# Homework #2

## Due on Thursday, February 3 before class starts

**1.** For each of the following functions, find a function $g(n)$ such that $f(n) \in \Theta(g(n))$. You **must use the simplest** $g(n)$ possible in your answers such as $n$, $logn$, $nlogn$, $n^2$, $n^3$, $a^n$, and product of them. Prove your assertion. (20pts)
**Hints:** You can either use the definition of big- $\Theta$ or use the limit.
**a.** $(n^3+1)^2$
**b.** $\sqrt{9n} + 9\log n$
**c.** $2n\log(n^2) + (n+1)^2\log n$
**d.** $3^{n+2} + 4^{n-2}$

**2**. Prove that every polynomial of degree $k$, $p(n) = a_k n^k + a_{k-1}n^{k-1} + \cdots + a_0$ , with $a_k > 0$ belongs to $\Theta(n^k)$. (10pts)
**b**. Prove that exponential functions $a^n$ have different orders of growth for different values of base $a > 0$. (10pts)
**Hints:** Use either definition or limit to prove the assertions.

**3.** Find the order of growth of the following sums. You need to indicate the class $\Theta(g(n))$ the function belongs to. You must use the simplest $g(n)$ possible in your answers. (20pts)

**a.** $\sum_{i=0}^{n}(i^2+1)^2$
**b.** $\sum_{i=1}^{n} \text{n} \ \lg(i^2)$
**c.** $\sum_{i=0}^{n}(i+1)2^i$
**d.** $\sum_{i=0}^{n}\sum_{j=0}^{i-1}(i+j)$
**Hints:** You may refer to **Appendix A** and the properties we discussed in class.

**4** Consider the following algorithm.
**Algorithm** Enigma(A[1...n, 1...n])
//Input: A matrix A[1…n, 1...n] of real numbers
**for** i ← 1 to n **do**
  **for** j ← 1 to n **do**
    **if** A[i, j] ≠ A[j, i]
      **return** false
**return** true
**a.** What does this algorithm compute? (5pts)
**b.** What is its basic operation and what is the efficiency class ($\Theta(g(n))$ the function belongs to) of this algorithm? (5pts)

**c.** Make as many improvements as you can in this algorithm. You must write down the pseudo code for your new algorithm. What is the efficiency class ($\Theta(g(n))$ the function belongs to) of your new algorithm? If you cannot improve this algorithm, explain why you cannot do it. (10pts)

**5.** Solve the following recurrence relations. **Give the particular solution to the problem.**

a. $x(n) = x(n-1) + 3$ for $n > 0, x(0) = 2$ (5pts)
b. $x(n) = x(n-1) + 3n$ for $n > 1, x(1) = 1$ (5pts)
c. $x(n) = x(n/4) + n$ for $n > 1, x(1) = 1$ (solve for $n = 4^k$) (5pts)
d. $x(n) = 2x(n-1) - x(n-2)$ for $n > 1, x(0) = 0, x(1) = 1$ (5pts)