

Today's Agenda

- **Image Degradation and Restoration**

Reminder: Proposal of Final Project

Due: 11:59 pm, Feb. 21.

Late submission penalty applies.

Include

- Title and names of the team members
- Topic: a research project or a survey
- Brief introduction on the background
- Timeline and project management for a teamwork
- An initial list of papers being reviewed (Survey project only)

At most one page

Each team only needs one abstract

Reminder: Paper Reading

Presentation days:

- Wednesday, March 13
- Monday, March 18
- Wednesday, March 20

Send me an email (tongy@cse.sc.edu) by **11:59pm of Feb. 21**, which includes:

- The paper you are going to present
 - Title, authors, where and when it was published, pages
 - Example: Sing Bing Kang, Ashish Kapoor, Dani Lischinski ,
“Personalization of Image Enhancement ”, in *Proceedings of IEEE Conference on computer vision and Pattern Recognition (CVPR)*, 2010
- Your name and preference of these three days in a decreasing order.
Earlier email has higher priority in choosing the day

I will provide feedback (approve/suggest to change) to your selected paper

Where to Find the Paper

The paper you choose must be published in an official journal or conference!

A journal paper is preferred!

You can find papers from journals

IEEE Transactions on Pattern Analysis and Machine Intelligence

<http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?reload=true&punumber=34>

IEEE Transactions on Image Processing

<http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=83>

Other premier conferences or journals, CVPR, ICCV, ECCV, IEEE Trans. on Medical Imaging ...

Deadline for your email: **11:59pm, Feb. 21**

Image Degradation and Restoration



http://fireoracleproductions.com/services__samp

Image degradation due to

- noise in transmission
- imperfect image acquisition
 - environmental condition
 - quality of sensor

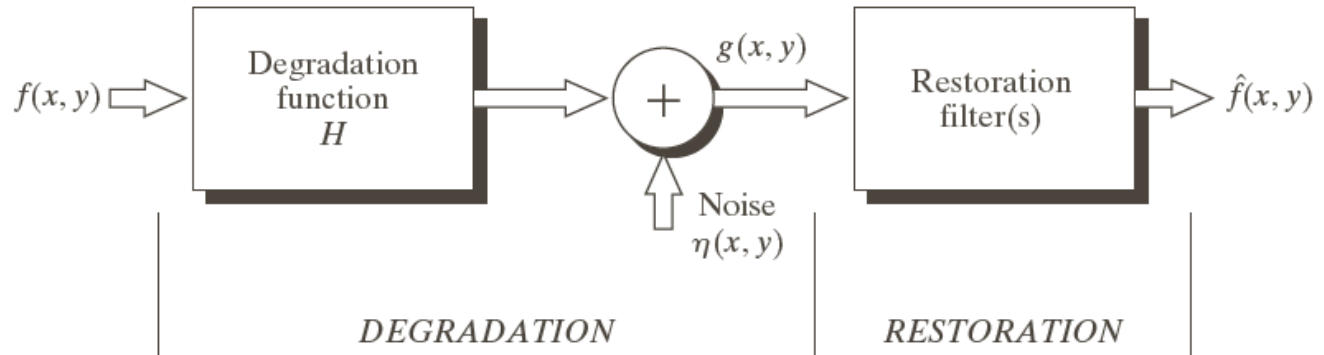


Google image

Image Degradation and Restoration

FIGURE 5.1

A model of the image degradation/restoration process.



$$g(x, y) = h(x, y) \otimes f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

Identity $H(u, v) \rightarrow$ degradation only comes from additive noise

Image Restoration with Additive Noise

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

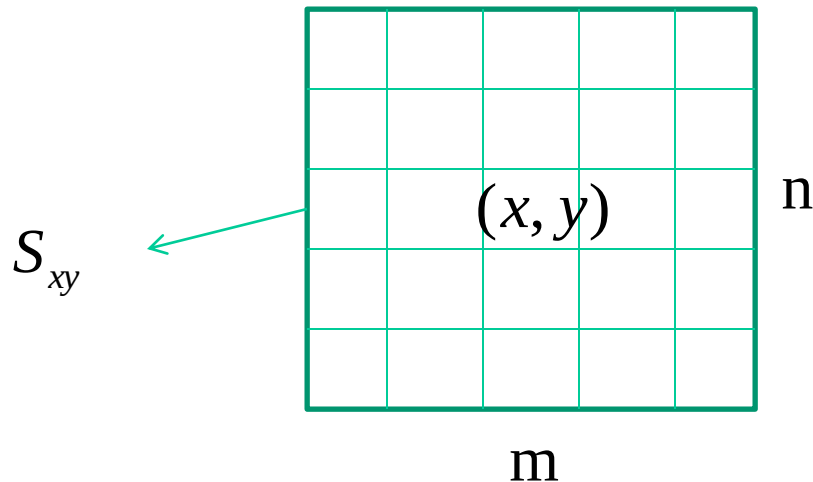
Noise models:

- Impulse noise: pepper and salt
- Continuous noise model:
 - Gaussian, Rayleigh, Gamma, Exponential, Uniform

Properties of Noise

- **Spatial properties**
 - Spatially periodic noise
 - Spatially independent noise
- **Frequency properties**
 - White noise – noise containing all frequencies within a bandwidth

Mean Filters for Continuous Noise Models



Arithmetic Mean Filter: a linear filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

