Announcement

Homework #4 has been posted on Blackboard and course website.

Due 2:20pm, Wednesday, Feb. 28th

Today’s Agenda

- Filtering in Frequency domain

Image Smoothing Using Frequency Domain Filters – Ideal Lowpass Filter

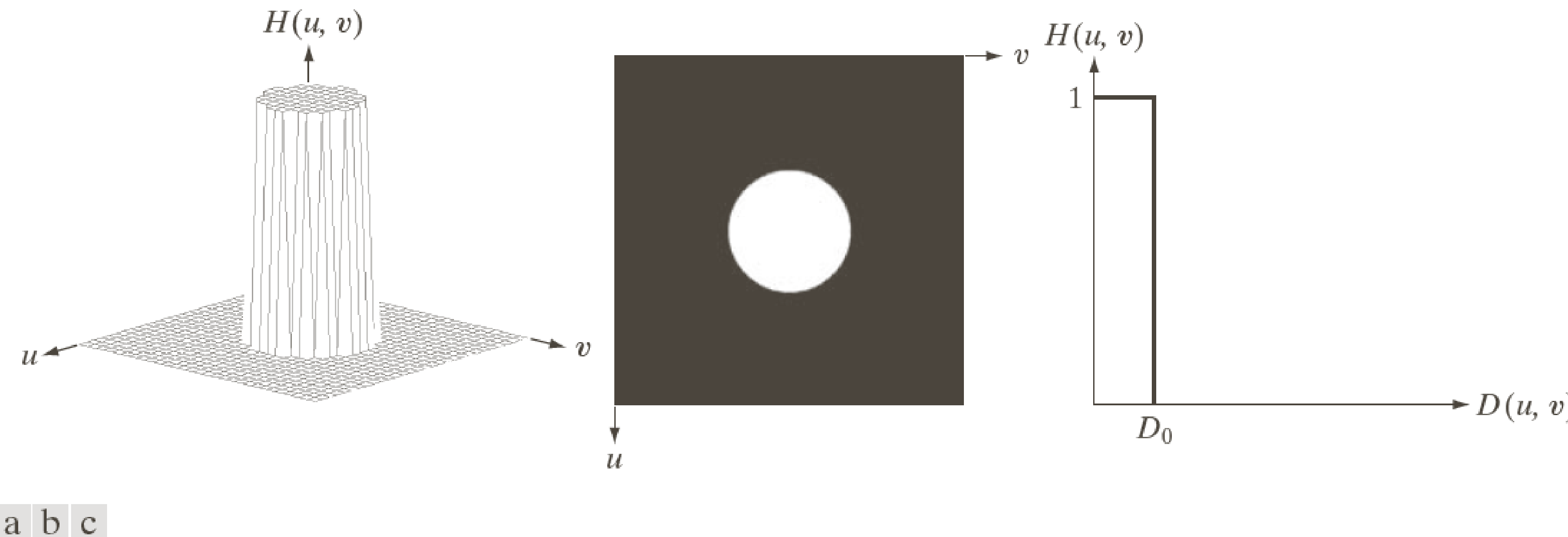
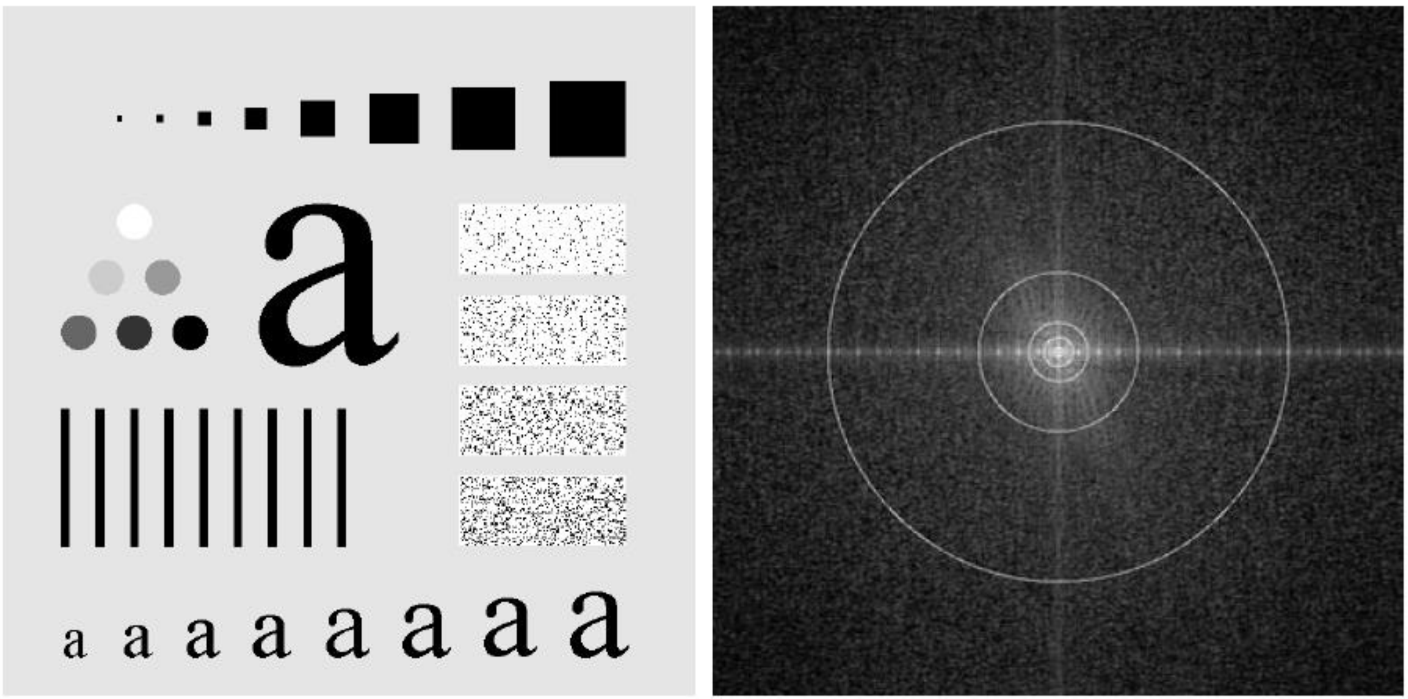


FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

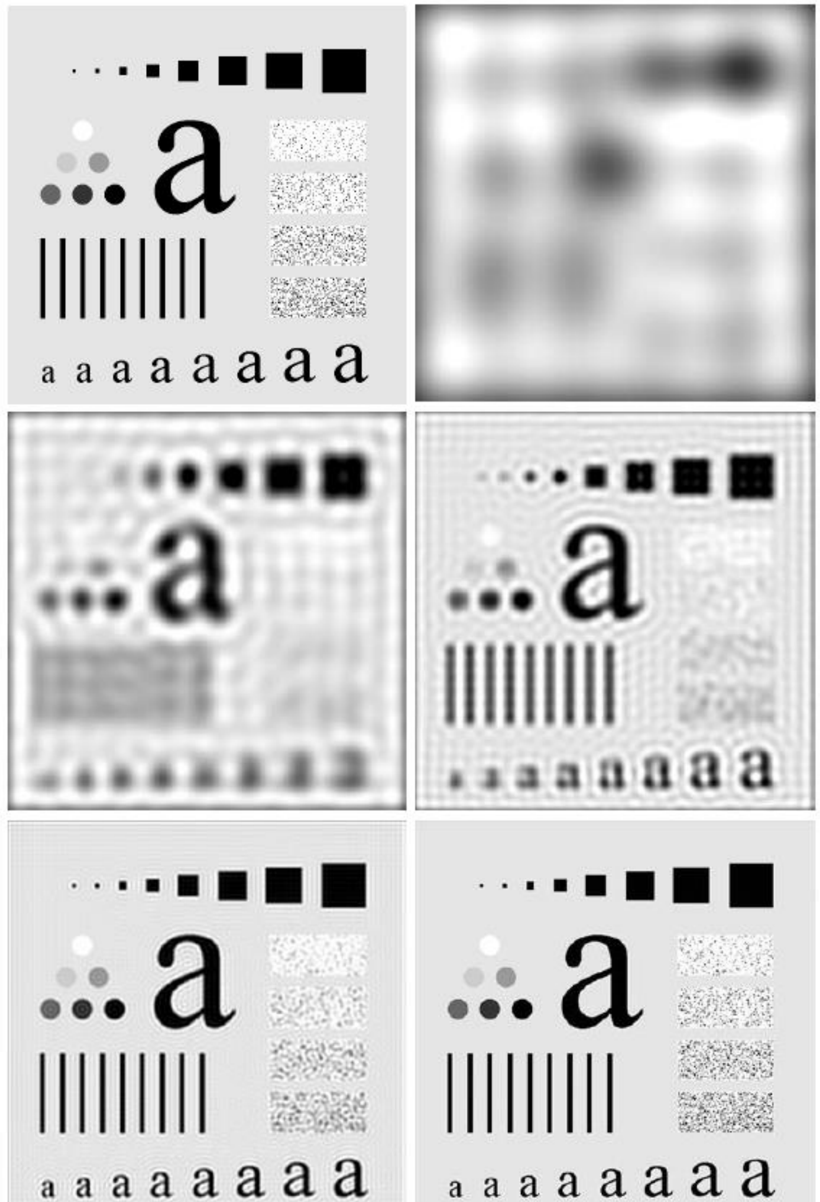
Locating the Cut-Off Frequency



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Applying the ILPF – Blurring and Ringing



