Note Title 4/3/2009

Choose a paper for presentation in pairs from the special section on Bayesian Modelling in International Journal of Approximate Reasoning, Vol. 50, No. 3 (March 2009), edited by Linda van der Gaag and Russell Almond

1) Planning for success: the interdisciplinary approach to building Bayesian models
2) Verifying monotonicity of Bayesian networks with domain experts
3) Modeling human reasoning about meta-information
4) Bayesian networks: ~~A~~ teacher's view

5) Preventing knowledge transfer errors: Probabilistic decision support systems through the users' eyes.

Your choice?

* Ben Fine & Nick Stiffler ; Paper #4 April 15

1. ***Reflexivity.*** For any lottery A, $A \succeq A$

2. ***Completeness.*** For any pair (A, B) of lotteries, $A \succeq B$ or $B \succeq A$.

3. ***Transitivity.*** If $A \succeq B$ and $B \succeq C$, then $A \succeq C$.

4. ***Preference increasing with probability.*** If $A \succeq B$ then $\alpha A + (1-\alpha)B \succeq \beta A + (1-\beta)B$ if and only if $\alpha \geq \beta$.

5. ***Continuity.*** If $A \succeq B \succeq C$ then there exists $\alpha \in [0,1]$ such that $B \sim \alpha A + (1-\alpha)C$

6. ***Independence.*** If $C = \alpha A + (1-\alpha)B$ and $A \sim D$, then $C \sim (\alpha D + (1-\alpha)B)$.

Theorem: For an individual who acts according to a preference ordering satisfying rules 1-6 above, there exists a utility function over the outcomes s.t. the expected utility is maximized.

We choose the worst outcome (price), x_W
" " " best " ("), x_B

We set $U(x_B)=1$ and $U(x_W)=0$.

Using the axiom of continuity (5), we can write

$$[x] \sim \alpha [x_B] + (1-\alpha)[x_W], \quad \text{where} \quad \alpha \in [0,1]$$

It follows that any lottery A can be written in the form

$$A = \sum_i t_i [x_i] \quad , \quad \text{where the } t_i\text{s are probabilities}$$

As done in the grade effort example, we choose

$U(x) = \alpha$ (in the lottery with x_B and x_W)

$$EU(A) = \sum_i t_i U(x_i)$$

But, by the determination $U(x_i)$,

$$[x_i] \sim \underset{\alpha}{U(x_i)}[x_B] + \underset{(1-\alpha)}{(1-U(x_i))}[x_W] \quad \circledast$$

By axiom 6,

$$A \sim \sum_i t_i \left(\underbrace{U(x_i)[x_B] + (1-U(x_i))[x_W]}\right)$$

equal to $[x_i]$ by axiom 6 and $\circledast$

Since $U(x_i)$ is independent of t_i,

$$A \sim \left(\sum_i t_i U(x_i)\right)[x_B] + \left(\sum_i t_i (1-U(x_i))\right)[x_W]$$

Hence, for all lotteries A, we have (axiom 3):

$$A \sim \alpha[x_B] + (1-\alpha)[x_W], \quad \text{where } \alpha = EU(A)$$

Now let $A \sim \alpha[x_B] + (1-\alpha)[x_W]$ and $B \sim \beta[x_B] + (1-\beta)[x_W]$

same

same

By axiom [4], $A \succsim B$ iff $\alpha \geq \beta$ iff $EU(A) \geq EU(B)$

<u>Monotonicity</u>

$$A \succ B \;\Rightarrow\; (p \geq q \;\Leftrightarrow\; [p, A;\ 1-p, B] \succsim [q, A;\ 1-q, B])$$

$\alpha \quad 1-\alpha \qquad \beta \quad 1-\beta$