

2013-02-14

Note Title

2013-02-14

Simplified version of Theorem 9.7 [Yeshenko].

Let P be a propositional theory (based on the language $L(P_1)$) whose rules of inference include modus ponens. If the theorems of P include all tautologies and a contradiction, then for all formulas $A \in F(P_1)$, $\vdash_P A$.

Proof.

Let C be a contradiction that is a theorem of P .

- (1) $\vdash_P C$ C is a theorem
- (2) $\vdash_P \neg C$ $\neg C$ is a tautology
- (3) $\vdash_P (\neg C \supset (C \supset A))$. a tautology (since it is lemma 9.1(3))
(comment: $\neg(\neg A \wedge C) \supset A$)
- (4) $\vdash_P (C \supset A)$ m.p. on (2) and (3)
- (5) $\vdash_P A$ m.p. on (1) and (4)

Theorem 9.7 [Yasukawa] (full version) :

Replace "include a contradiction" with
"include more than those formulas that are
tautologies"

Proof Let C be a non-tautology. Then as a theorem

$(C \supset A)$ holds
when either (1) A holds
or (2) $\neg C$ holds

C	A	$C \supset A$
t	t	t
t	f	f
f	t	t
f	f	f

Define B_1, \dots, B_r as in Theorem 9.5 (Lemma 9.1)
for $(c > A)$.

In the first case (A holds)

$B_1, \dots, B_r \vdash_p (c > A)$. (using the construction
of Lemma 9.2)

$B_1, \dots, B_r \vdash_p C$ is a theorem

$B_1, \dots, B_r \vdash_p A$ by m.p

In the second case, (αC holds)

$B_1, \dots, B_K \vdash_p (\alpha \supset A)$ using the construction of
Lemma 9.2

$B_1, \dots, B_K \vdash_p C$ (is a theorem)

$B_1, \dots, B_K \vdash A$ \vdash_p

Now, eliminate B_K using the construction

of Thm. 9.5,

You can eliminate all B_i in the same way
(again, as in Thm. 9.5), \square

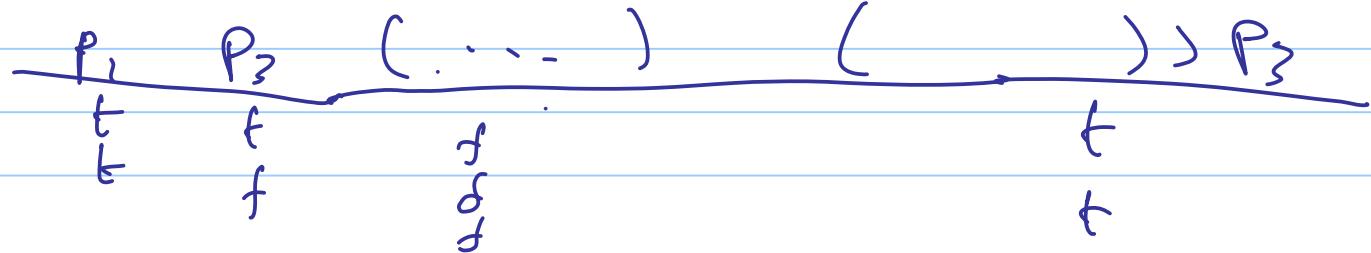
$$C = \neg p_1 \wedge p_1, \text{ and } \vdash_p C$$

Let A be an arbitrary formula, $\uparrow\downarrow A = p_3$

(1) $\vdash_p \neg p_1 \wedge p_1$ (b/c it is a theorem)

(2) $\vdash_p \neg (\neg p_1 \wedge p_1)$ b/c it is a tautology

(3) $\vdash_p ((\neg p_1 \wedge p_1) \wedge \neg (\neg p_1 \wedge p_1)) \Rightarrow p_3$ b/c it is a tautology



f f t
f f

f.

t
t

(4) $\vdash_p ((\neg p_1 \wedge p_1) \Rightarrow (\neg(\neg p_1 \wedge p_1) \Rightarrow p_3))$ b/c it is
a tautology

(5) $\vdash_p (\neg(\neg p_1 \wedge p_1) \Rightarrow p_3)$ m.p on 1, 4

(6) $\vdash_p p_3$ m.p on 2, 5

End of example

Notes from 731, 2011-01-27
as of now.