Non-linear Mean Filters

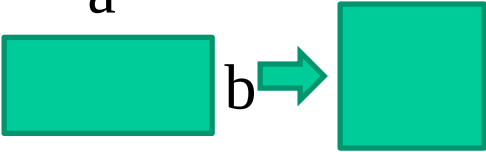
Geometric Mean Filter

Harmonic Mean Filter

Contraharmonic Mean Filter

Mean Filter

Geometric Mean Filter

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$


- Removing the salt noise
- Fail in the pepper noise

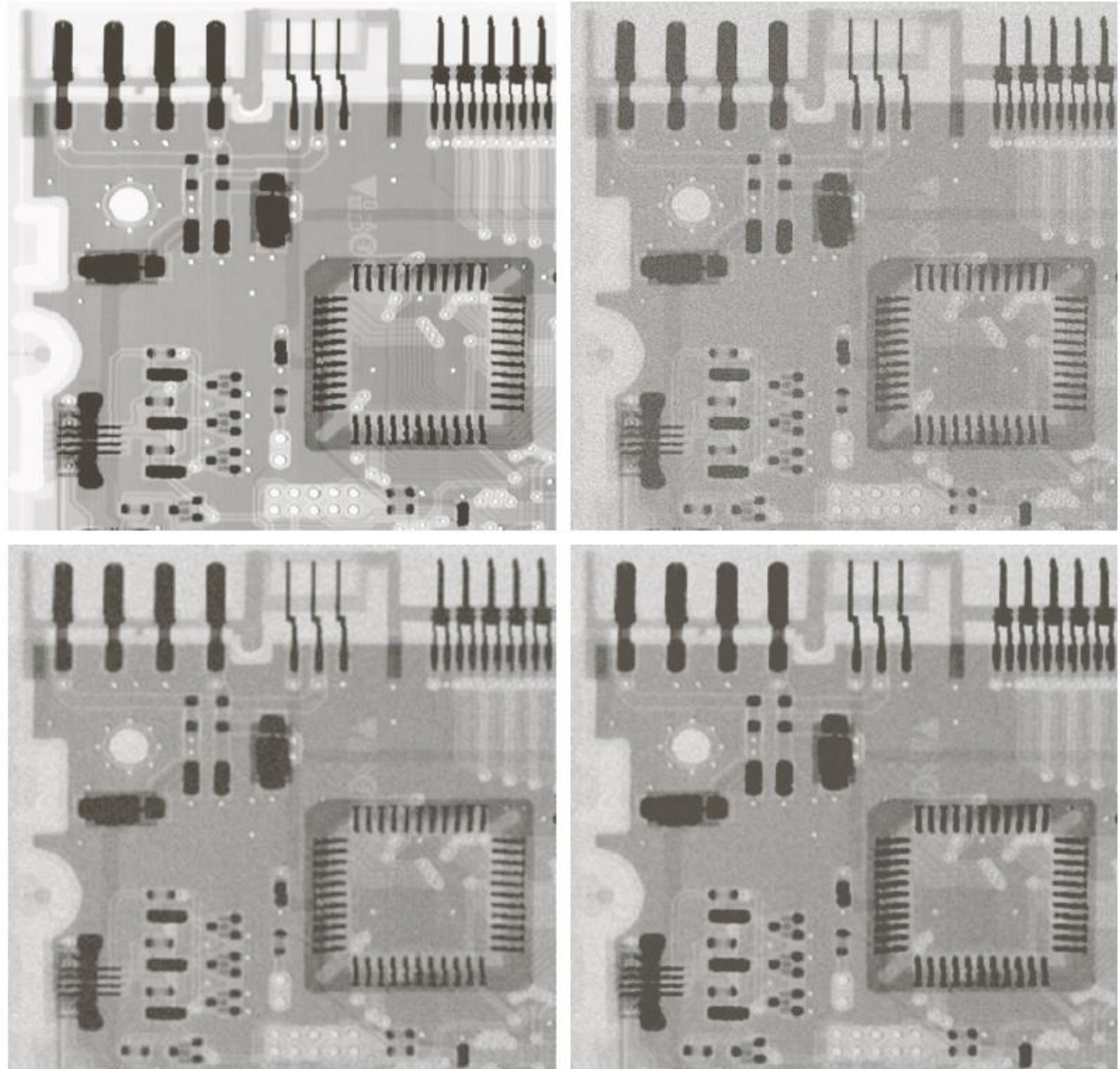
An Example

a	b
c	d

FIGURE 5.7

(a) X-ray image.
(b) Image corrupted by additive Gaussian noise.
(c) Result of filtering with an arithmetic mean filter of size 3×3 .
(d) Result of filtering with a geometric mean filter of the same size.

(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Mean Filter

Harmonic Mean Filter

$$\hat{f}(x, y) = \frac{1}{\frac{1}{mn} \sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

- Removing the salt noise
- Fail in the pepper noise

Mean Filter

Contraharmonic Mean Filter

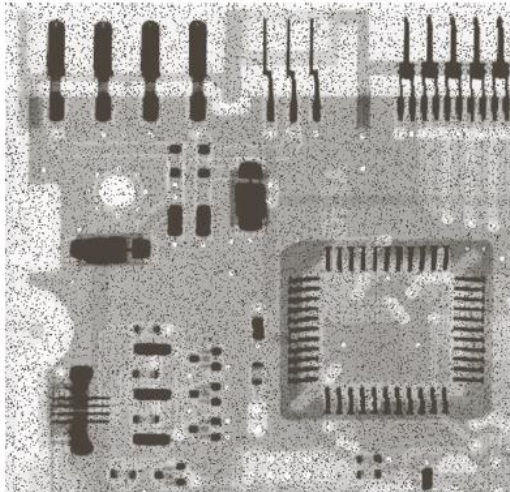
$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