Original

ILPF, cutoff 10,
Energy 87%

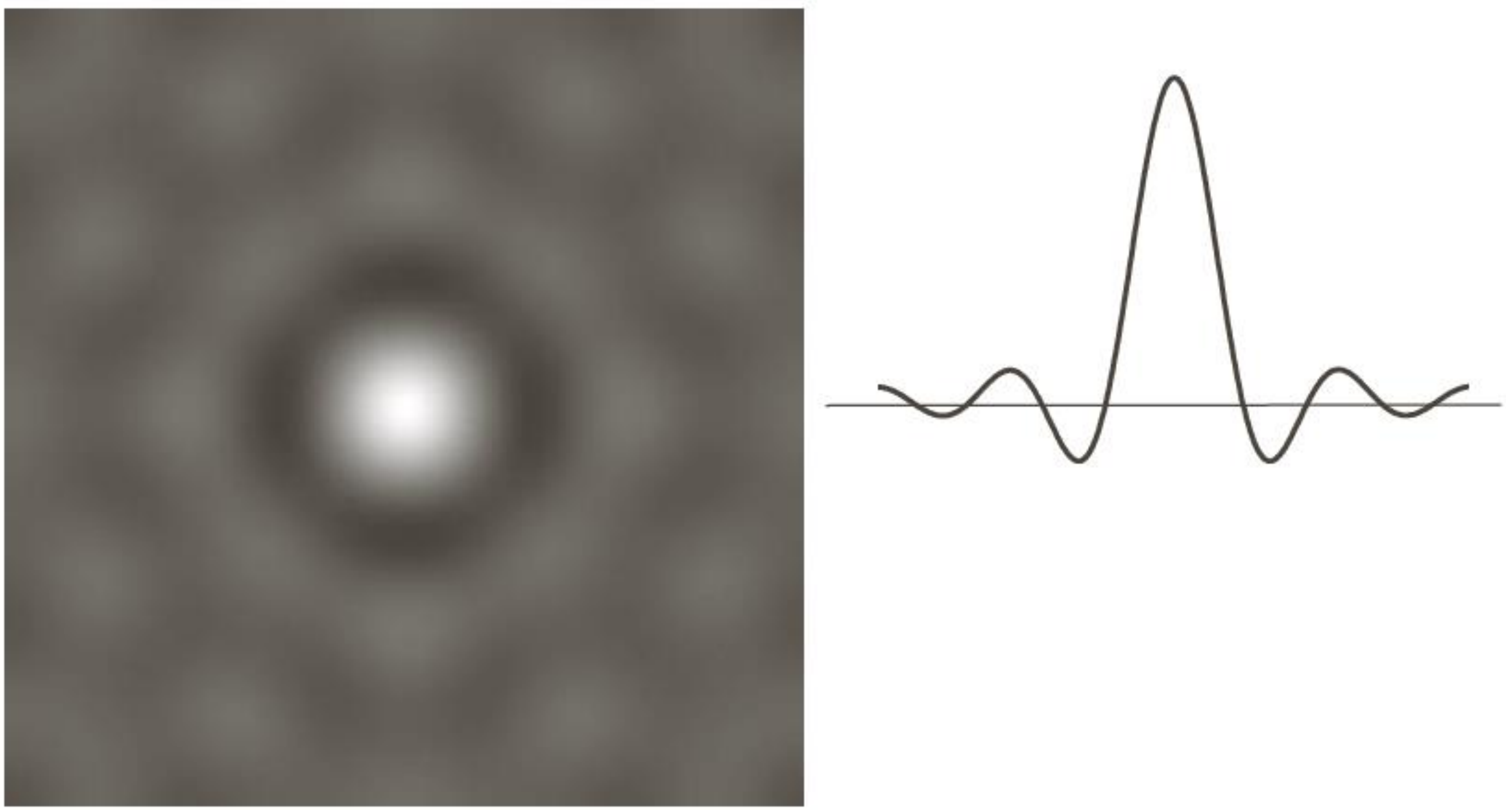
ILPF, cutoff 30
Energy 93.1%

ILPF, cutoff 60
Energy 95.7%

ILPF, cutoff 160
Energy 97.8%

ILPF, cutoff 460
Energy 99.2%

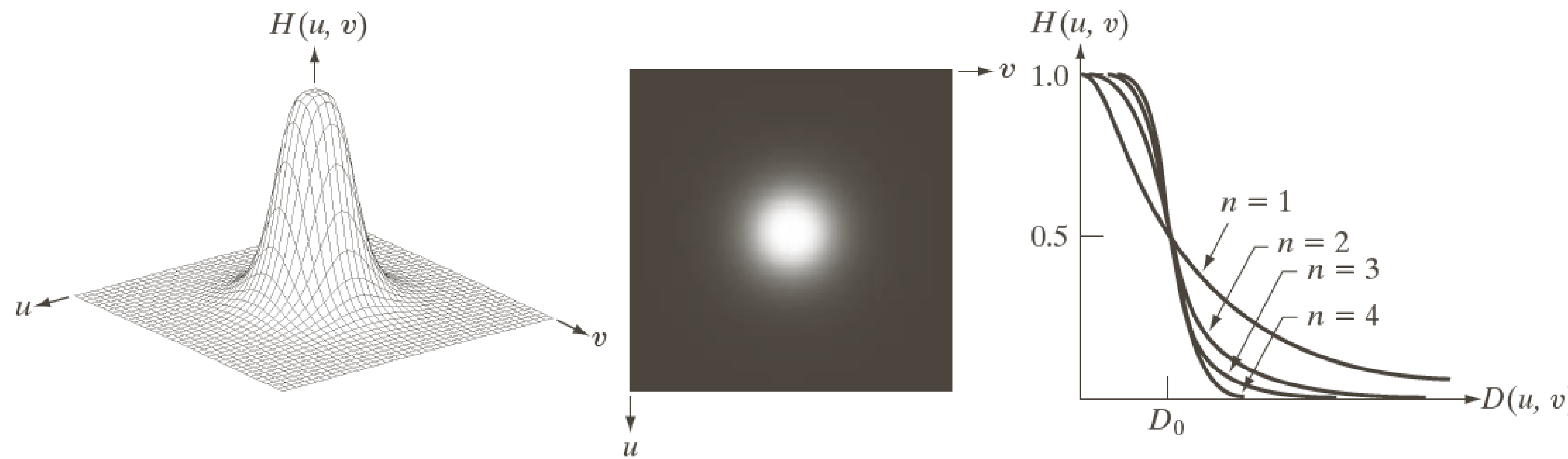
Why?



a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Butterworth Lowpass Filters (BLPF)



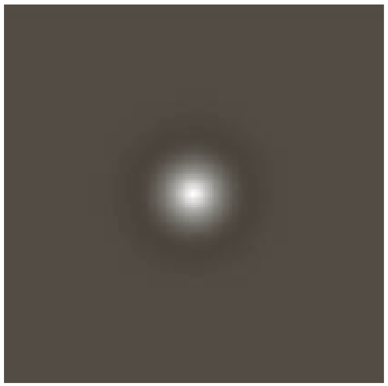
a b c

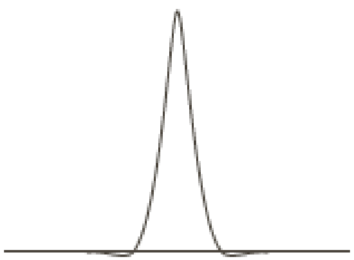
FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$ → order

