tristimulus values of a color: The amounts of the three primary color to match a test color



The CIE 1931 RGB Color matching functions.

wavelength

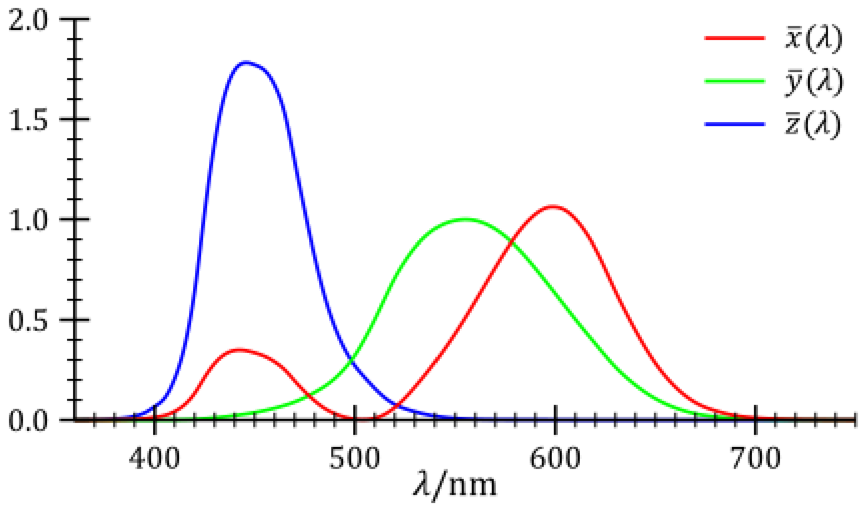
CIE (International Commission on Illumination) RGB matching function

$$R = \int_0^\infty I(\lambda)\bar{r}(\lambda)d\lambda \qquad G = \int_0^\infty I(\lambda)\bar{g}(\lambda)d\lambda \qquad B = \int_0^\infty I(\lambda)\bar{b}(\lambda)d\lambda$$

$$\int_0^\infty \bar{r}(\lambda)d\lambda = \int_0^\infty \bar{g}(\lambda)d\lambda = \int_0^\infty \bar{b}(\lambda)d\lambda$$

Chromaticity

Tristimulus values of XYZ space



CIE XYZ matching function

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$X = \int_0^\infty I(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_0^\infty I(\lambda)\bar{y}(\lambda)d\lambda$$ ⇨ **Luminance**

$$Z = \int_0^\infty I(\lambda)\bar{z}(\lambda)d\lambda$$

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

} **Chromaticity**

$$z = \frac{Z}{X+Y+Z}$$

$$\Rightarrow z = 1 - x - y$$

xy-Y space

Chromaticity Diagram

x and y to represent colors

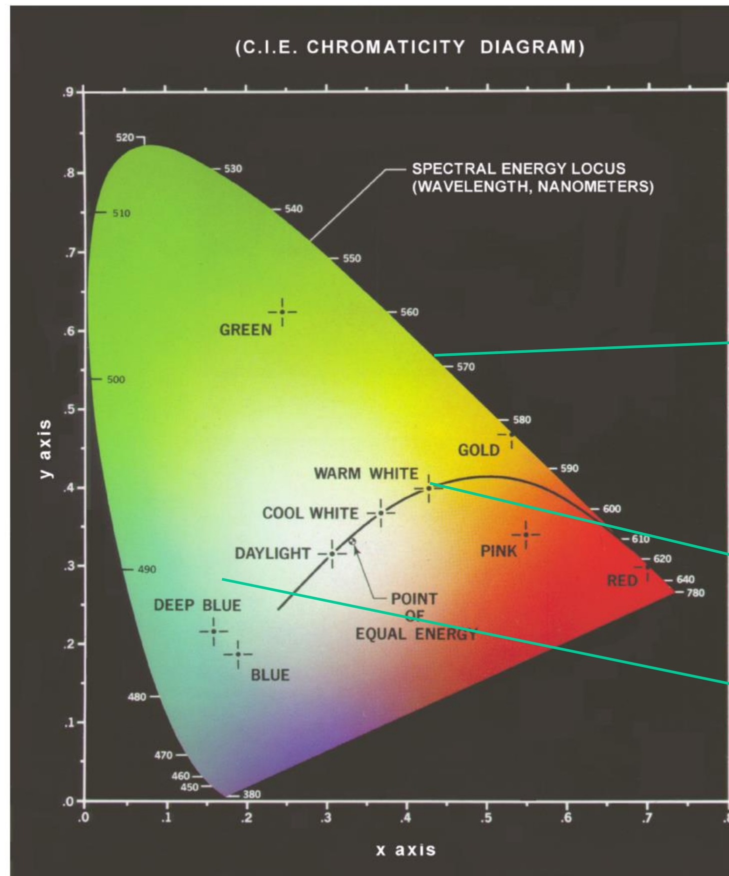


FIGURE 6.5 Chromaticity diagram. (Courtesy of the General Electric Co., Lamp Business Division.)