Q is the order of the filter

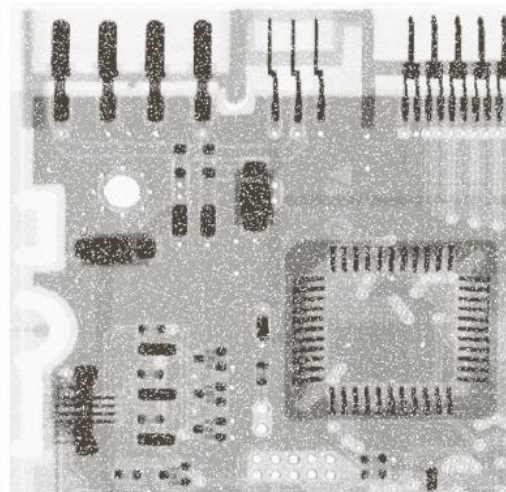
- positive Q removes pepper noise
- negative Q removes salt noise
- Special cases: Q=0, Q=-1

An Example

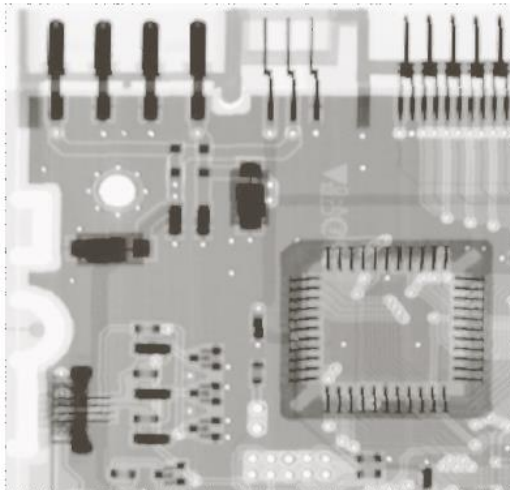
Pepper noise



Salt noise



$Q=1.5$

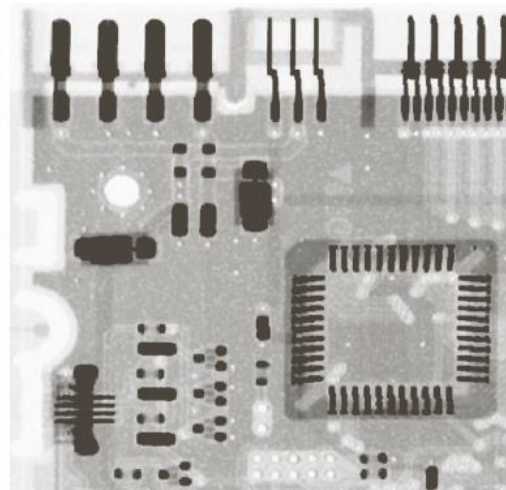


a	b
c	d

FIGURE 5.8

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

$Q=-1.5$



A Failed Case with Wrong Sign of Contraharmonic Filter

a b

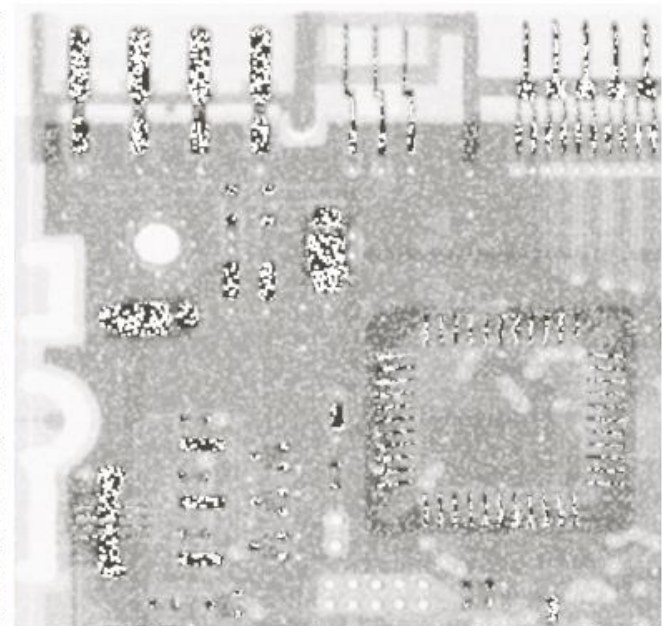
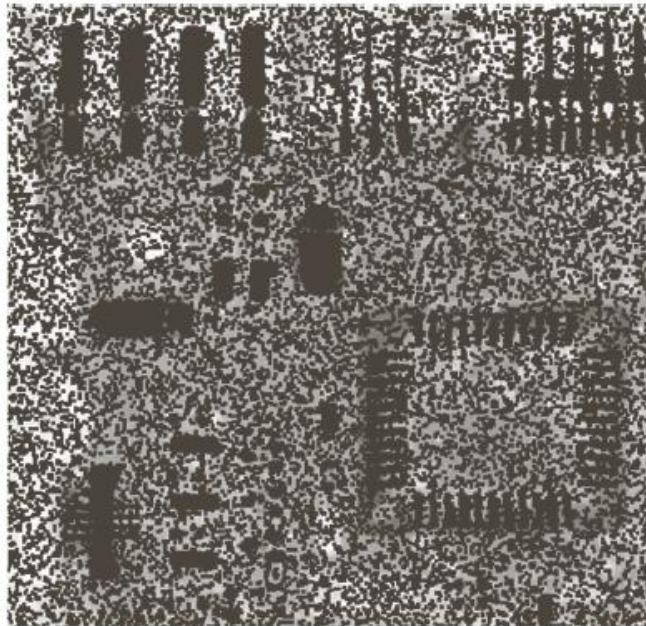
FIGURE 5.9

Results of selecting the wrong sign in contraharmonic filtering.

(a) Result of filtering

Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.

(b) Result of filtering 5.8(b) with $Q = 1.5$.



