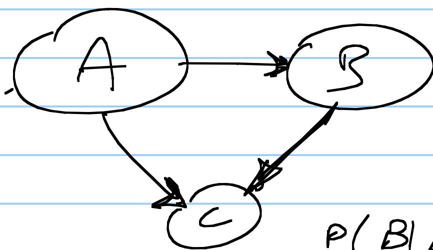
Note Title 2/12/2009

HW 3 CSCE 582 due 2009-02-14 All exercises from [J07]

2.17



$A = a_2$
$(e = (0,1)$ on $A)$

$P(A) = (0.9, 0.1)$

$P(B|A)$

B \ A	a_1	a_2
b_1	0.3	0.6
b_2	0.7	0.4

$P(C|A,B)$

B \ A	a_1	a_2
b_1	(0.1, 0.9)	(0.1, 0.9)
b_2	(0.2, 0.8)	(0.2, 0.8)

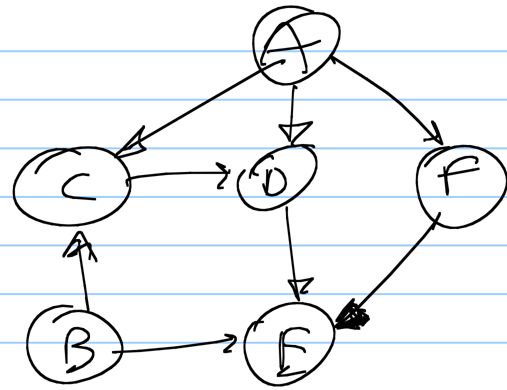
$$P(B,C,e) = \sum_{A} \prod_{v \in A,B,C} P(v|pa(v)) \cdot e = P(A=a_2) \cdot P(B|A=a_2) \cdot P(C|A=a_2, B) =$$

$$0.1 \times \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix} \times \begin{array}{c|cc} C \backslash B & b_1 & b_2 \\ \hline c_1 & 0.1 & 0.2 \\ c_2 & 0.9 & 0.8 \end{array}$$

=

C \ B	b_1	b_2
c_1	$0.1 \times 0.6 \times 0.1$ 0.006	$0.1 \times 0.4 \times 0.2$ 0.008
c_2	$0.1 \times 0.6 \times 0.9$ 0.054	$0.1 \times 0.4 \times 0.8$ 0.032

2.18



$P(A)$
$P(B)$
$P(C|A,B)$
$P(D|A,C)$
$P(E|B,D,F)$
$P(F|A)$

Need: $P(F|C=c_1)$

$$\sum_{A,B,D,E} P(A)P(B)P(c_1|A,B)P(D|A,c_1)P(E|B,D,F)P(F|A) =$$

$$= \sum_{A,B,D} P(A)P(B)P(C_1, A,B)P(D|A,c_1)P(F|A)\sum_{E} P(E|B,D,F) =$$

$$= \sum_{A,B} P(A)P(B)P(c_1|A,B)P(F|A)\sum_{D} P(D|A,c_1) = (*) =$$

$$= \sum_{B} P(B)\sum_{A} P(A)P(c_1|A,B)P(F|A) = \sum_{B} P(B)\,H_A(B,F) =$$

$= H_B(F)$. The answer is $\dfrac{H_B(F)}{\sum_F H_B(F)}$, i.e., the result of normalizing $H_B(F)$.

The largest table has size 1,000, and it is used to compute $H_A(B,F)$.

Note that we do not need to work with the 10^4-sized table $P(E|D,B,F)$, because $\sum_E P(E|D,B,F)$ is one.

We could save computation time and space by summing over B before summing over A:

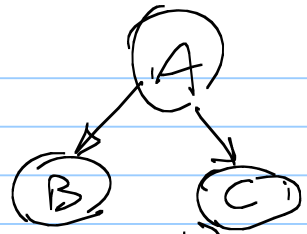
$$\circledast = \sum_A P(A)\,P(F|A) \sum_B P(B)\,P(c_1|A,B) =$$

$$= \sum_A P(A)\,P(F|A)\, H_B(A) = H_A(F).$$

The answer is $\frac{H_A(F)}{\sum_F H_A(F)}$, i.e., the result of normalizing $H_A(F)$.