Pure color and fully saturated

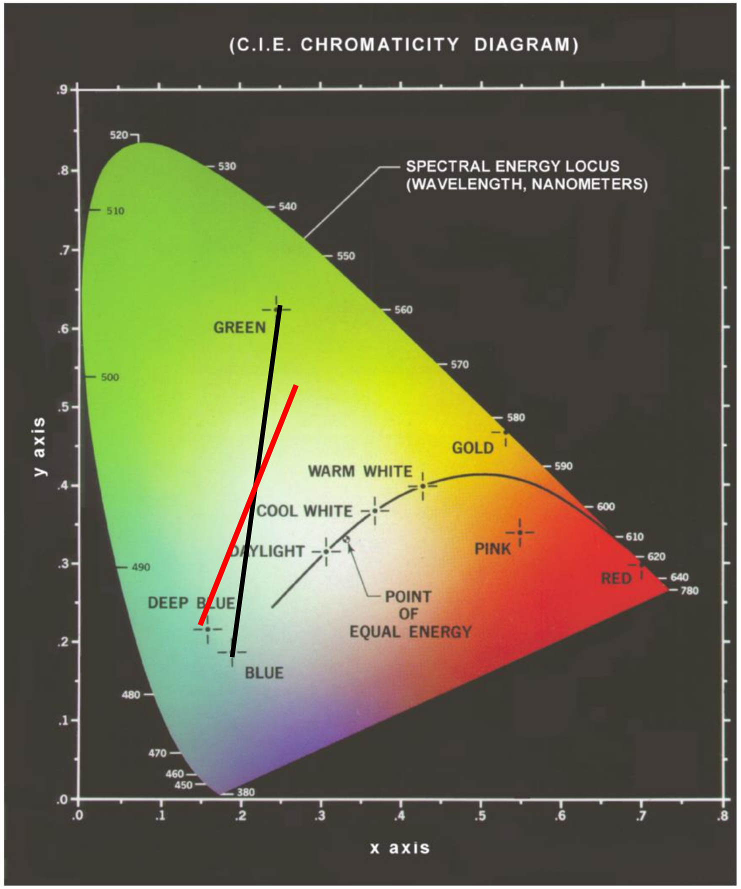
Equal energy with zero saturation

Mixed color with less saturation

Chromaticity Diagram (Cont'd)

Color mixing: any color on a line segment can be generated by the two ending points in the color diagram

Metamerism: the same color can be generated with different combinations of source colors with the same tristimulus values



Color Gamut

- **Color gamut:** a complete subset of colors can be displayed on a device or represented by a color space.
- The color represented by 3 given colors resides in the triangle formed by the 3 points
- Not all colors can be represented by 3 primary colors

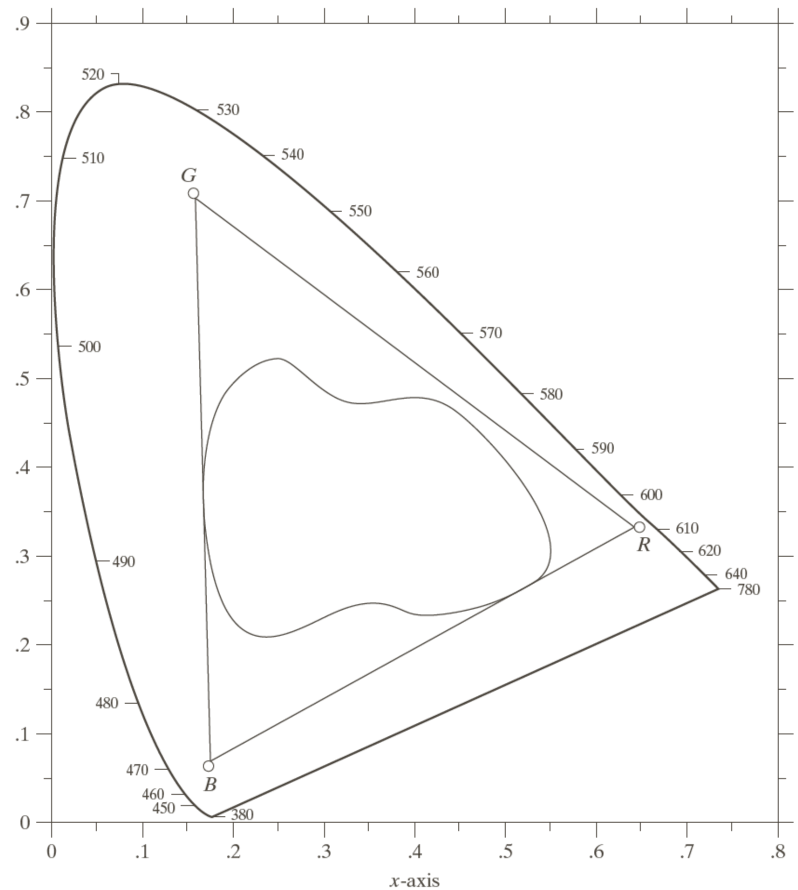


FIGURE 6.6 Typical color gamut of color monitors (triangle) and color printing devices (irregular region).