Summary: Mean Filters for Continuous Noise Models

Arithmetic Mean Filter: a linear filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- Work well for continuous noise

Summary: Non-linear Mean Filters

Geometric Mean Filter

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

Work well for

- Continuous noise
- Salt noise

Harmonic Mean Filter

$$\hat{f}(x, y) = \frac{1}{\frac{1}{mn} \sum_{(s,t) \in S_{xy}} \frac{1}{g(s, t)}}$$

Fail for the pepper noise

Contraharmonic Mean Filter

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

Q is the order of the filter

- positive Q removes pepper noise
- negative Q removes salt noise
- Special cases: Q=0, Q=-1

Order-Statistic Filters -- Median Filter

a	b
c	d

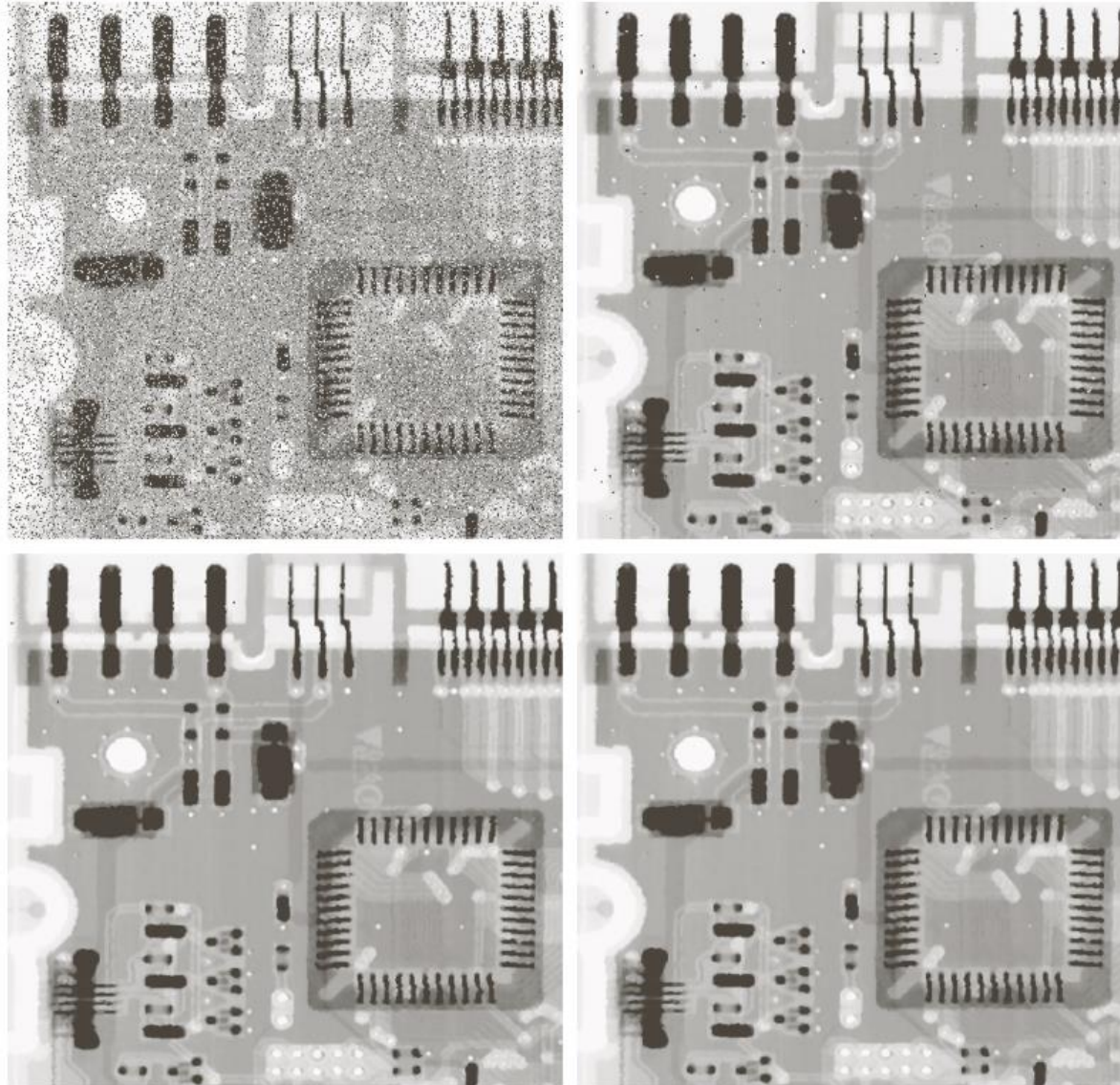
FIGURE 5.10

(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.

(b) Result of one pass with a median filter of size 3×3 .

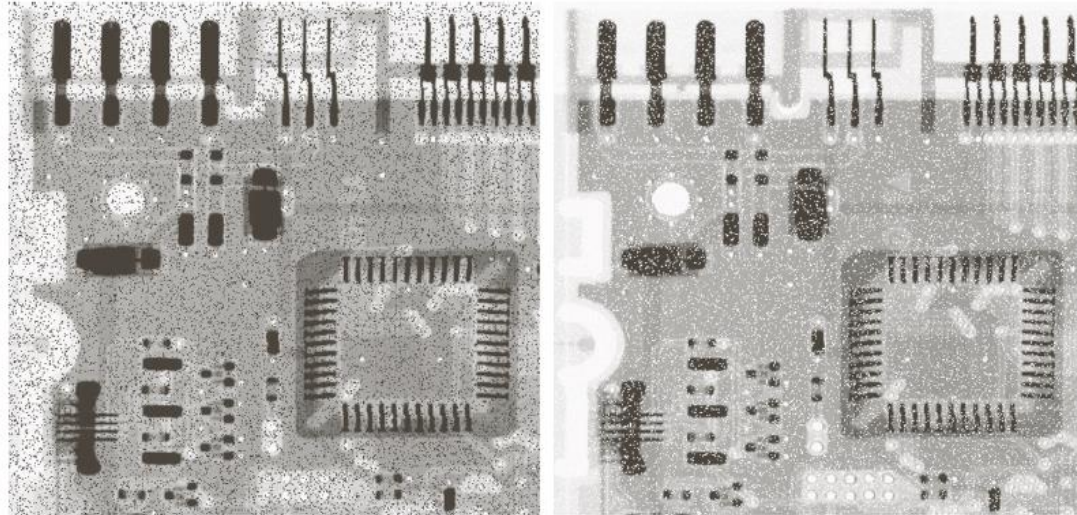
(c) Result of processing (b) with this filter.