D_0 is the cutoff frequency

Applying the BLPF





Order 2

Original

ILPF, cutoff 10
Energy 87%

ILPF, cutoff 30
Energy 93.1%

ILPF, cutoff 60
Energy 95.7%

ILPF, cutoff 160
Energy 97.8%

ILPF, cutoff 460
Energy 99.2%

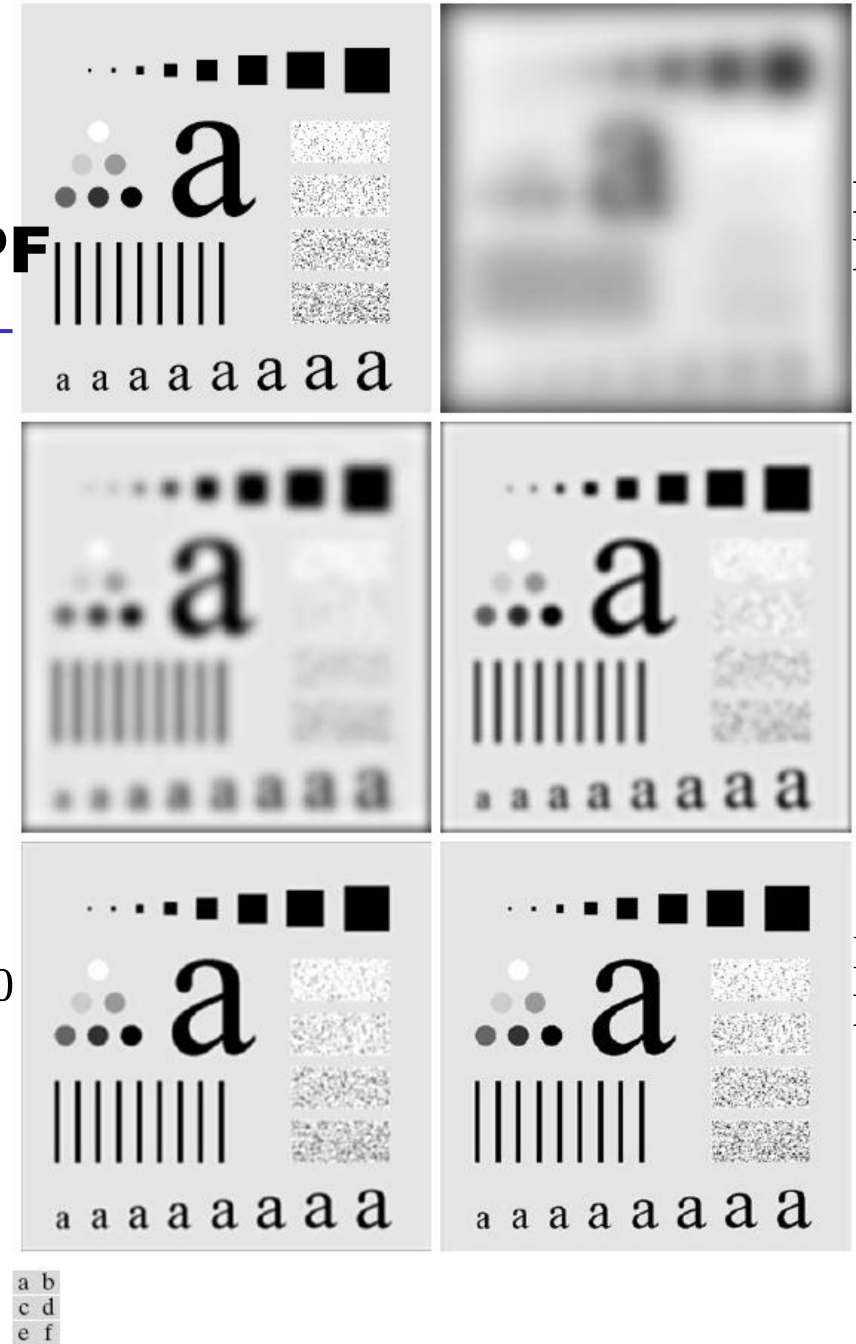


FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Different-Order BLPF

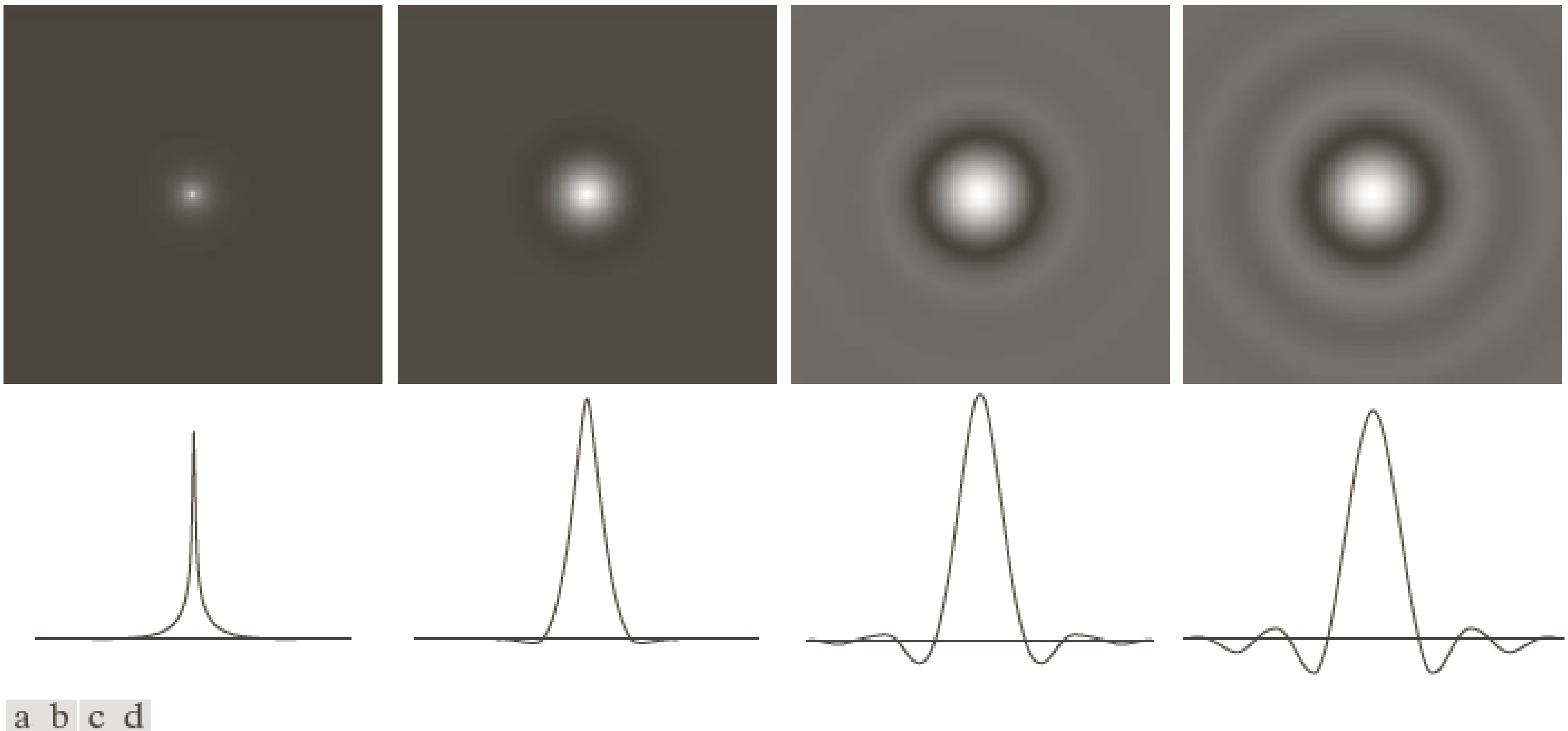
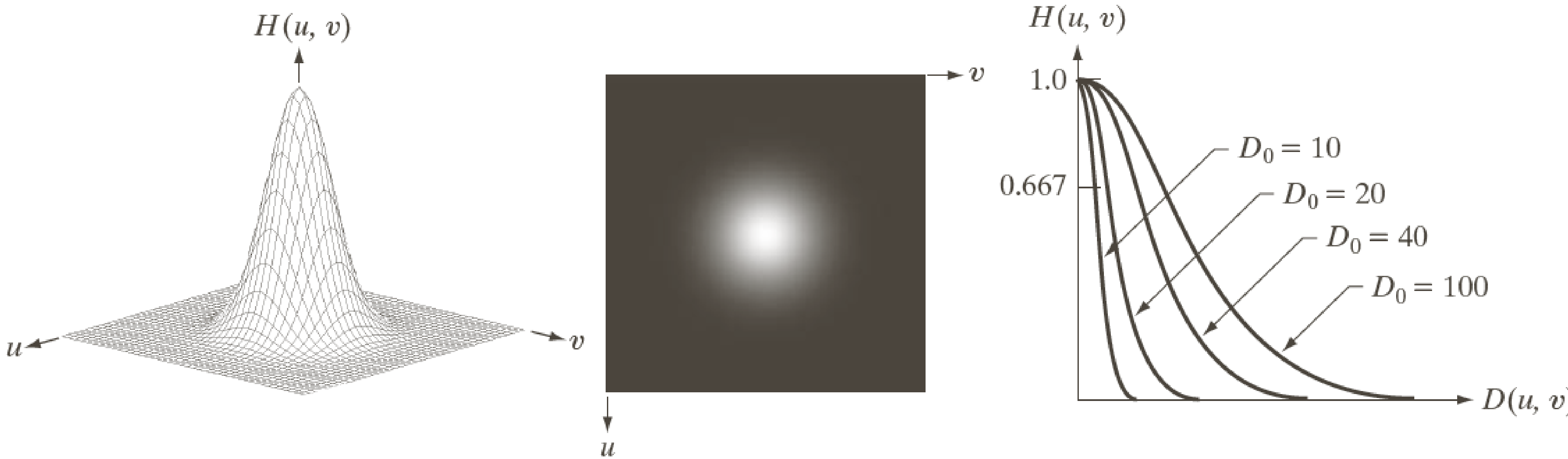


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000×1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Gaussian Lowpass Filters



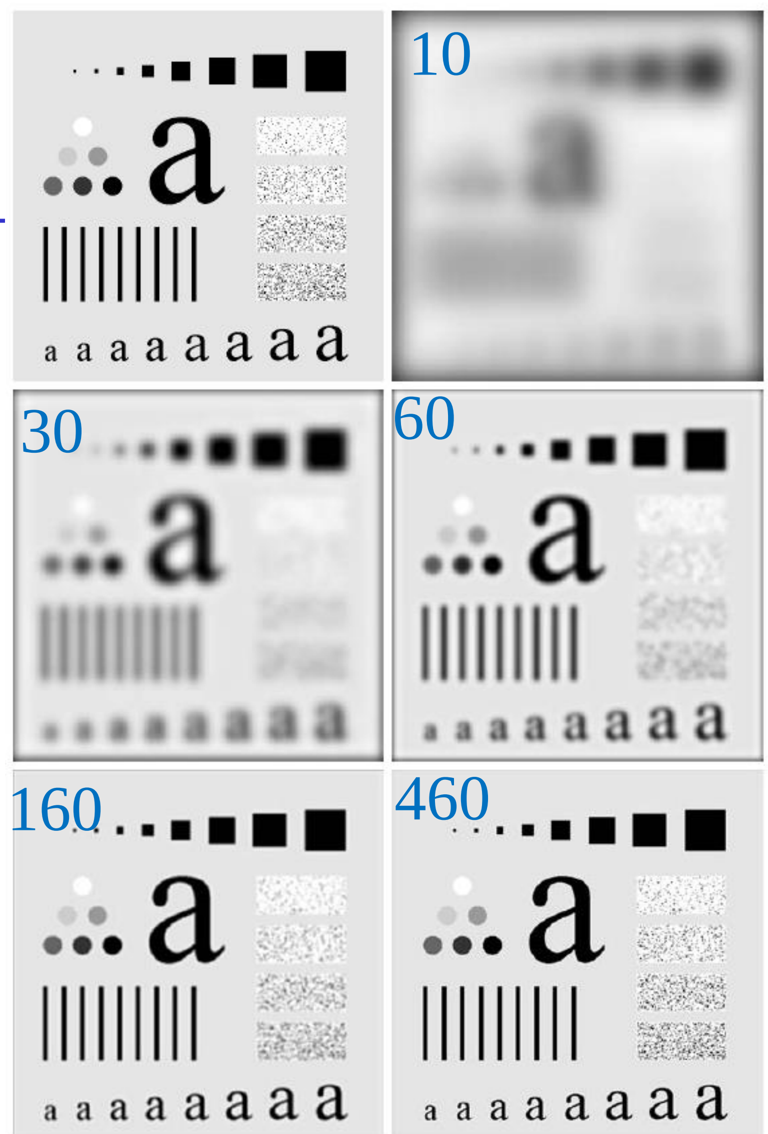
a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Applying the GLPF

