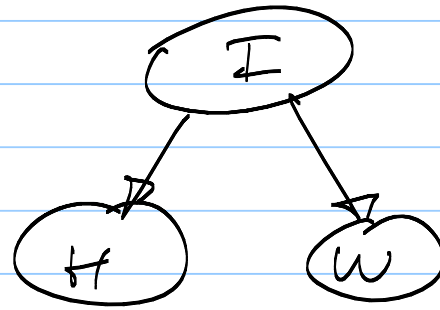
582 2012-01-24 (edited 2014-02-11)

Note Title 2012-01-19

[first part from 1-19]

Icy Roads,

(In this variation, Holmes and Watson are equally bad drivers.)



$P(I) = (\underset{y}{0.7}, \underset{n}{0.3})$

$P(H|I) =$

H \ I	y	n
y	0.8	0.1
n	0.2	0.9

$P(W|I) =$

W \ I	y	n
y	0.8	0.1
n	0.2	0.9

Compute the initial probabilities of W and I.

$P(H,I) = P(H|I)\,P(I) =$

H \ I	y	n
y	0.56	0.03
n	0.14	0.27

$= P(W,I)$

Check: $\sum_H \sum_I P(H,I) = 1$ ✓

So, the initial probability of the crashes is.

$P(H) = (0.59, 0.41) = \sum_I P(H, I) = P(W)$

Check: $\sum_H P(H) = 1$ ✓

Now, we update $P(I)$ using the information (evidence) that Watson crashed $(W = y)$

$P(I \mid W=y) =$ (use Bayes' rule) $= \dfrac{P(W=y \mid I)\, P(I)}{P(W=y)} =$

$= \dfrac{1}{0.59} \times \overset{\quad y \qquad\; n}{(0.8, 0.1)} \times \overset{\quad y \qquad\; n}{(0.7, 0.3)} = \dfrac{1}{0.59} \times (0.56, 0.03) =$

pointwise table multiplication

$= (0.95,\ 0.05)$ (approx.) = "updated $P(I)$"

Now, we update the probability that Holmes crashed.

$P(H|I) \times P(I)$ to compute a joint probability. Then, marginalize to H: $\sum_I P(H,I)$

$P(H,I) =$

H \ I	y	n
y	0.8×0.95	0.1×0.05
n	0.2×0.95	0.9×0.05

$\sum_I P(H,I) = (0.765, 0.235)$

Smith's belief that Holmes crashed based on Watson's having crashed.

Then Smith is told that the roads are not icy

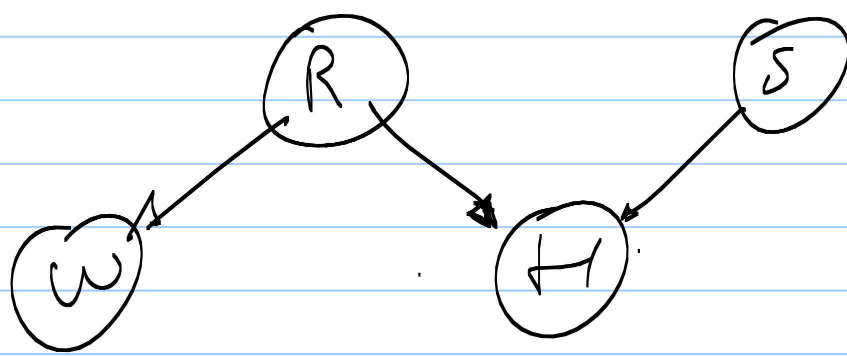
So, $P(H \mid I=n) = (0.1, 0.9)$

$P(W|R) =$

W \ R	y	n
y	1	0.2
n	0	0.8

$P(H|R,S) =$

S \ R	y	n
y	(1, 0)	(0.9, 0.1)
n	(1, 0)	(0, 1)



$P(R) = (0.2, 0.8)$ (y, n)

$P(S) = (\quad , 0.9)$

Wet grass example - computations using tables

The initial (prior, i.e. in the absence of evidence) prob. that Watson's lawn is wet is obtained from