(d) Result of processing (c) with the same filter.



Repeating median filter will remove most of the noise while increase image blurring

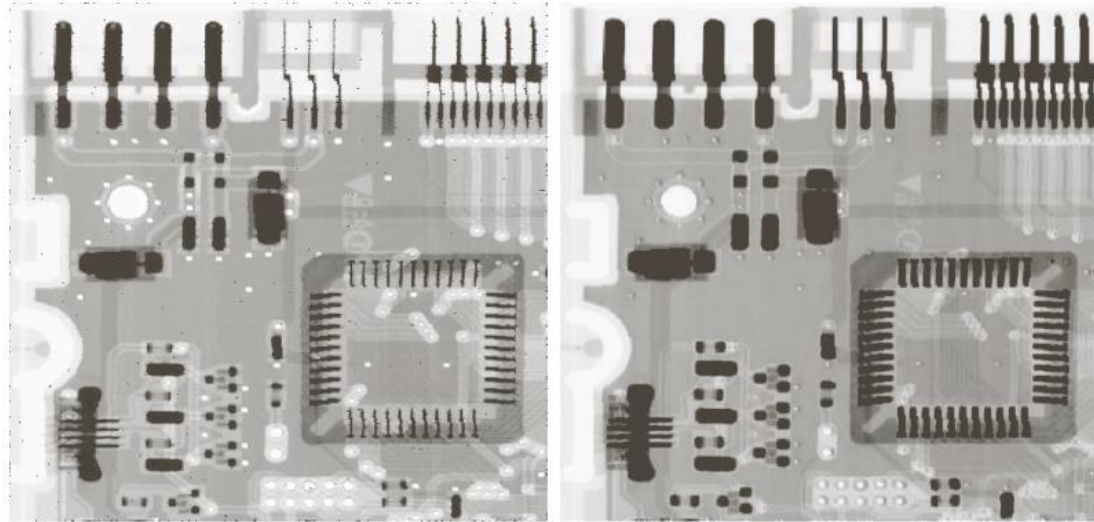
Order-Statistic Filters -- Max/Min Filters



a b

FIGURE 5.11

(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.



- Find the extreme points
- Remove the targeting impulse noise

Order-Statistic Filters

Midpoint filter

- Combine order statistics and averaging
- Works best for randomly distributed noise, like Gaussian or uniform noise
- Not suitable for impulse noise and blur the boundary

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(s,t) \in S_{xy}} \{g(s,t)\} + \min_{(s,t) \in S_{xy}} \{g(s,t)\} \right]$$