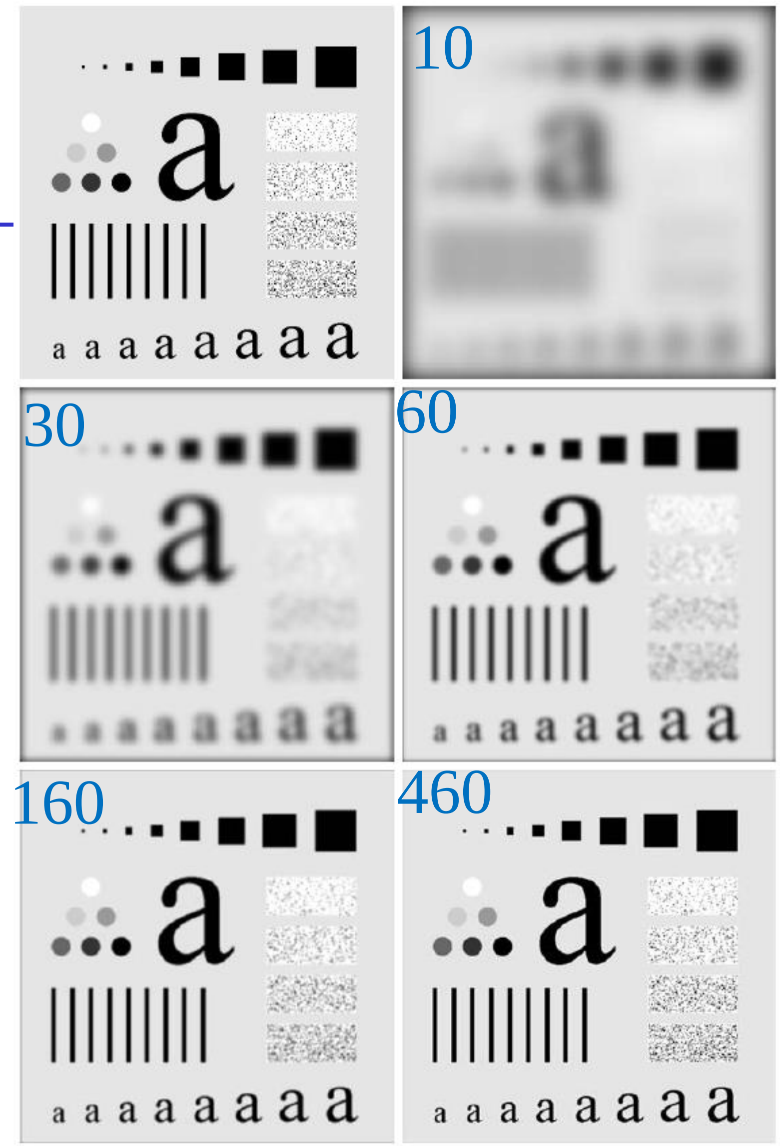
D0 = cutoff radius



a b
c d
e f

BLPF with order 2

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.



a b
c d
e f

GLPF

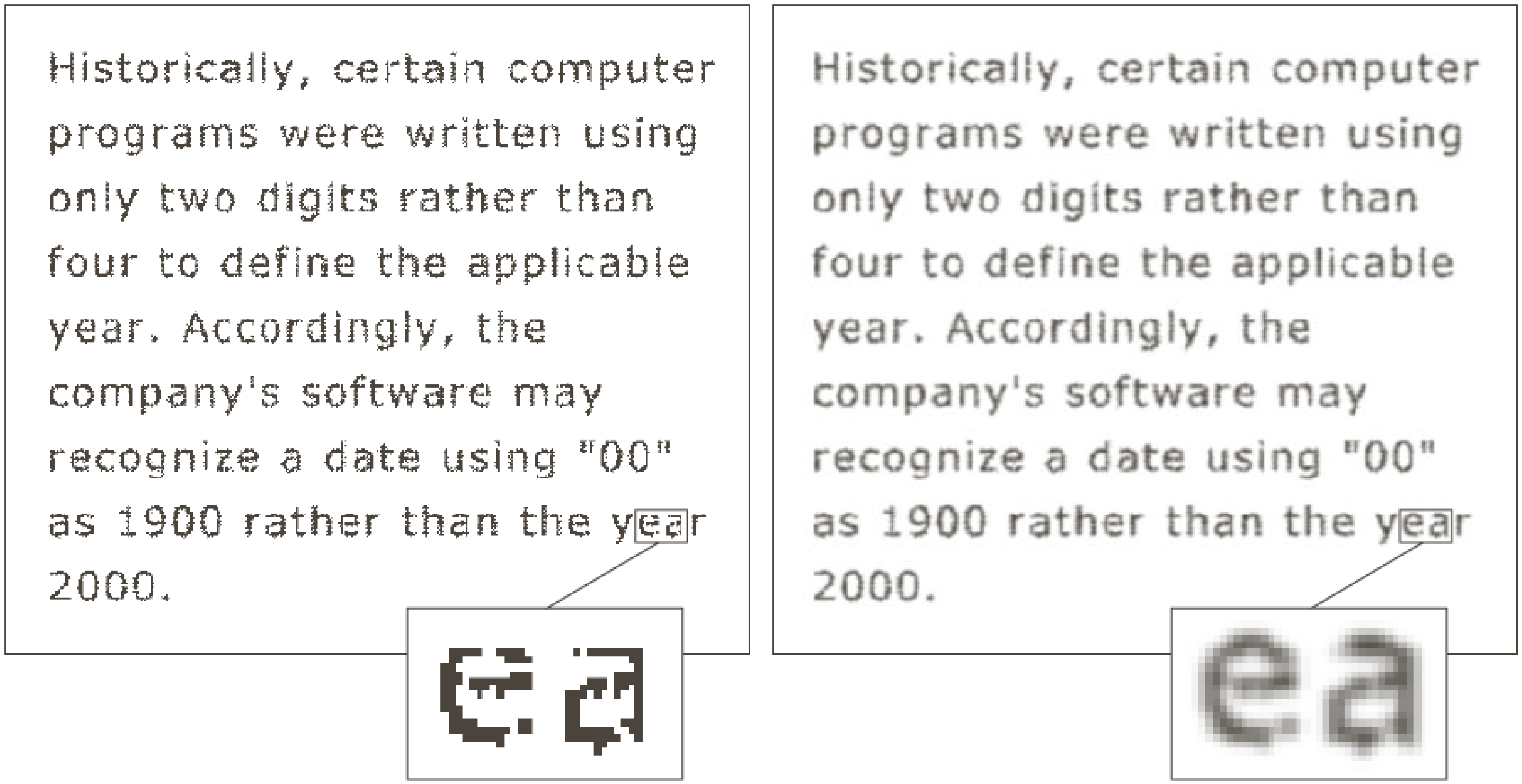
FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Summary of Lowpass Filters

TABLE 4.4
Lowpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \leq D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D(u,v)/D_0]^{2n}}$	$H(u,v) = e^{-D^2(u,v)/2D_0^2}$

Applications



a b

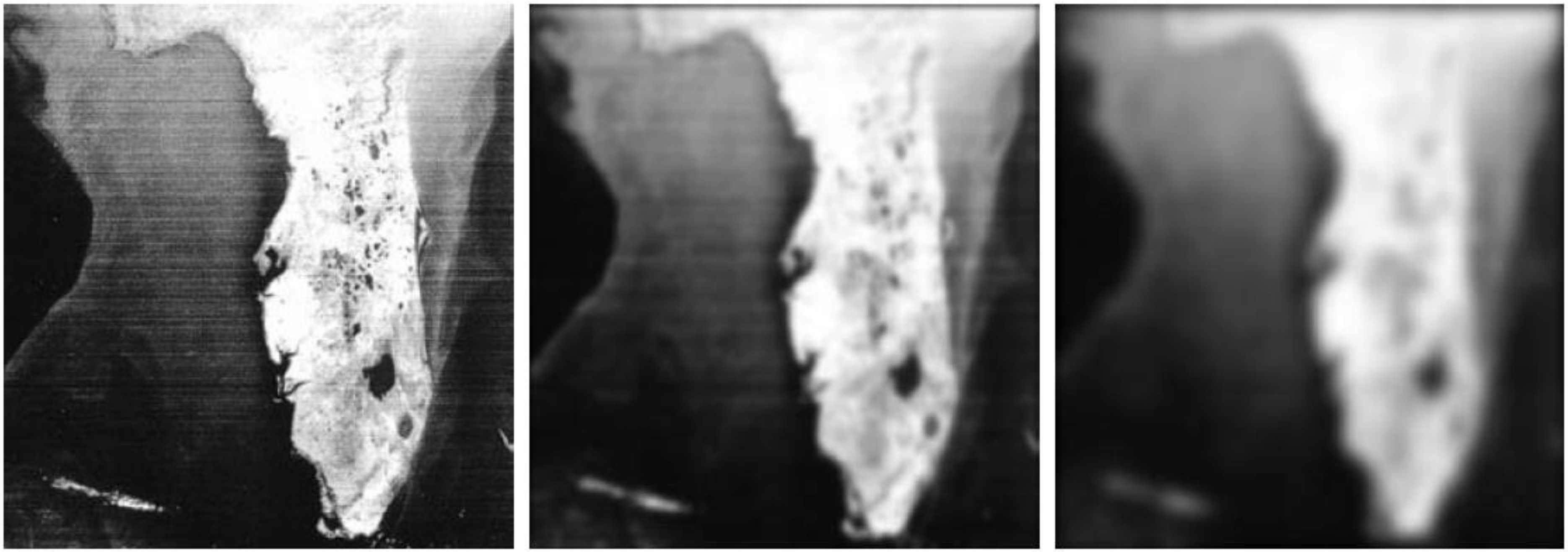
FIGURE 4.49 (a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Applications

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Applications



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Highpass Filters in Frequency Domain

