Theorem 3.27. Let $G = (V, E)$ be a directed acyclic graph (dag).
For any probability distribution $P$ on the variables of $G$, the following conditions are equivalent:

1. $P$ admits a recursive factorization according to $G$.
   \[ P(V) = \prod_{v \in V} P(v | pa(v)) \]

2. $P$ admits the directed global Markov property, relative to $G$.
   If $A$ and $B$ are d-separated by $C$, then $A$ and $B$ are independent given $C$.

3. $P$ admits the directed local Markov property, relative to $G$.
   Each variable in $G$ is independent of its non-descendants given its parents.