CSCI 590–001 Fall 2000

MIDTERM EXAM–CLOSED BOOK (50 MINUTES)

Wednesday 00/10/4

Answers

1. State the fundamental rule [5 points]. **Answer:** $P(A, B) = P(A \mid B)P(B)$.

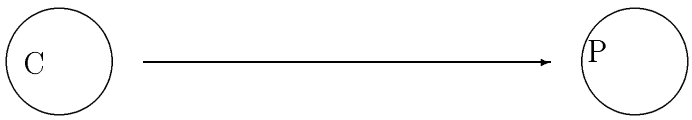
2. State Bayes' rule [5 points]. **Answer:** $\frac{P(D|S)=P(S|D)P(D)}{P(S)}$. (Why did I use the letters S and D?)

3. Define conditional independence [5 points]. Start like this: "Variables A and B are conditionally independent given C if and only if" ... **Answer:** Variables A and B are conditionally independent given C if and only if $P(A \mid B, C) = P(A \mid C)$. (Also acceptable, even though this is not the definition, is: Variables A and B are conditionally independent given C if and only if $P(A, B \mid C) = P(A \mid C)P(B \mid C)$).

4. State the chain rule for Bayesian networks [5 points]. Start like this: "Let BN be a Bayesian network over $U = \{A_1, \ldots, A_n\}$. Then the joint probability distribution $P(U)$ is" ... **Answer:** Let BN be a Bayesian network over $U = \{A_1, \ldots, A_n\}$. Then the joint probability distribution $P(U)$ is $\prod_{i=1}^{n} P(A_i \mid pa(A_i))$, where $pa(A_i)$ is the set of parents of A_i.

5. These questions concerning d-separation refer to the Dyspnoea example in the handout [15 points in total].
   (a) Suppose that we have no evidence [5 points].
      i. Are α and ξ d-connected? **Answer:** yes.
      ii. Are τ and λ d-connected? **Answer:** no.
   (b) Suppose that we have evidence on ϵ [5 points].
      i. Are α and ξ d-connected? **Answer:** no.
      ii. Are τ and λ d-connected? **Answer:** yes.
   (c) We observe that a patient has dyspnoea (shortness of breadth). Suppose that we also get to know that the patient is a smoker. Should our belief in the patient having tubercolosis change? Does d-separation in the network capture your intuition? Please explain your answer [5 points]. **Answer:** Our belief in tubercolosis should change, because it is more likely that a smoking patient has bronchitis or lung cancer and these are (directly or indirecty) competing (with respect to tubercolosis) causes of dyspnoea. D-separation captures this intuition, because when there is evidence on δ, σ is d-connected to τ.

6. (Adapted from J. Pearl via Exercise 14.11 of: Russell, Stuart and Peter Norvig. *Artificial Intelligence: A Modern Approach.* Englewood Cliffs: Prentice Hall, 1995.) You are a witness of a night-time hit-and-run accident involving a taxi in Athens. Half of the taxis in Athens are blue, while



half are green. You testify, under oath, that the taxi was blue. Extensive testing under dim lighting conditions indicates that the probability of seeing a blue taxi when the taxi is actually blue is 75% and the probability of seeing a green taxi when the taxi is actually green is also 75% [10 points in total].

(a) Draw a DAG modeling the story. The DAG has two nodes, representing the true color of the taxi (call it C) and the perceived color of the taxi (call it P) [5 points]. **Answer:** See figure.

(b) Quantify the network by giving the two required probability tables [5 points]. **Answer:** $P(C) = (0.5, 0.5)$, and $P(P \mid C)$ as given in the following table:

	blue	green
blue	0.75	0.25
green	0.75	0.25

(c) Compute the probability that the taxi you saw is actually blue [0 points]. (Do this if you have time.) **Answer:** $P(C = blue | P = blue) = 0.75$.