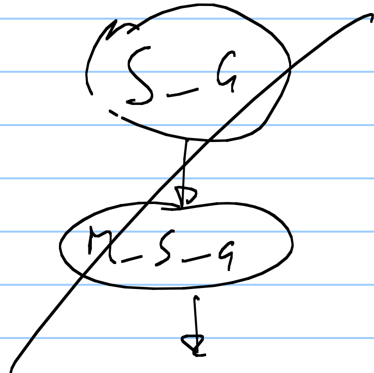
Note Title 3/18/2009



A node is simplicial if its family is a clique (i.e., if it has a complete neighbor set).

Section 4.3

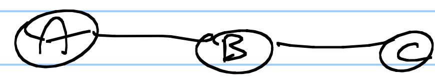
Proposition 4.4: Let G be a triangulated graph, X be a simplicial node. Let G' be G with X eliminated. Then G' is triangulated. (This is just Prop. 4.2 stated using new definitions.)

Theorem 4.1 A triangulated graph with at least two nodes has at least two simplicial nodes.

With two nodes, A — B, both A and B are simplicial.
Trivial also for complete graphs.

stat An incomplete triangulated graph with at least three nodes has at least two non-adjacent simplicial nodes.

By induction.

Base case: 

Ind. step. Assume stat is true for graphs w/ fewer than n nodes.

We need to show that any incomplete triang. graph of n nodes (say G) has two non-adj. simplicial nodes.

Let X be the first node in the perfect elimination sequence (which exists b/c G is triangulated. Let G' be G after eliminating X. G' is triangulated (b/c G has a perfect elim ordering and we removed the first node in the ordering).

Two cases: G' is complete — choose any other node

G' is not complete. Then it meets the ind. hypo, i.e., G' is a non-complete triangulated graph of $< n$ (exactly $n-1$) nodes. ✓

<u>Corollary 4.1</u> In a triangulated graph, each variable A has a perfect elimination sequence ending in A.

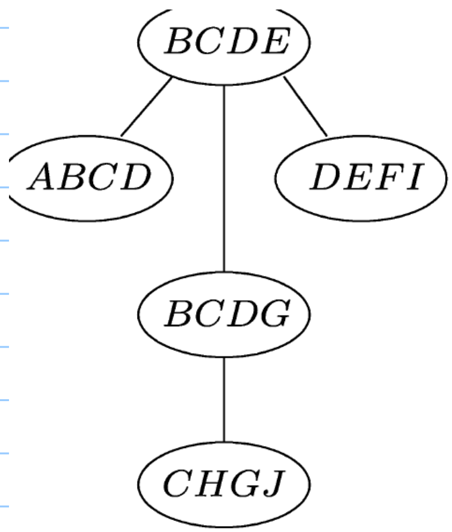
Test for whether a graph (G) is triangulated is constructible: just try eliminating simplicial nodes — if you can obtain the empty graph, then G is triangulated; otherwise it is not.

<u>Alg. 4.1</u> To determine the set of cliques of a triang. graph do the following. (Read it!)

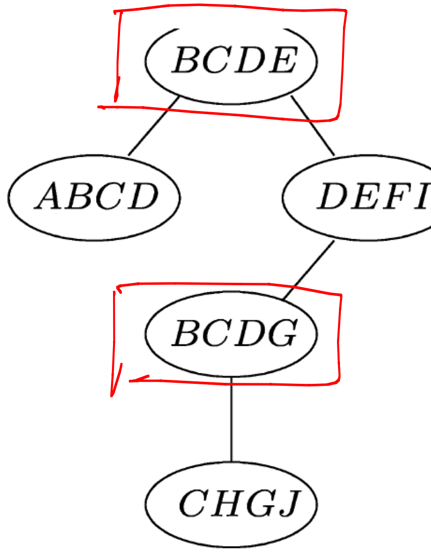
Join trees

(often called junction trees, but in [JJ07] junction trees are more complicated entities)

A *join tree* for a triangulated graph G is a tree $T = (V, E)$, where V is the set of cliques of G and, if there is a path from v_1 to v_2 in T, $v_1 \cap v_2$ is contained in every node on the path.



(a)



(b)

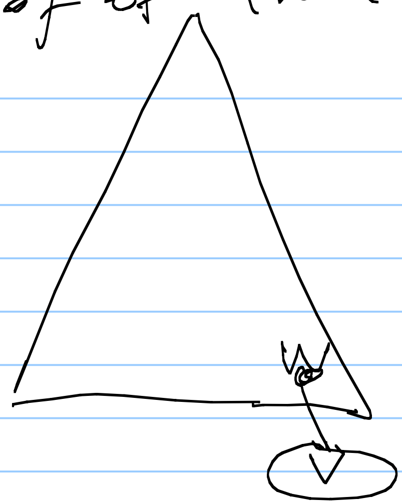
(a) is a join tree (for the graph of Fig. 4.14)

(b) is not a join tree, b/c $\{B, C, D, E\} \cap \{B, C, D, G\} = \{B, C, D\}$ but node DEFI doesn't contain $\{B, C, D\}$!

<u>Theorem 4.3</u> If the cliques of an undirected graph G can be organized into a join tree, then G is triangulated

<u>Theorem 4.4</u> If the undirected graph G is triangulated, then G can be organized into a join tree

Proof of Thm 4.3



$V \setminus W \neq \{\}$

Remove every variable in $V \setminus W$. Each of these can be eliminated w/out creating fill-ins, b/c all of their neighbors are in the clique W. Repeat!

$\square$