Now, the largest tables have size 100. They are used to compute $H_B(A)$ and $H_A(F)$.

2.19



[See errata!!]

$P(B|A,C) = P(B|A)$ because A d-separates B and C and d-separation implies conditional independence.

Also, you can write $P(A,B,C) =$ (chain rule for BNs) $= P(A)P(B|A)P(C|A)$

So, $P(A,C) = \sum_B P(A)P(B|A)P(C|A) = P(A)P(C|A)\sum_B P(B|A) = P(A)P(C|A)$

$$P(A,B) = \sum_C P(A)P(B|A)P(C|A) = P(A)P(B|A)$$

$$P(A) = \sum_{B,C} P(A)P(B|A)P(C|A) = P(A)$$

So,

$$P(B|A,C) = \frac{\cancel{P(A)}P(B|A)\cancel{P(C|A)}}{\cancel{P(A)}\cancel{P(C|A)}} = P(B|A) = \frac{\cancel{P(A)}P(B|A)}{\cancel{P(A)}}$$

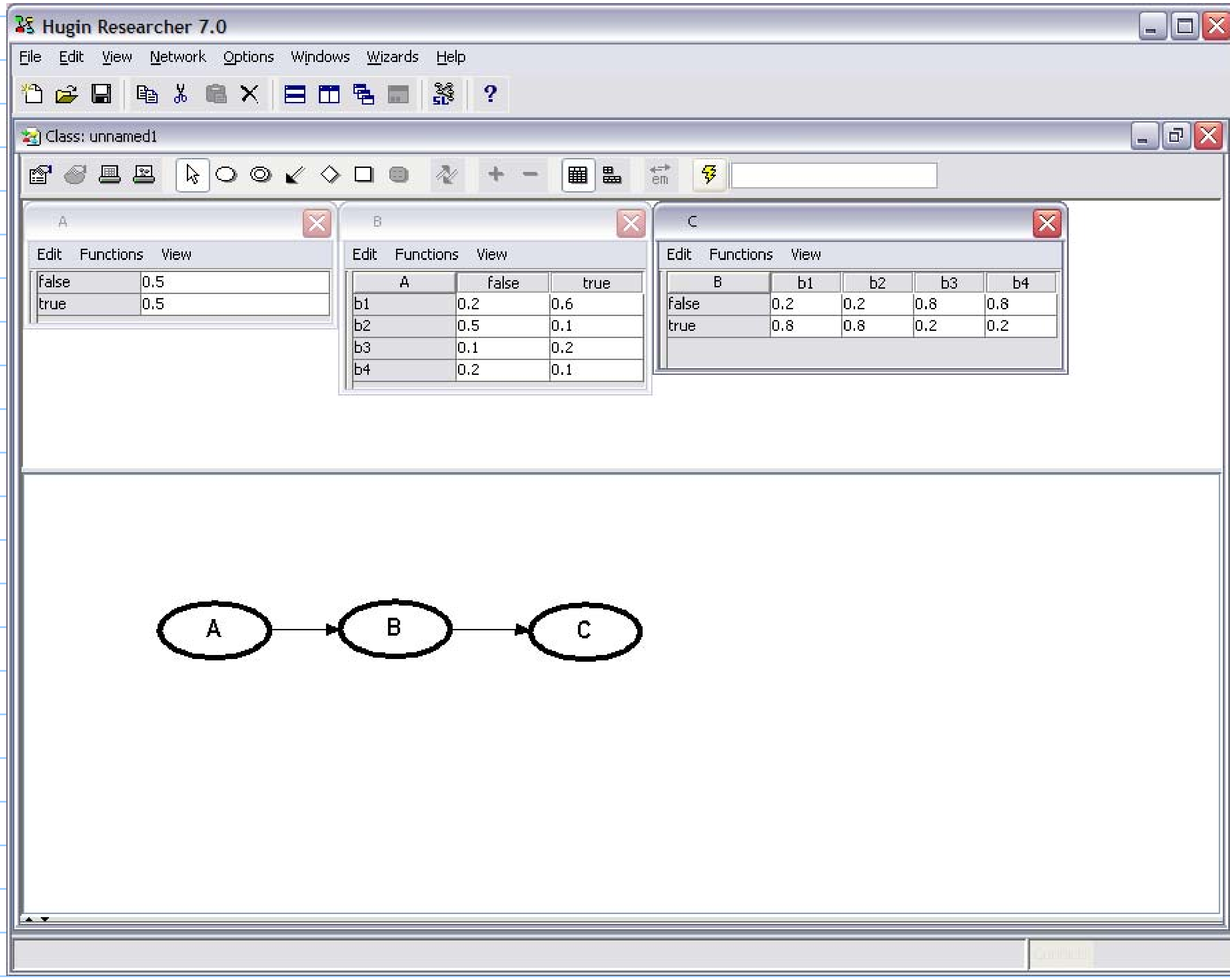
$$P(A,B,C) = P(A)\,P(B|A)\,P(C|A) = \begin{array}{cc} a_1 & a_2 \\ (0.1, & 0.9) \end{array} \times \begin{array}{c|cc} & a_1 & a_2 \\ \hline b_1 & 0.2 & 0.3 \\ b_2 & 0.8 & 0.7 \end{array} \times \begin{array}{c|cc} & a_1 & a_2 \\ \hline c_1 & 0.5 & 0.6 \\ c_2 & 0.5 & 0.4 \end{array} =$$

