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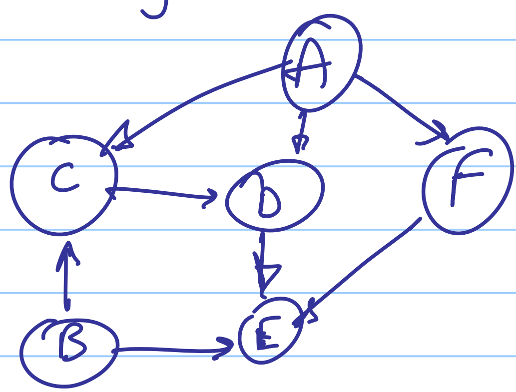
Note Title 2014-02-20

Exercise 2.12

To specify a BN, in addition to its structure (a DAG), one needs to provide conditional probability tables for each variable given its parents.



$P(A)$

$P(B)$

$P(C \mid A, B)$

$P(D \mid A, C)$

$P(E \mid B, D, F)$ $P(F \mid A)$

Bad error is to write $P(C|A)$ and $P(E|B)$,

or $P(E|B)$, $P(E|D)$, $P(E,F)$

Exercise 2.14

$P(A,B,C,D,E,F) = P(A)P(B)\,P(C|A,B)\,P(D|A,C)\,P(E|B,D,F)$
$P(F|A)$ $\left(= P(V) = \prod_{v \in V} P(v|pa(v))\right)$

$= (*)$

You need to show that B and D are conditionally independent given A and C.

You may do so by showing that:

(1) $P(B \mid D, A, C) = P(B \mid A, C)$
(2) $P(D \mid B, A, C) = P(D \mid A, C)$ } defn.

(3) $P(B, D \mid A, C) = P(B \mid A, C)\, P(D \mid A, C)$] necessary and sufficient condition

We are going to show 1 in two steps

(a) compute $P(A, B, C, D)$

(b) compute $P(B \mid D, A, C) = \dfrac{P(B, C, D, A)}{P(A, C, D)}$ and show that it is equal to $P(B|A,C) = \dfrac{P(A,B,C)}{P(A,C)}$.

(a) $P(A,B,C,D) = \sum_E \sum_F P(A,B,C,D,E,F) =$

$= \sum_E \sum_F (*) = \sum_E \sum_F P(A)P(B)P(C|A,B)P(D|A,C)$
$P(E|B,D,F)P(F|A) =$

$= P(A)P(B)P(C|A,B)P(D|A,C) \sum_E \sum_F P(E|B,D,F)P(F|A) =$

$= P(A)P(B)P(C|A,B)P(D|A,C) \sum_F \sum_E P(F|A)P(E|B,D,F)$.

$$= P(A)P(B)P(C|A,B)P(D|A,C)\sum_F P(F|A)\sum_E P(E|B,D,F) =$$

$$= P(A)P(B)P(C|A,B)P(D|A,C)\sum_F P(F|A) =$$

$$= P(A)P(B)P(C|A,B)P(D|A,C)$$

(b) $$P(B|A,C,D) = \frac{P(A,B,C,D)}{P(A,C,D)} = \frac{P(A)P(B)P(C|A,B)P(D|A,C)}{\sum_B P(A)P(B)P(C|A,B)P(D|A,C)} =$$

$$\sum_B P(A,B,C,D) = P(A,C,D)$$

$$= \frac{\cancel{P(A)}P(B)P(C|A,B)\cancel{P(D|A,C)}}{\cancel{P(A)}\cancel{P(D|A,C)}\sum_C P(B)P(C|A,B)} = \frac{P(C|A,B)P(B)}{\sum_B \underbrace{P(B)P(C|A,B)}} =$$

$$= \frac{P(B,C|A)}{\sum_B P(B,C|A)} = \frac{P(B,C|A)}{P(C|A)} = P(B|A,C) \checkmark$$

<u>Note</u> how much easier to show that $\{A,C\}$ d-separates B from D.