582 2010-09-01

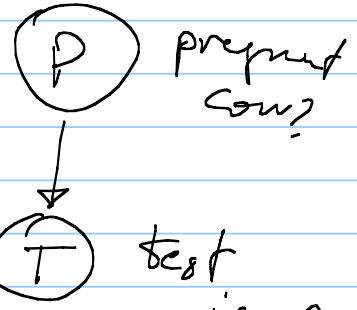
Note Title 9/1/2010

1.7

$P(T|P)$

T \ P	y	n
y	1	0
n	0	1

(perfect test)



In this exercise

T \ P	p	~p
t	0.98	0.001
~t	0.02	0.999

We will compute $P(P,T)$

P \ T	t	~t
p	0.049	0.001
~p	0.00095	0.94905

0.04995

T \ P	p	~p
y		error
n	error	

(1) $P(p,t) = P(t|p) \times P(p) = 0.98 \times 0.05 = 0.049$

(2) $P(p,\sim t) = P(\sim t|p) \times P(p) = 0.02 \times 0.05 = 0.001$

(3) $P(\sim p, t) = P(t|\sim p) \times P(\sim p) = 0.001 \times 0.95 = 0.00095$

(4) $P(\sim p, \sim t) = P(\sim t|\sim p) \times P(\sim p) = 0.999 \times 0.95 = 0.94905$

$$P(p|t) = \frac{P(p,t)}{P(t)} = \frac{P(p,t)}{\sum_{P \in \{p,\sim p\}} P(P,t)} = \frac{0.049}{0.04995} = 0.981$$

$$= \frac{P(t|p) \times P(p)}{P(t)} = \frac{0.98 \times 0.05}{P(t|p)P(p) + P(t|\sim p) \times P(\sim p)} =$$

1.11 $P(A) = \sum_{B \in \{b_1, b_2, b_3\}} P(A, B) = (0.2, 0.4, 0.4)$

A \ B	b_1	b_2	b_3
a_1	0.05	0.10	0.05
a_2	0.15	0.00	0.25
a_3	0.10	0.20	0.10

$P(B) = \sum_{A \in \{a_1, a_2, a_3\}} P(A, B) = (0.3, 0.3, 0.4)$

$P(A|B) = \frac{P(A,B)}{P(B)} =$

	b_1	b_2	b_3
a_1	0.05 / 0.3	0.10 / 0.3	0.05 / 0.4
a_2	0.15 / 0.3	0.00 / 0.3	0.25 / 0.4
a_3	0.10 / 0.3	0.20 / 0.3	0.10 / 0.3

=

	b_1	b_2	b_3
a_1	.167	.333	.125
a_2	.5	0	.625
a_3	.333	.667	.25

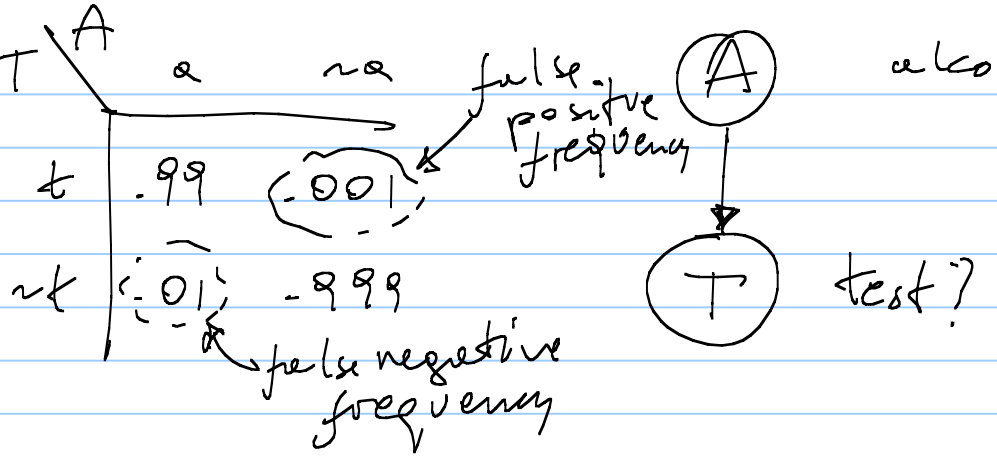
$P(B|A) = \frac{P(A,B)}{P(A)} =$

	a_1	a_2	a_3
b_1	0.05 / 0.2	0.15 / 0.4	0.10 / 0.4
b_2	0.10 / 0.2	0.00 / 0.4	0.20 / 0.4
b_3	0.05 / 0.2	0.25 / 0.4	0.10 / 0.2

=

	a_1	a_2	a_3
b_1	.25	.375	.25
b_2	.5	0	.5
b_3	.25	.625	.25

1.12 $P(T|A)$



(i) $P(A) = (.2, .8)$

$$P(a|t) = \frac{P(t|a)P(a)}{P(t)} = \frac{.99 \times .2}{P(t|a)P(a) + P(t|\neg a)P(\neg a)} = \frac{.99 \times .2}{.99 \times .2 + .001 \times .8} \approx .996$$ (approx.)

(ii) $P(A) = (.001, .999)$

$$P(a|t) = \frac{P(t|a)P(a)}{P(t|a)P(a) + P(t|\neg a)P(\neg a)} = \frac{.99 \times .001}{.99 \times .001 + .001 \times .999} = .498 \text{ (approx)}$$

1.13

$$P(B,C) = \sum_{A \in \{a_1, a_2\}} P(A,B,C) =$$

	b_1	b_2
c_1	0.02	0.08
c_2	0.18	0.72

$$P(B) = \sum_{C \in \{c_1, c_2\}} P(B,C) = (0.2, 0.8)$$

$$\frac{P(A, B, C)}{\sum_{A \in \{a_1, a_2\}} P(A, B, C)}$$

$$P(A|B) \overset{?}{=} P(A|B,C)$$

$P(A|B,C)_2$

A \ (B,C)	(b_1, c_1)	(b_1, c_2)	(b_2, c_1)	(b_2, c_2)
a_1	$\frac{0.006}{0.02} = \frac{0.054}{0.18}$ ✓		$\frac{0.048}{0.08} = \frac{0.126}{0.72}$ ✓	
a_2	$\frac{0.014}{0.02} = \frac{0.126}{0.18}$ ✓		$\frac{0.032}{0.08} = \frac{0.288}{0.72}$ ✓	

yes