$$= \begin{array}{c|cc} & a_1 & a_2 \\ \hline b_1 & (0.1\times0.2\times0.5,\ 0.1\times0.2\times0.5) & (0.9\times0.3\times0.6,\ 0.9\times0.3\times0.4) \\ & 0.01 \qquad 0.01 & 0.162 \qquad 0.108 \\ b_2 & (0.1\times0.8\times0.5,\ 0.1\times0.8\times0.5) & (0.9\times0.7\times0.6,\ 0.9\times0.7\times0.4) \\ & 0.04 \qquad 0.04 & 0.378 \qquad 0.252 \end{array}$$

$$\begin{array}{r} 18\times \\ 9 \\ \hline 162 \end{array} \qquad \begin{array}{r} 12\times \\ 9 \\ \hline 108 \end{array} \qquad \begin{array}{r} 42\times \\ 9 \\ \hline 378 \end{array} \qquad \begin{array}{r} 28\times \\ 9 \\ \hline 252 \end{array}$$

Check: $\underbrace{0.01+0.01+0.04+0.04}_{0.1} + \underbrace{0.162+0.108}_{0.27} + \underbrace{0.378+0.252}_{0.63} = 1$ ✓

2,23



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true 0.5
B
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b1 0.2 0.6
b2 0.5 0.1
b3 0.1 0.2
b4 0.2 0.1
C
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A
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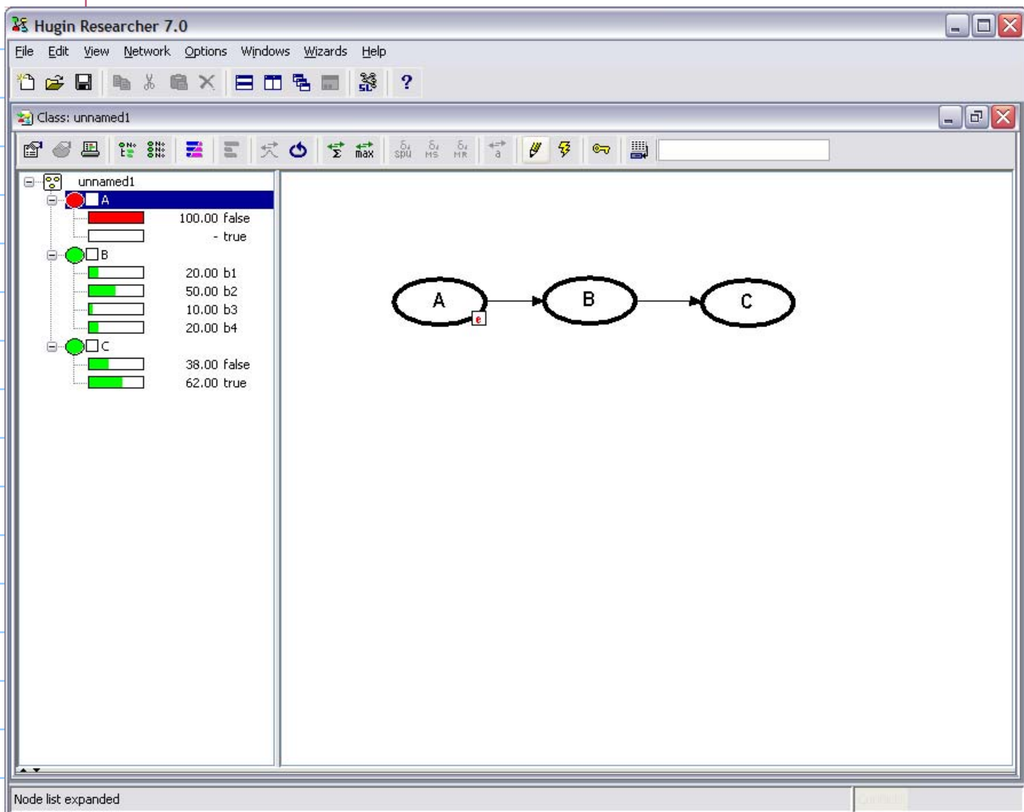

