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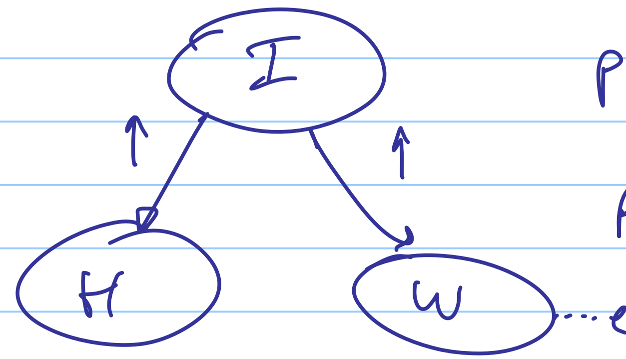
Note Title 2014-02-11

Icy Roads



$$P(I) = \begin{array}{cc} y & n \\ (0.7, & 0.3) \end{array}$$

$P(H|I) =$

H \ I	y	n
y	0.8	0.1
n	0.2	0.9

$P(W|I) =$

W \ I	y	n
y	0.8	0.1
n	0.2	0.9

(In this variation, Holmes and Watson are equally bad drivers.)

Compute the initial probabilities of W and H:

$$P(H, I) = P(H|I)\,P(I) = \begin{array}{c|cc} H \backslash I & y & n \\ \hline y & 0.56 & 0.03 \\ n & 0.14 & 0.27 \end{array} = P(W, I)$$

So, the initial probability of crashes is:

$$P(H) = (\overset{y}{0.59}, \overset{n}{0.41}) = P(W)$$

We now update $P(I)$ using the evidence: Watson has crashed $(W=y)$

$$P(I|W=y) = (\text{use Bayes' rule}) = \frac{P(W=y|I)\,P(I)}{P(W=y)} =$$

$$= \frac{1}{0.59} \times \begin{array}{cc} \multicolumn{2}{c}{I} \\ y & n \\ \hline \boxed{0.8} & \boxed{0.1} \end{array} \times \begin{array}{cc} \multicolumn{2}{c}{I} \\ y & n \\ \hline \boxed{0.7} & \boxed{0.3} \end{array} = \frac{1}{0.59}(0.56,\ 0.03) =$$

pointwise multiplication

approx

$= (0.95, 0.05)$: this is the "updated $P(I)$" $= P^*(I)$

We use this to update the probability that Holmes crashes:

$$P^*(H, I) = P(H \mid I) \times P^*(I) = \begin{array}{c|cc} H \backslash I & y & n \\ \hline y & 0.8 \times 0.95 & 0.1 \times 0.05 \\ n & 0.2 \times 0.95 & 0.9 \times 0.05 \end{array}$$

$P^*(H)$ = updated prob. that Holmes crashes, given that Watson crashes ("updated belief on the part of inspector Smith") =

$$\sum_{I} P^*(H, I) = (0.765, 0.235)$$

Then, Smith is told that the roads are not icy.

$$P^{**}(I) = (\overset{\text{yes}}{0}, \overset{\text{no}}{1})$$

$$P^{**}(H, I) = \begin{array}{c|cc} & \text{yes} & \text{no} \\ \hline y & 0 & 0.1 \\ n & 0 & 0.9 \end{array} = \begin{array}{cc} y & n \\ \Big(0.1, & \underline{\underline{0.9}}\Big) \end{array}$$

$$= P(H \mid I = n) = \begin{array}{cc} y & n \\ \Big(0.1, & 0.9\Big) \end{array}$$