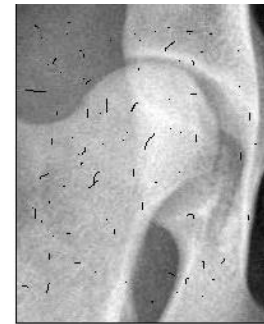
Random noise



Salt noise



Pepper noise



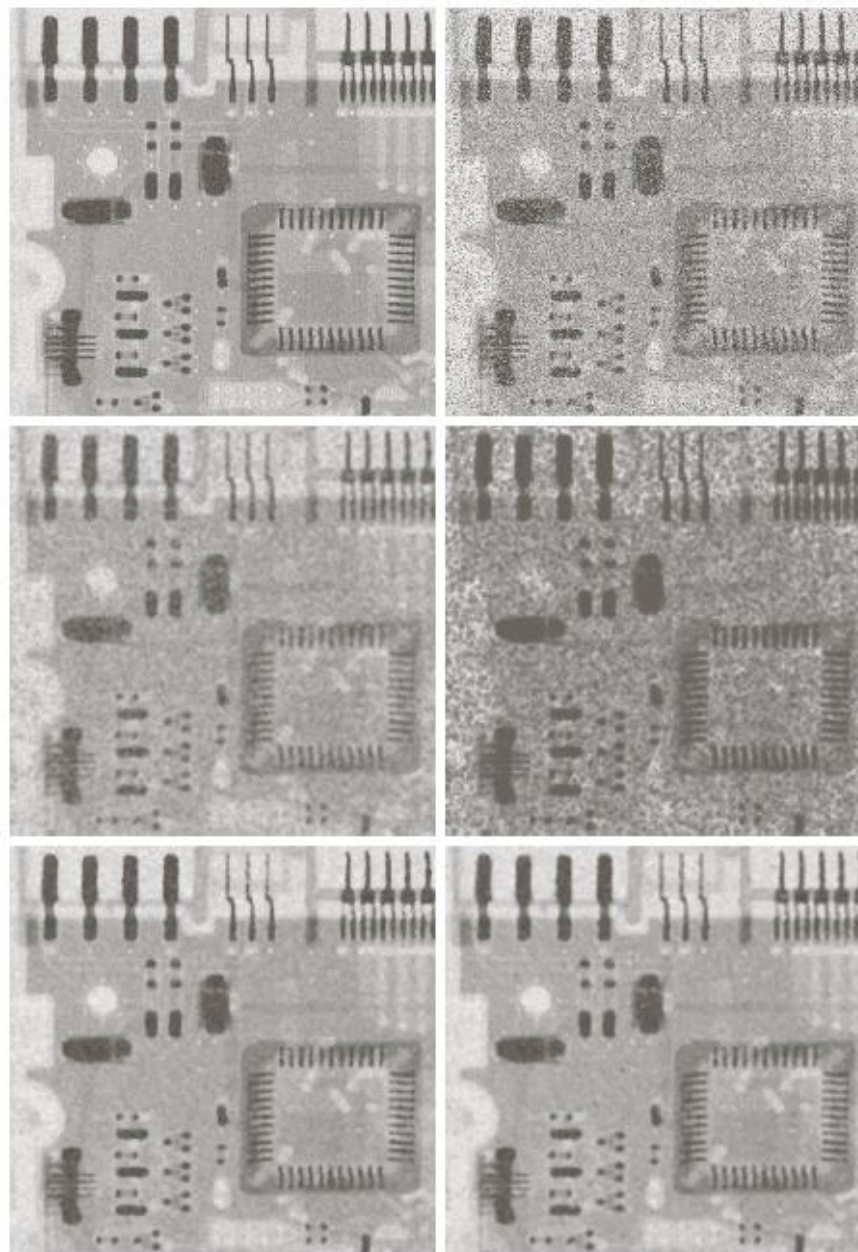
Order-Statistic Filters

Alpha-trimmed mean filter

- Delete $d/2$ lowest and $d/2$ highest intensity values
- A balance between arithmetic mean filter and median filter
- Suitable for combined salt-and-pepper and Gaussian noise

$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

Example



a	b
c	d
e	f

FIGURE 5.12

(a) Image corrupted by additive uniform noise.

(b) Image additionally corrupted by additive salt-and-pepper noise.

Image (b) filtered with a 5×5 ;

(c) arithmetic mean filter;

(d) geometric mean filter;

(e) median filter;

and (f) alpha-trimmed mean filter with $d = 5$.

Adaptive Filters

Adaptive local noise reduction filter

Key elements:

- the intensity value $g(x, y)$
- the variance of the noise σ_η^2
- the local mean of the neighborhood m_L
- the local variance of the neighborhood σ_L^2

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2} [g(x, y) - m_L]$$

Properties:

- If $\sigma_\eta^2 = 0$, $\hat{f}(x, y) = g(x, y) \rightarrow$ ideal case
- If $\sigma_\eta^2 / \sigma_L^2$ is low, preserve the edge information $\hat{f}(x, y) \approx g(x, y)$

Examples

a	b
c	d

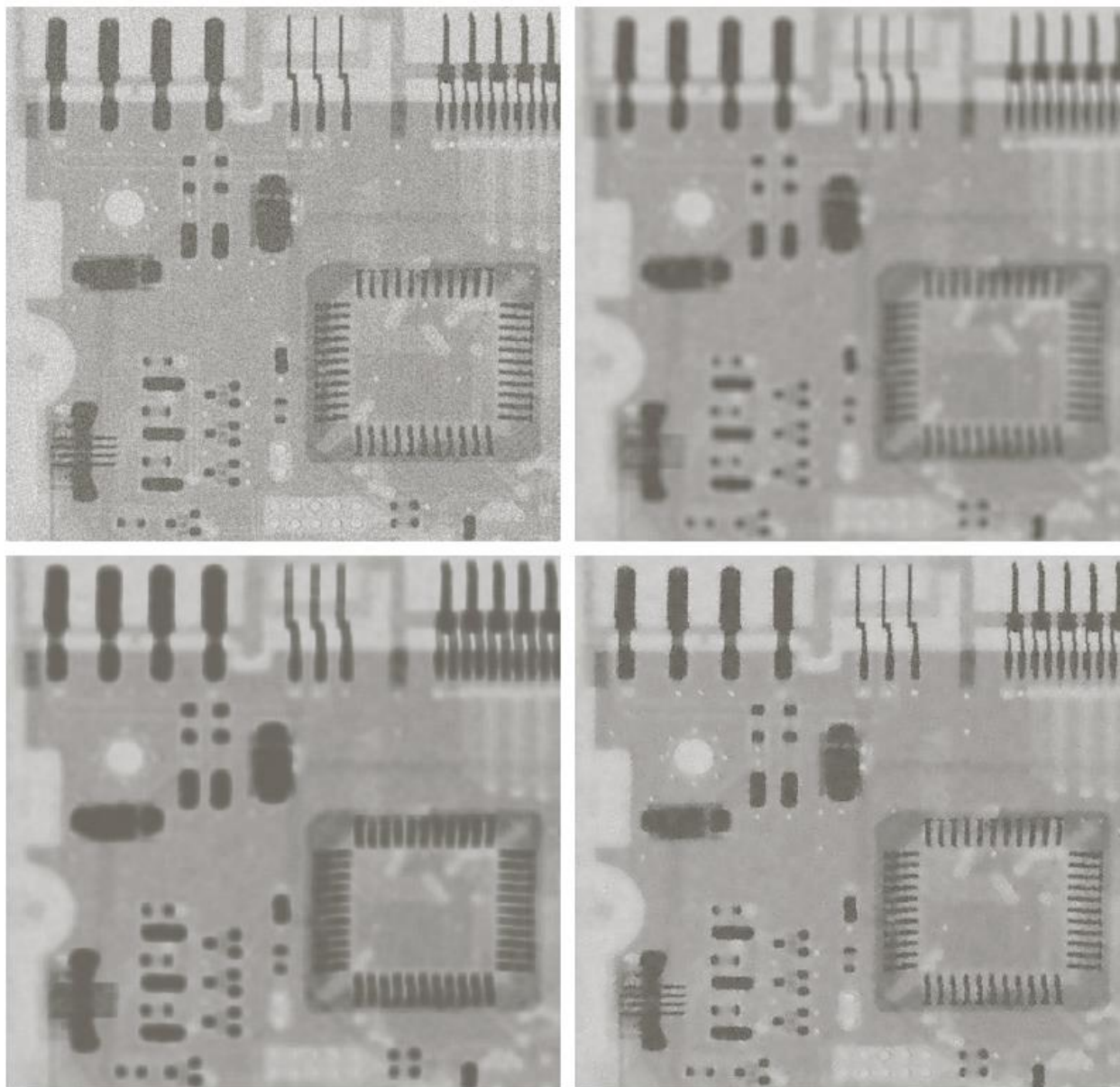
FIGURE 5.13

(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.

