

Choose a paper for presentation in parts from the special section on Bayesian Modelling in International Journal of Approximate Reasoning, Vol. 50, No. 3 (March 2009), edited by Linda van der Gaag and Russell Almond

- 1) Planning for success: the interdisciplinary approach to building Bayesian models
- 2) Verifying monotonicity of Bayesian networks with domain experts
- 3) Modeling human reasoning about meta-information
- 4) Bayesian networks: A teacher's view

5) Preventing knowledge transfer errors: Probabilistic decision support systems through the users' eyes.
Your choice?

* Ben Fine & Nick Stiffler : paper #4 April 15

1. **Reflexivity.** For any lottery A , $A \succeq A$

2. **Completeness.** For any pair (A, B) of lotteries, $A \succeq B$ or $B \succeq A$.

3. **Transitivity.** If $A \succeq B$ and $B \succeq C$, then $A \succeq C$.

4. **Preference increasing with probability.** If $A \succeq B$ then $\alpha A + (1 - \alpha)B \succeq \beta A + (1 - \beta)B$ if and only if $\alpha \geq \beta$.

5. **Continuity.** If $A \succeq B \succeq C$ then there exists $\alpha \in [0, 1]$ such that $B \sim \alpha A + (1 - \alpha)C$

6. **Independence.** If $C = \alpha A + (1 - \alpha)B$ and $A \sim D$, then $C \sim (\alpha D + (1 - \alpha)B)$.

Theorem: For an individual who acts according to a preference ordering satisfying rules 1-6 above, there exists a utility function over the outcomes s.t. the expected utility is maximized.

We choose the worst outcome (price), x_w
~ ~ ~ best ~ (~), x_B

We set $U(x_B) = 1$ and $U(x_w) = 0$.

Using the axiom of continuity (5), we can write

$$[x] \sim \alpha [x_B] + (1-\alpha) [x_w], \text{ where } \alpha \in [0, 1]$$

It follows that any lottery A can be written in the form

$$A = \sum_i t_i [x_i], \text{ where the } t_i \text{ are probabilities}$$

As done in the grade effort example, we choose

$$U(x) = \alpha \text{ (in the lottery with } x_B \text{ and } x_w)$$

$$EU(A) = \sum_i t_i U(x_i)$$

But, by the determination $U(x_i)$,

$$[x_i] \sim \underbrace{U(x_i)}_{\alpha} [x_B] + \underbrace{(1-U(x_i))}_{(1-\alpha)} [x_w] \quad (*)$$

By axiom 6,

$$A \sim \sum_i t_i \left(U(x_i) [x_B] + (1-U(x_i)) [x_w] \right)$$

(equal to $[x_i]$
by axiom 6
and $(*)$)

Since $U(x_i)$ is independent of t_i ,

$$A \sim \left(\sum_i t_i U(x_i) \right) [x_B] + \left(\sum_i t_i (1-U(x_i)) \right) [x_w]$$

Hence, for all lotteries A , we have (axiom 3):

$$A \sim \alpha [x_B] + (1-\alpha) [x_w], \text{ where } \alpha = EU(A)$$

Now let $A \sim \alpha [x_B] + (1-\alpha) [x_w]$ and $B \sim \beta [x_B] + (1-\beta) [x_w]$

$\xrightarrow{\text{same}} \quad \xrightarrow{\text{same}}$

By axiom [4], $A \succ B$ iff $\alpha \succ \beta$ iff $EU(A) \geq EU(B)$.

Monotonicity

$$A \succ B \Rightarrow (p \geq q \Leftrightarrow [p, A; 1-p, B] \succeq [q, A; 1-q, B])$$

$$\alpha \quad 1-\alpha \quad \beta \quad 1-\beta$$

Which of the following two lotteries would you prefer?:

- Lottery A = [\$1mill.],
- Lottery B = $0.1[\$5\text{mill.}] + 0.89[\$1\text{mill.}] + 0.01[\$0]$.

What about these two?:

- Lottery C = $0.11[\$1\text{mill.}] + 0.89[\$0]$,
- Lottery D = $0.1[\$5\text{mill.}] + 0.9[\$0]$.

$$U(\$0) = 0$$

$$U(\$5\text{mill.}) = 1$$

$$U(\$1) = u, \quad 0 \leq u \leq 1$$

If you prefer A to B, by Thm 9.1,

$$u > 0.1 \times 1 + 0.89 \times u + 0.01 \times 0$$

$$u > 0.1 + 0.89u, \quad \text{so } u > \frac{0.10}{0.11} = \frac{10}{11}$$

$$\text{Therefore } EU(C) = 0.11 \cdot u + 0.89 \cdot 0 = 0.11u > 0.11 \cdot \frac{10}{11} = 0.1$$

$$EU(D) = 0.1 \cdot 1 + 0.9 \cdot 0 = 0.1$$

So $EU(C) > EU(D)$. Therefore, by Thm 9.1, C must be preferred to D

If you prefer B to A, by Thm 9.1,

$$u < \frac{10}{11}$$

$$EU(C) < 0.1$$

$$EU(D) = 0.1$$

So, $EU(D) > EU(C)$. Therefore, D must be preferred to C.