$P(R,W) = P(W|R)\,P(R)$ and marginalization

$$P(W) = \sum_R P(R,W)$$

$$P(W,R) = \begin{array}{c|cc} W \backslash R & y & n \\ \hline y & 1 & 0.2 \\ n & 0 & 0.8 \end{array} \times \begin{array}{cc} y & n \\ (0.2 & 0.8) \end{array} = \begin{array}{c|cc} W \backslash R & y & n \\ \hline y & 0.2 & 0.16 \\ n & 0 & 0.64 \end{array}$$

$$P(W) = \sum_R P(R,W) = (\overset{y}{0.36}, \overset{n}{0.64})$$

Do the same for $P(H)$ (the prior probability that Holmes's lawn is wet).

$$P(H,R,S) \underset{\substack{\uparrow \\ \text{chain rule}}}{=} P(H|R,S)\,P(R,S) \underset{\substack{\uparrow \\ \text{independence of } R \text{ and } S \\ \text{in the absence of} \\ \text{evidence on } H \text{ or} \\ \text{of its descendents}}}{=} P(H|R,S)\,P(R)\,P(S)$$

$$P(H,R,S) = \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & (1,0) & (0.9,0.1) \\ n & (1,0) & (0,1) \end{array} \times \overset{\begin{array}{cc} y & n \end{array}}{(0.2,\ 0.8)} \times \overset{\begin{array}{cc} y & n \end{array}}{(0.1,\ 0.9)} =$$

$$\begin{array}{c|cc} S \backslash R & y & n \\ \hline y & (0.02,0) & (0.072,0.008) \\ n & (0.18,0) & (0,0.72) \end{array}$$

$$P(H) = \sum_{R,S} P(H,R,S) = \overset{\begin{array}{cc} y & n \end{array}}{(0.272,\ 0.728)}$$

Compute $P(R,S \mid H=y)$ and from this $P(R \mid H=y)$ and $P(S \mid H=y)$

$$P(R,S \mid H=y) = \frac{P(R,S,H)}{P(H=y)} = \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & \frac{0.02}{0.272} & \frac{0.072}{0.272} \\ n & \frac{0.18}{0.272} & \frac{0}{0.272} \end{array} \approx \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & 0.074 & 0.265 \\ n & 0.662 & 0 \end{array}$$

$$P(R \mid H=y) = \sum_S P(R,S \mid H=y) = (0.736,\ 0.265) \text{ (approx)}$$

$$P(S \mid H=y) = \sum_R P(R,S \mid H=y) = (0.339,\ 0.662) \text{ (approx)}$$

Update $P(W)$, i.e., compute $P(W \mid H=y)$

$$P(W,R \mid H=y) = P(W \mid R, H=y)\, P(R \mid H=y) = P(W \mid R)\, P(R \mid H=y) =$$

chain rule (conditional version with context $H=y$)

W is independent of H given R

$$= \begin{array}{c|cc} W \backslash R & y & n \\ \hline y & 1 & 0.2 \\ n & 0 & 0.8 \end{array} \quad \begin{array}{cc} \multicolumn{2}{c}{R} \\ y & n \\ \multicolumn{2}{c}{(0.736,\ 0.265)} \end{array} \quad = \begin{array}{c|cc} W \backslash R & y & n \\ \hline y & .736 & .053 \\ n & 0 & .2120 \end{array}$$

$$P(W \mid H=y) = \sum_R P(W, R \mid H=y) = \begin{array}{cc} y & n \\ (.789, & .212) \end{array}$$

The evidence $W=y$ arrives. We compute $P(R \mid W=y, H=y)$ using Bayes' rule

$$P(R \mid W=y, H=y) = \frac{P(W \mid H=y, R)\, P(R \mid H=y)}{P(W \mid H=y)} =$$

$$= \frac{P(W \mid R)\, P(R \mid H=y)}{P(W \mid H=y)} = \begin{array}{cc} y & n \\ (0.93, & 0.07) \end{array}$$

Holmes also wants $P(S \mid H=y, W=y)$.