(b) Result of arithmetic mean filtering.

(c) Result of geometric mean filtering.

(d) Result of adaptive noise reduction filtering. All filters were of size 7×7 .



Adaptive Median Filter

Stage A: check if the median value is an extreme value

$$A1 = z_{\text{med}} - z_{\text{min}}$$

$$A2 = z_{\text{med}} - z_{\text{max}}$$

If $A1 > 0$ AND $A2 < 0$, go to stage B

Else increase the window size

If window size $\leq S_{\text{max}}$ repeat stage A

Else output z_{med}

Goal 1: remove salt-and-pepper noise with higher probability

Goal 2: smoothing the noise other than impulses

Goal 3: reduce distortion

Stage B: check if the center pixel is an extreme value

$$B1 = z_{xy} - z_{\text{min}}$$

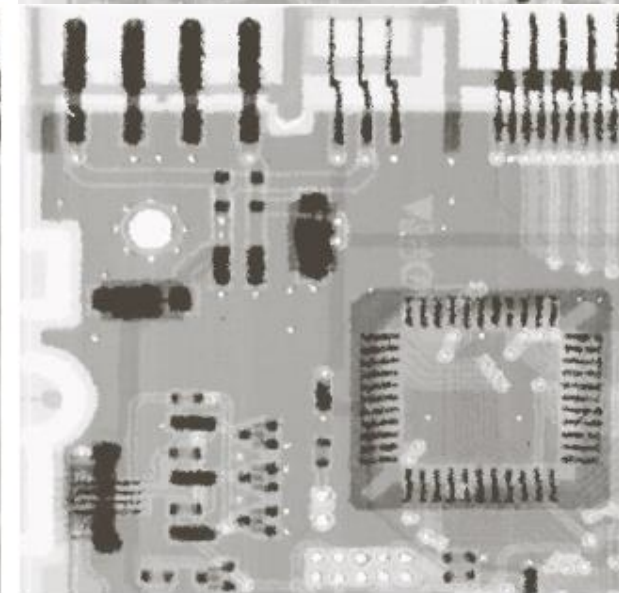
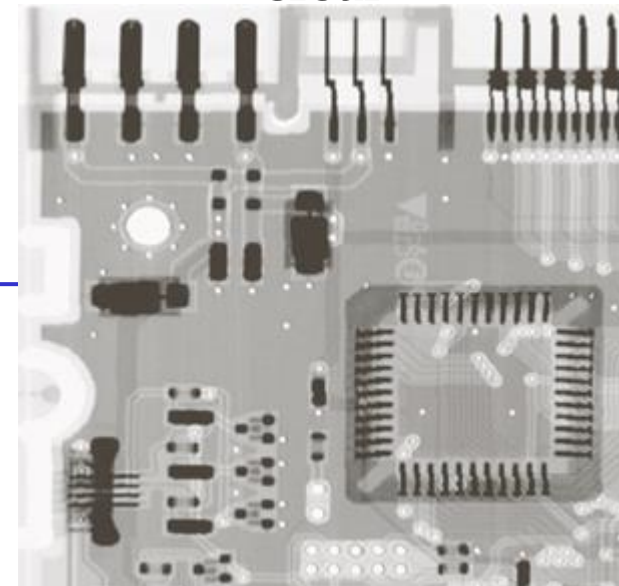
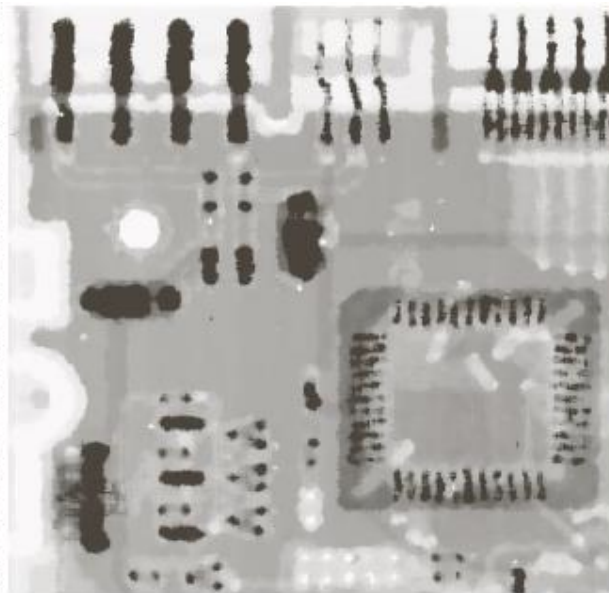
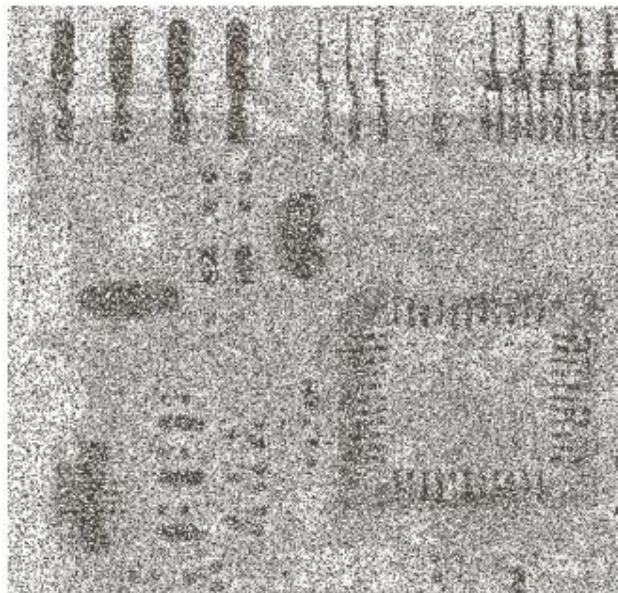
$$B2 = z_{xy} - z_{\text{max}}$$