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

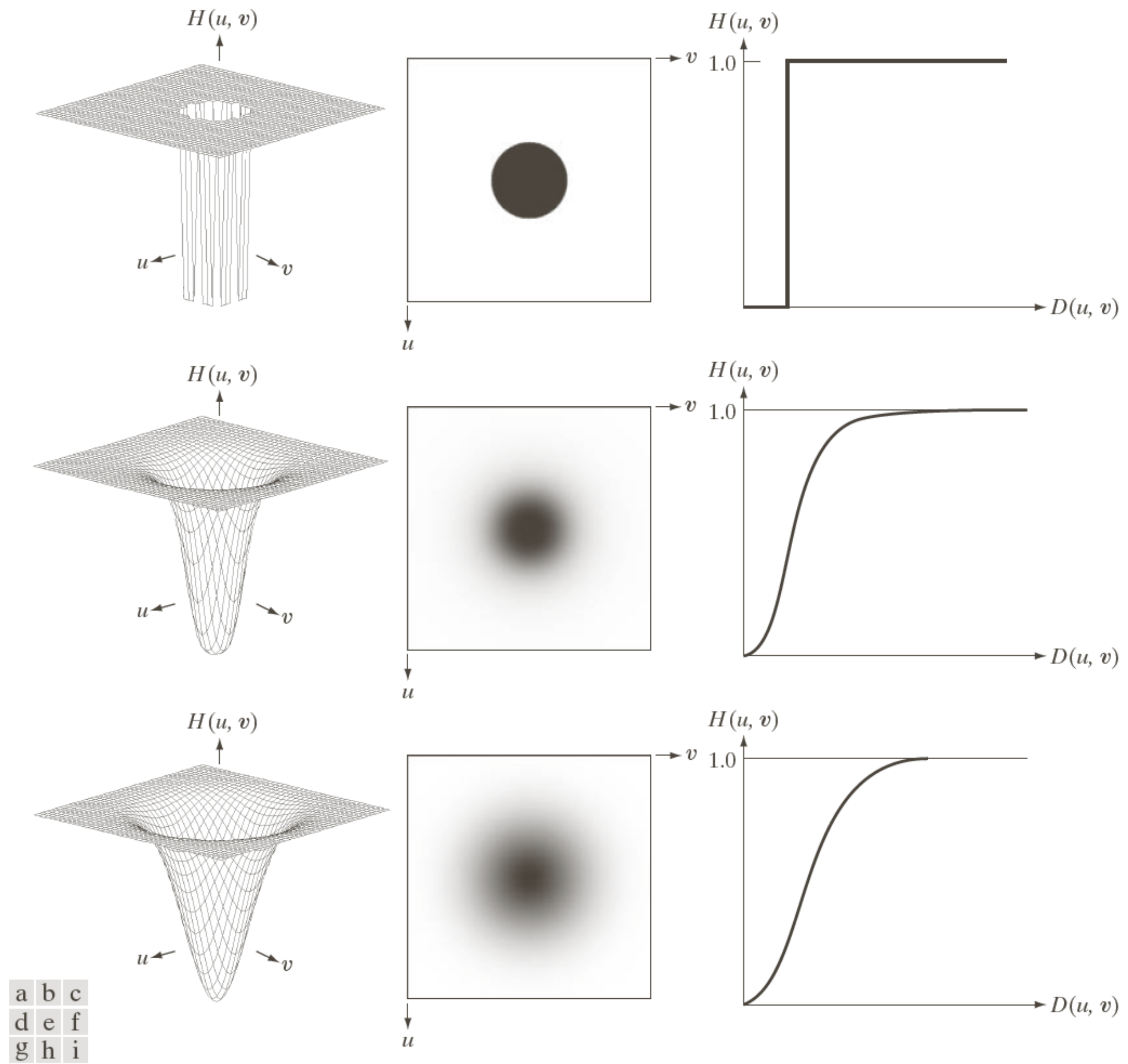


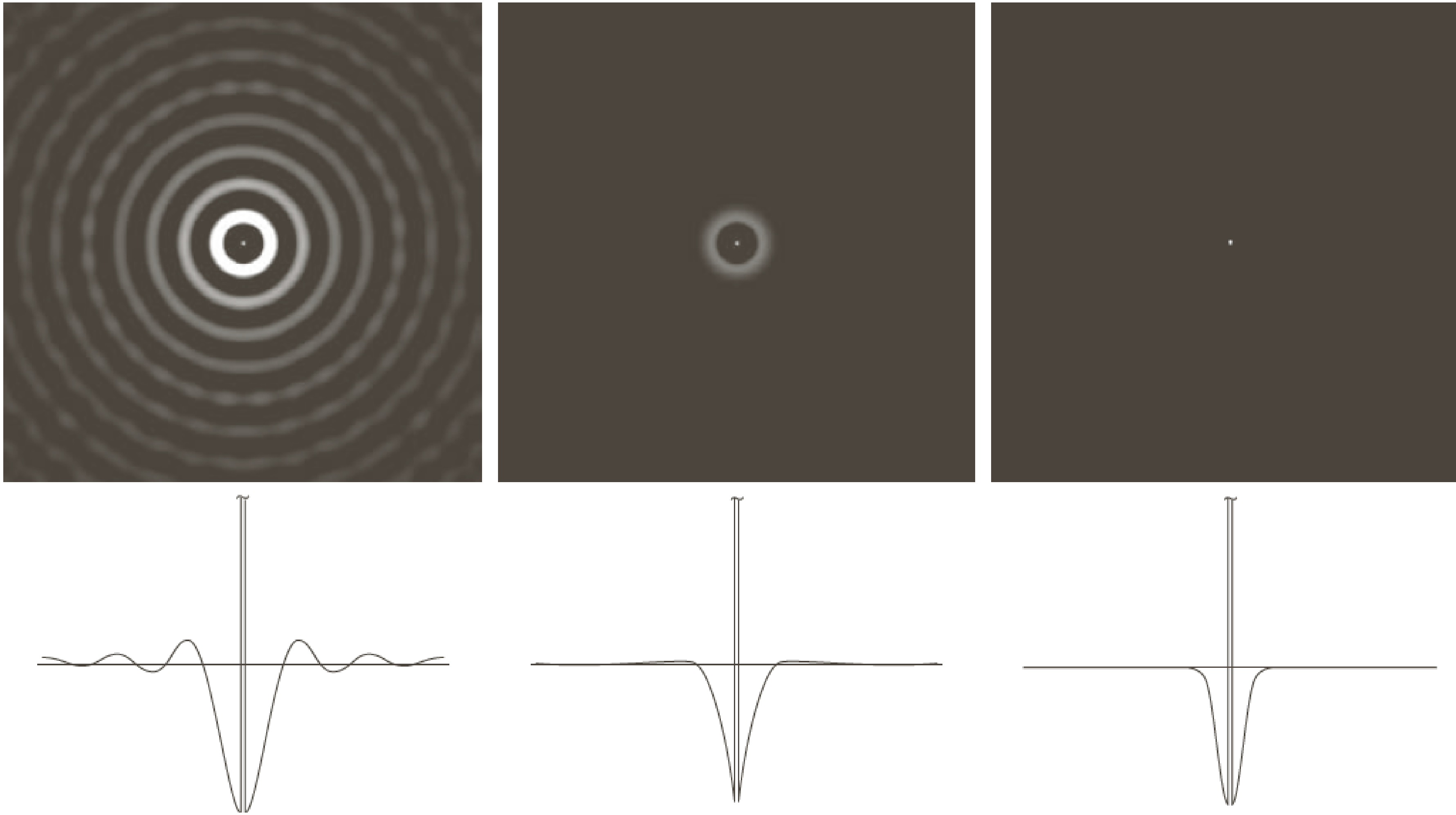
FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Highpass Filters

TABLE 4.5
Highpass filters. D_0 is the cutoff frequency and n is the order of the Butterworth filter.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \leq D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$	$H(u,v) = \dfrac{1}{1 + [D_0/D(u,v)]^{2n}}$	$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$

Highpass Filters In Spatial Domain



a b c

FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Applying IHPF

a b c

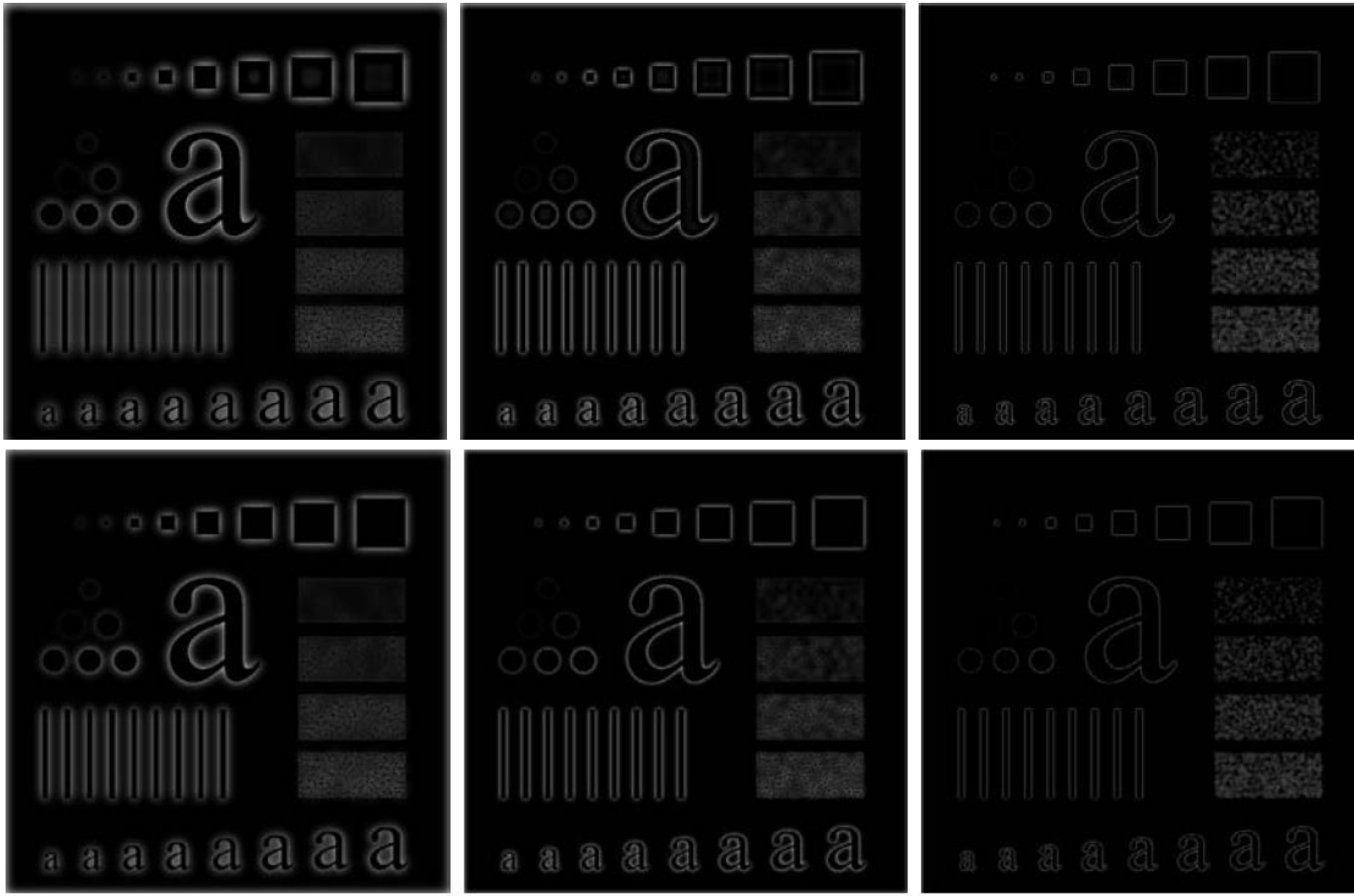
FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60,$ and 160.