Compute $P(R, S \mid H=y, W=y)$

$$P(S \mid R, H=y) = \frac{P(R, S \mid H=y)}{P(S \mid H=y)} = \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & \frac{0.074}{0.736} & \frac{0.265}{0.265} \\ n & \frac{0.662}{0.736} & \frac{0}{0.265} \end{array} =$$

$$= \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & 0.1 & 1 \\ n & 0.9 & 0 \end{array}$$

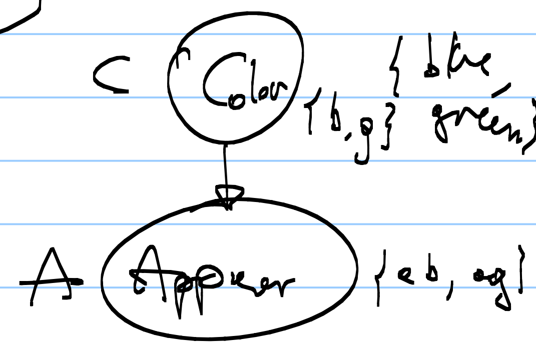
chain rule

$$P(R, S \mid H=y, W=y) = P(S \mid R, H=y, W=y)\, P(R \mid H=y, W=y) =$$

$$= P(S \mid R, H=y)\, P(R \mid H=y, W=y) = \begin{array}{c|cc} S \backslash R & y & n \\ \hline y & 0.093 & 0.07 \\ n & 0.837 & 0 \end{array}$$

$$P(S \mid H=y, W=y) = \sum_R P(R, S \mid H=y, W=y) \approx \begin{array}{cc} y & n \\ (\boxed{0.163}, & 0.837) \end{array}$$

larger than prior



Taxis in Athens.

$$P(\text{Color}) = \begin{array}{cc} b & g \\ (0.9, & 0.1) \end{array}$$

$P(A \mid C)$

A \ C	b	g
ab	.7	.2
ag	.3	.8

$P(A, C) = P(A \mid C)\, P(C) =$

A \ C	b	c
ab	.63	.02
ag	.27	.08

$$P(C \mid A=ag) = \frac{P(A, C, e)}{P(e)} = \left(\frac{.27}{.35}, \frac{.08}{.35}\right) \approx (.77, .23)$$

$$= \frac{P(A=ag \mid C)\, P(C)}{P(A=ag)} = \begin{array}{c|cc} & b & g \\ \hline ab & NA & NA \\ ag & \frac{.27}{.27+.08} & \frac{.08}{.27+.08} \end{array} =$$

Bayes' rule

$$P(C \mid A) = \frac{P(A \mid C)\, P(C)}{P(A)} = \begin{array}{c|cc} & b & g \\ \hline ab & \frac{.63}{.63+.02} & \frac{.02}{.63+.02} \\ ag & \frac{.27}{.27+.08} & \frac{.08}{.27+.08} \end{array}$$

The second row of this table is $P(C \mid A=ag)$.

Back to Taxis in Athens

$$\begin{array}{cl} T & \{B, G\} \\ \downarrow & \\ C & \{AB, AG\} \end{array}$$

$$P(T) = \begin{array}{c} B \quad\ \ G \\ (0.9, 0.1) \end{array} = \begin{array}{c} B \quad\ \ G \\ (\alpha, 1-\alpha) \end{array}$$

$C \backslash T$	B	G	
AB	$1-x$	y	false blue rate
AG	x	$1-y$	

false green rate

What are x and y, if

we want $P(T=G \mid C=AB) = P(G \mid AB) = 0.5$

$P(T, C) = P(C \mid T)\, P(T) =$

	B	G
AB	$\alpha(1-x)$	$y(1-\alpha)$
AG	αx	$(1-y)(1-\alpha)$

$$P(G|AB) = (\text{Bayes' rule}) = \frac{y(1-\alpha)}{\alpha(1-x) + y(1-\alpha)} = 0.5$$

$$\alpha(1-x) = y(1-\alpha) \Rightarrow \begin{cases} 1-x = 1-\alpha \\ y = \alpha \end{cases} \Rightarrow \begin{cases} x = \alpha \\ y = \alpha \end{cases}$$

(note: $\alpha = 1-\alpha \Rightarrow \alpha = 0.5$, imposs.)

For $\alpha = \cancel{0.1}$, $x = y = \cancel{0.1}$

0.9 $\qquad$ 0.9