If $B1 > 0$ AND $B2 < 0$, output z_{xy}

Else output z_{med}

clear

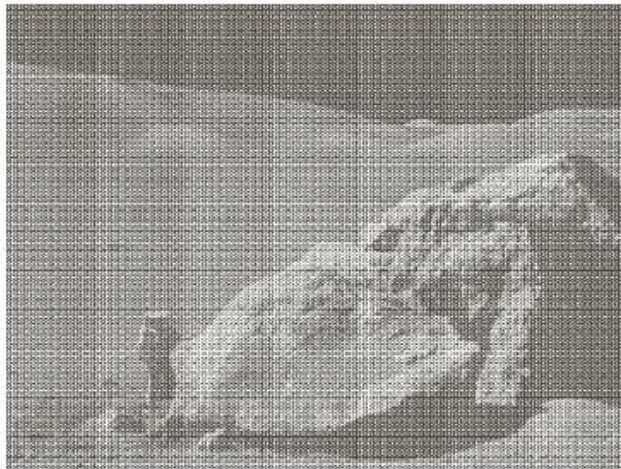
Adaptive Median Filter -- Example



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Periodical Noise



a b
c d

FIGURE 2.40

(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)

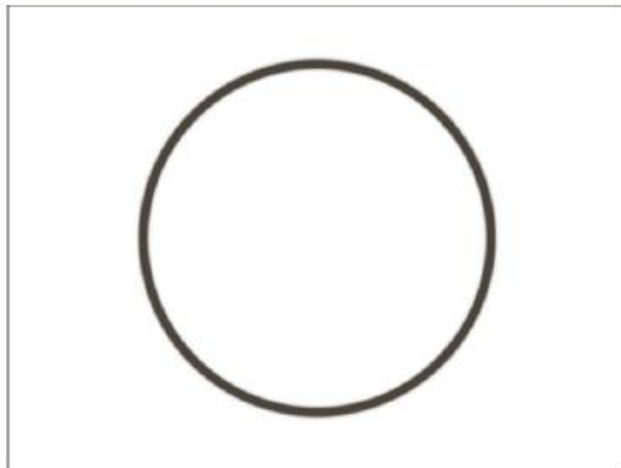


Image is corrupted by a set of sinusoidal noise of different frequencies

Linear, Position-Invariant Degradations – Noise Free Case

$$g(x, y) = H[f(x, y)]$$

Linearity $H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$

Position/space invariant $H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$



H does not depend on the location (x, y) ,
only represented by the input and output

Impulse Response for Linear H

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) \delta(x - \alpha, y - \beta) d\alpha d\beta \quad \leftarrow \text{Sifting property}$$

$$g(x, y) = H[f(x, y)]$$



$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) \boxed{H[\delta(x - \alpha, y - \beta)]} d\alpha d\beta$$

Impulse response

Impulse Response for Linear H

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) H[\delta(x - \alpha, y - \beta)] d\alpha d\beta$$

Impulse response $h(x, \alpha, y, \beta) = H[\delta(x - \alpha, y - \beta)]$

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta$$

If H is position invariant

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta$$



convolution

Image Degradations

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

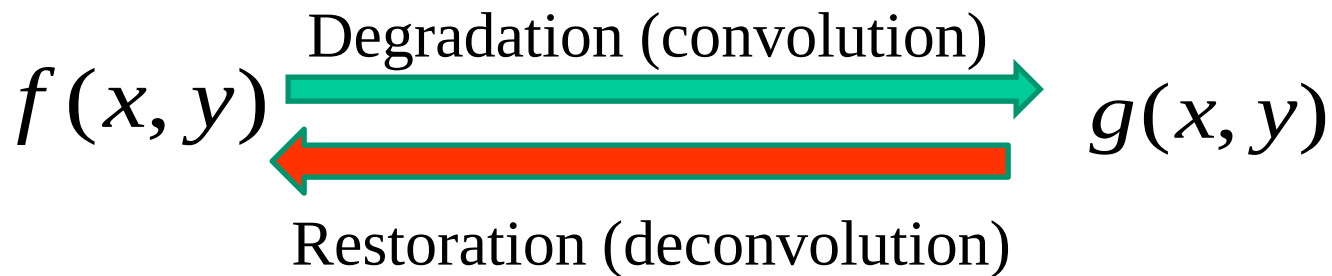


$$g(x, y) = h(x, y) \otimes f(x, y) + \eta(x, y)$$

Degradation VS Restoration

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta + \eta(x, y)$$

Note: a linear, position invariant degradation system with additive noise can be modeled as the convolution of the degradation function with the image plus the additive noise.



Estimate the Degradation Function

- **Observation**
- **Experimentation**
- **Mathematical modeling**

Estimate Degradation Function - Observation

Assumptions:

- The degradation function is linear and position-invariant
- No other knowledge about the degradation function

Estimation by image observation:

- Extract a subimage with strong signal → Higher signal-to-noise ratio
- Perform restoration on the subimage

degradation function in the subimage ← $H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$ → Observed subimage

↓ $H(u, v)$ → Restored subimage

Application: restoring old pictures

Estimate Degradation Function - Experimentation

Assumptions:

- A similar equipment is available
- Change the system setting can achieve similar degraded images

$$H(u, v) = \frac{G(u, v)}{A}$$

→ Observed image
→ Impulse signal

FIGURE 5.24
Degradation estimation by impulse characterization.
(a) An impulse of light (shown magnified).
(b) Imaged (degraded) impulse.