Applying BHPF/GHPF

$D_0 = 30$ $D_0 = 60$ $D_0 = 160$

BHPF

GHPF



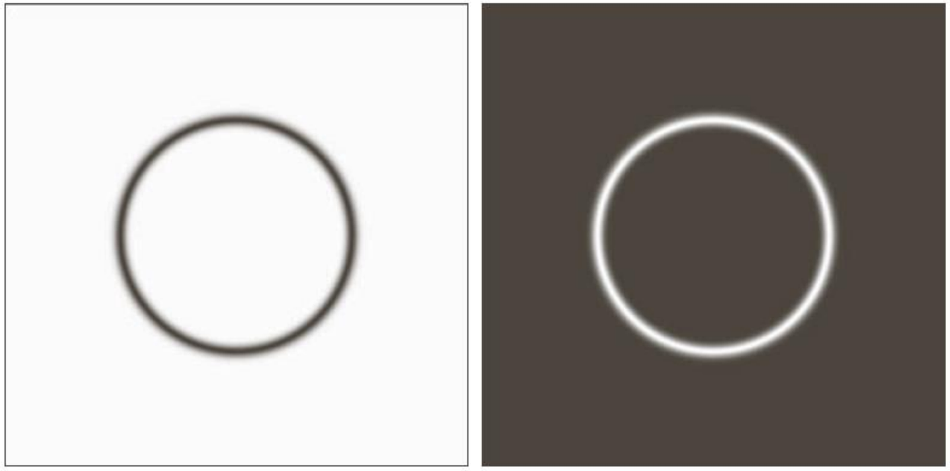
Applications

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Bandreject and Bandpass Filters

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$



a b

FIGURE 4.63
(a) Bandreject Gaussian filter. (b) Corresponding bandpass filter. The thin black border in (a) was added for clarity; it is not part of the data.

Example

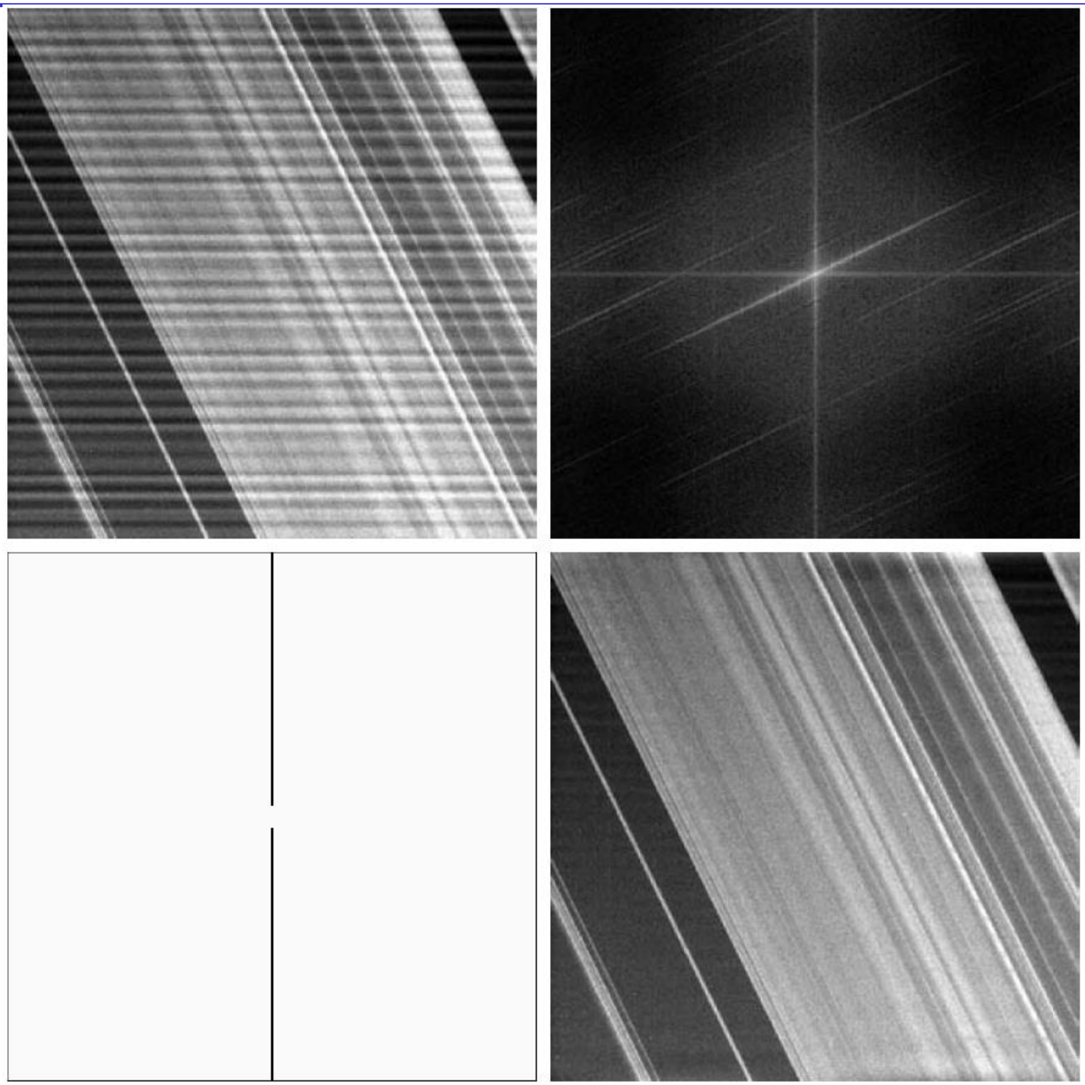
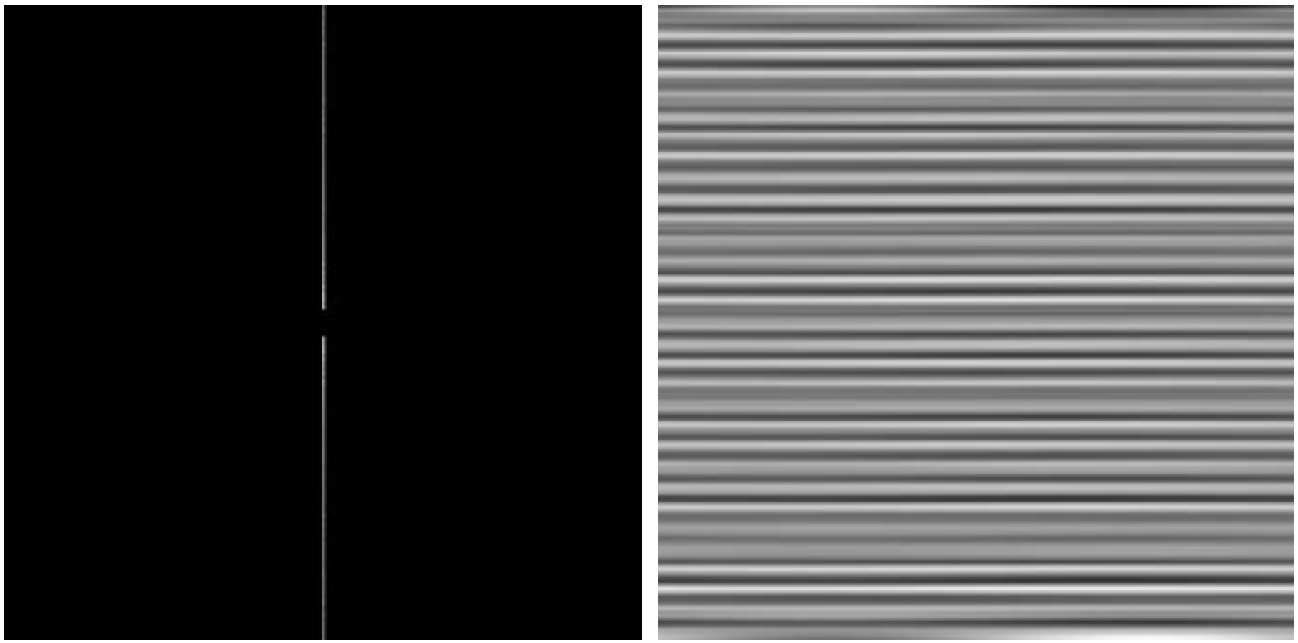


FIGURE 4.65 (a) 674 × 674 image of the Saturn rings showing nearly periodic interference. (b) Spectrum: The bursts of energy in the vertical axis near the origin correspond to the interference pattern. (c) A vertical notch reject filter. (d) Result of filtering. The thin black border in (c) was added for clarity; it is not part of the data. (Original image courtesy of Dr. Robert A. West, NASA/JPL.)

Example



a b

FIGURE 4.66
(a) Result (spectrum) of applying a notch pass filter to the DFT of Fig. 4.65(a). (b) Spatial pattern obtained by computing the IDFT of (a).