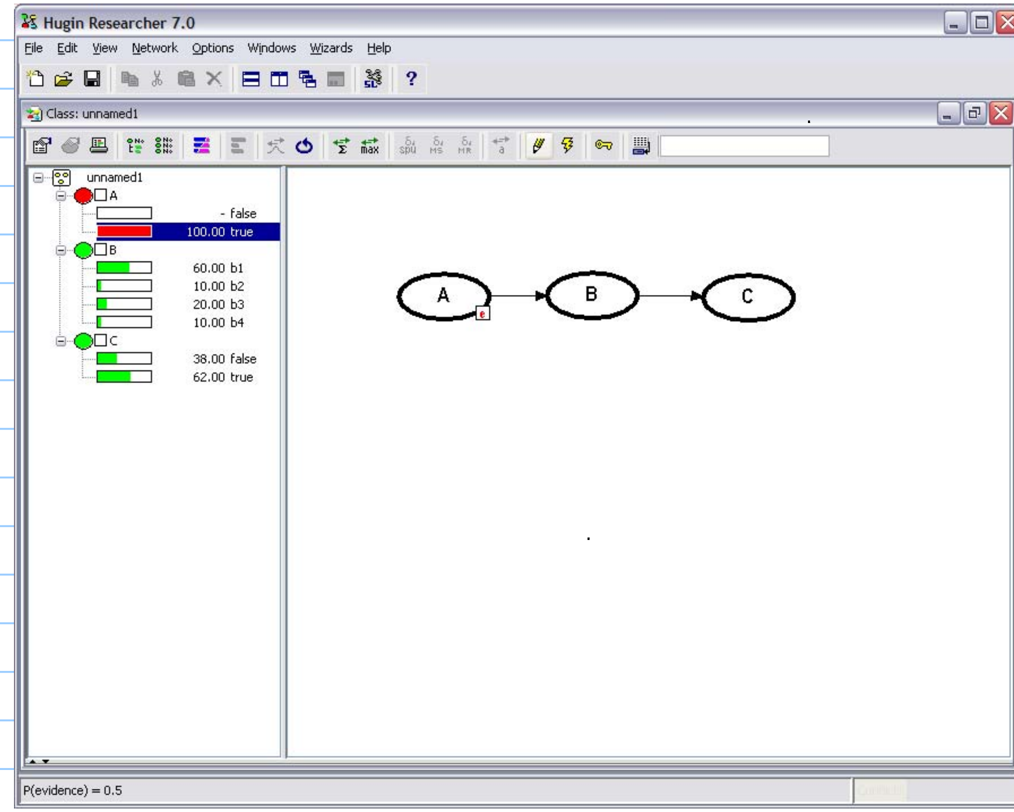

Note that d-separation implies conditional independence, but, as this example shows, the converse is not true!