Implementation

Separability of the 2D DFT

Computing the IDFT using a DFT algorithm

The Fast Fourier Transform (read Chapter 4.11.3)

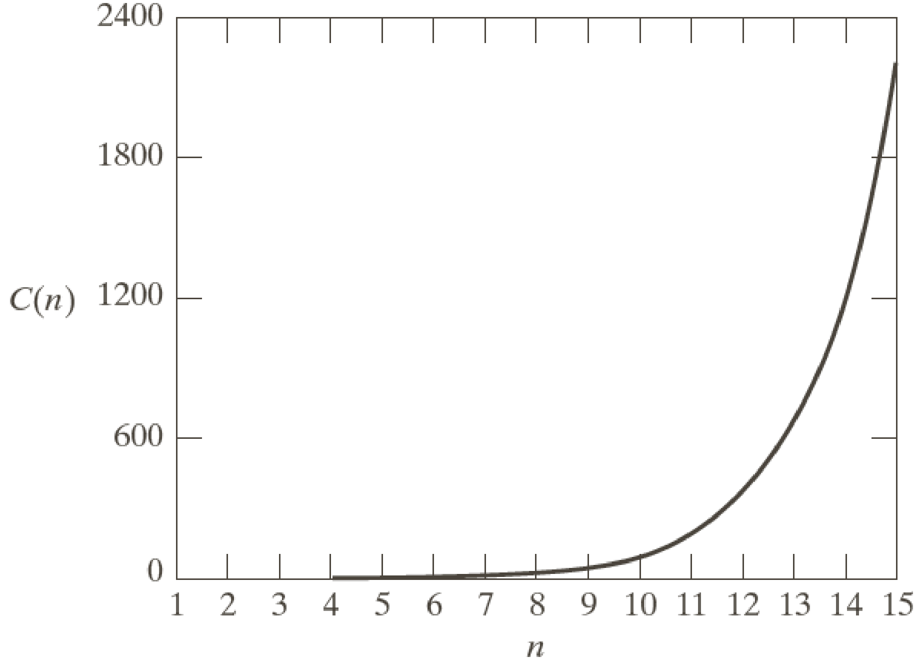


FIGURE 4.67 Computational advantage of the FFT over a direct implementation of the 1-D DFT. Note that the advantage increases rapidly as a function of n.

Reading Assignments

Chapter 4.3 – 4.11

Image Degradation and Restoration

http://fireoracleproductions.com/services__samp

Google image

Image degradation due to

- noise in transmission
- imperfect image acquisition
    - environmental condition
    - quality of sensor

Image Degradation and Restoration

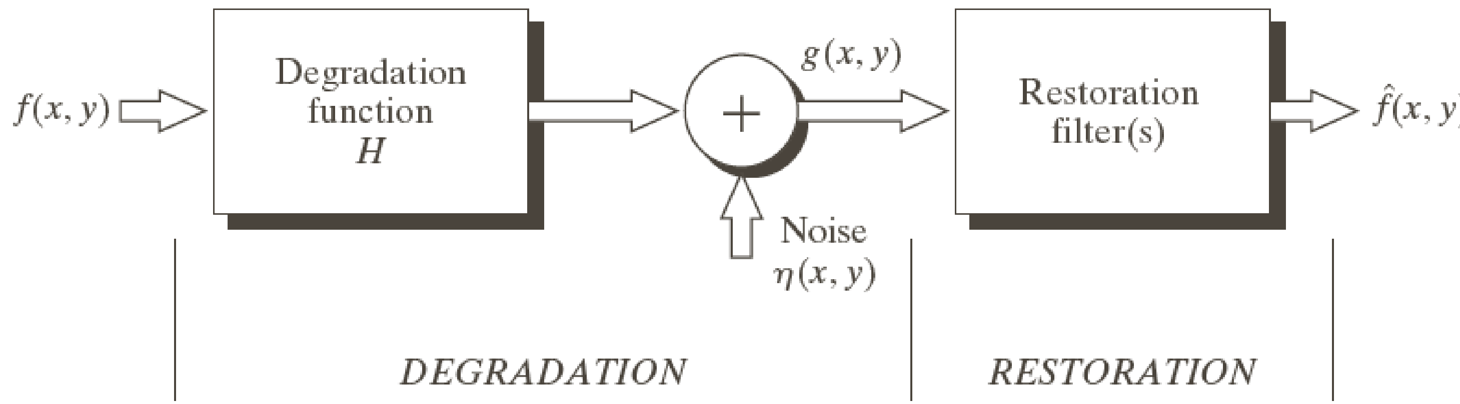


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x,y) = h(x,y) \otimes f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Identity *H*(*u*,*v*) → degradation only comes from additive noise

Restoration in the Presence of Noise Only – Spatial Filtering

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

Image Restoration with Additive Noise

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

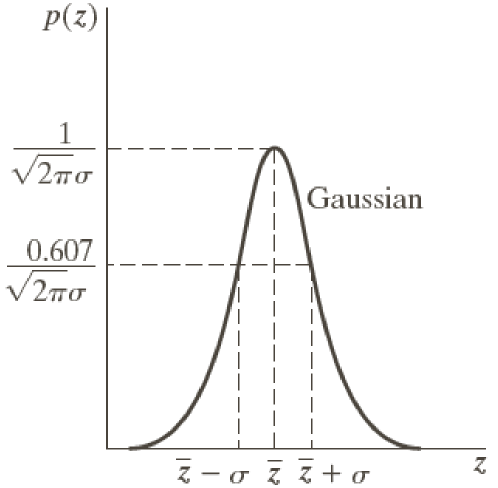
Noise models:

- Impulse noise: pepper and salt
- Continuous noise model:
  - Gaussian, Rayleigh, Gamma, Exponential, Uniform

Properties of Noise

- **Spatial properties**
  - Spatially periodic noise
  - Spatially independent noise
- **Frequency properties**
  - White noise – noise containing all frequencies within a bandwidth

Some Important Noise Model

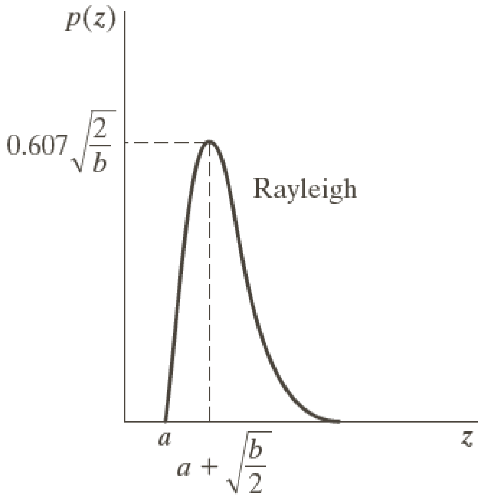


$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}}$$

http://www.gergltd.com/cse486/project2/

- Due to electronic circuit
- Due to the image sensor
    - poor illumination
    - high temperature

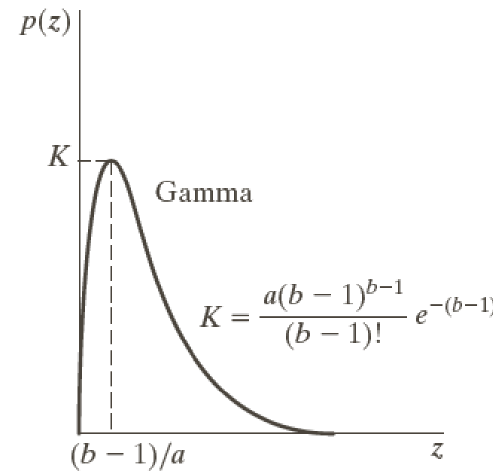
Some Important Noise Model



$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}} & z \geq a \\ 0 & z < a \end{cases}$$

Rayleigh noise

- range imaging
- Background model for Magnetic Resonance Imaging (MRI) images

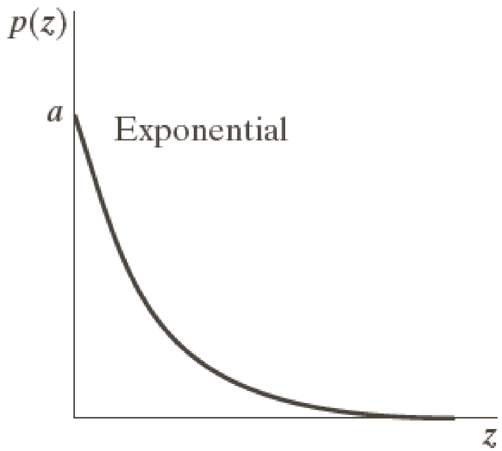


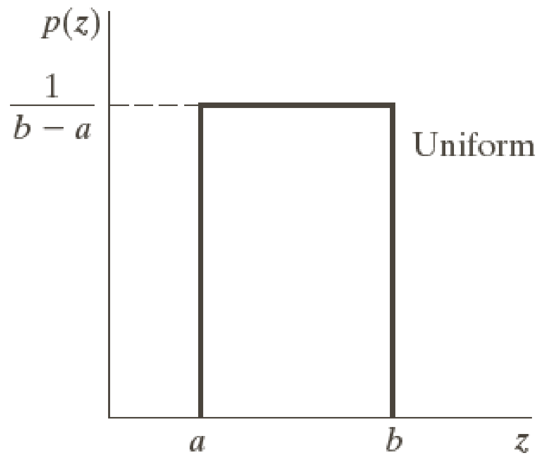
$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

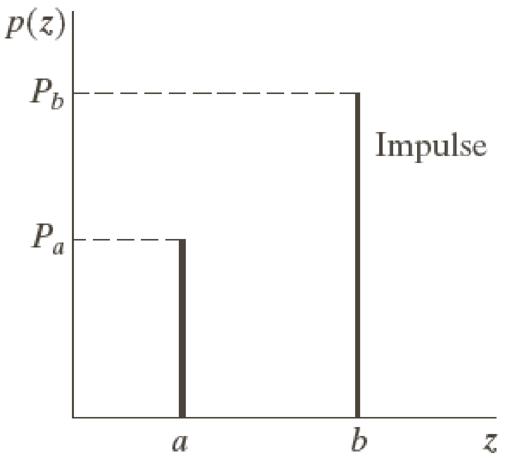
Gamma noise

- laser imaging

Some Important Noise Model







$$P(z) = \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$P(z) = \begin{cases} \frac{1}{b-a} & a \leq z \leq b \\ 0 & otherwise \end{cases}$$

$$P(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{Otherwise} \end{cases}$$

Exponential noise
- laser imaging

Uniform noise

Impulse noise
- salt and pepper noise
- A/D converter error
- bit error in transmission

An Example

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

What is its histogram?

An Example (cont.)

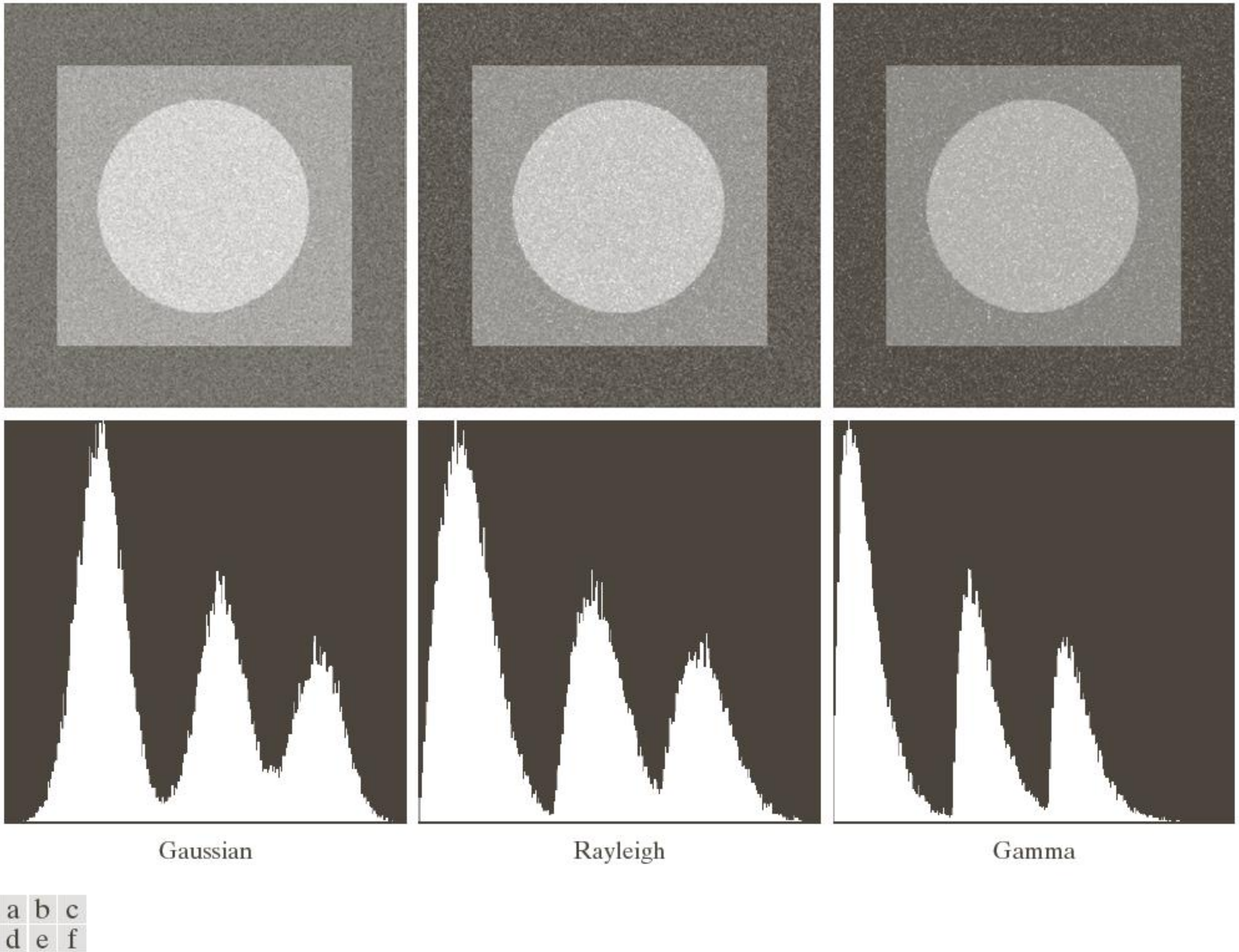


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

An Example (cont.)

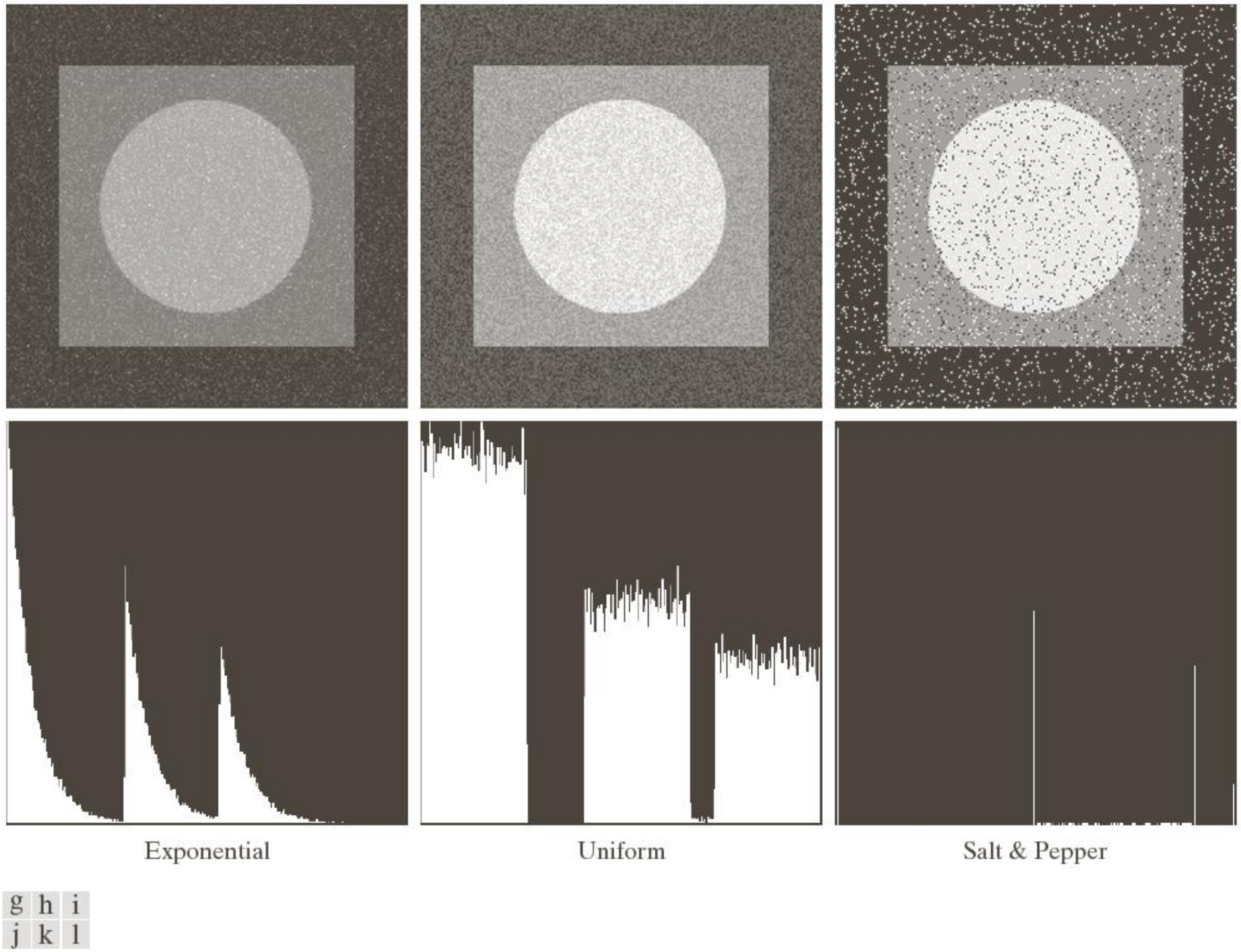


FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Estimation of Noise Parameters

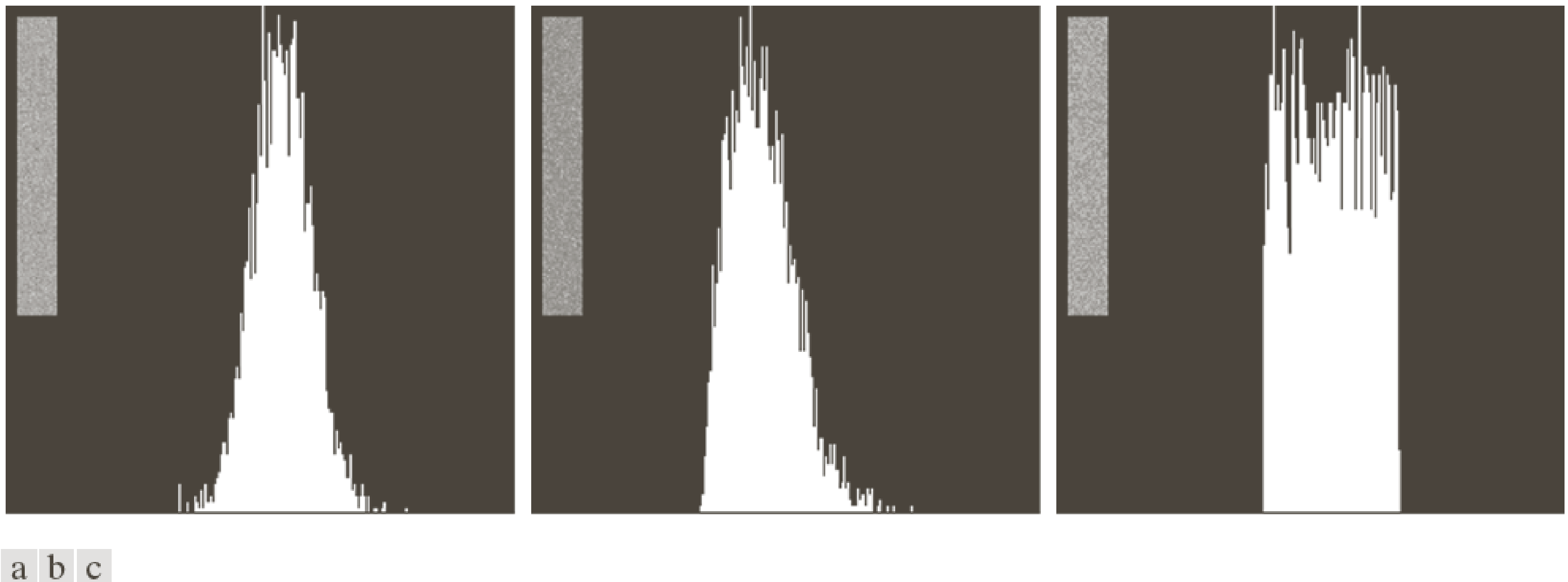


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Take a small stripe of the background, do statistics for the mean and variance.