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Note Title 10/19/2009

Uwe Schöning <u>Logic for Computer Scientists</u>.

Birkhäuser, 1989.

Algorithm: satisfiability of a Horn formula

1. Mark every occurrence of an atomic formula A in F if there is a subformula of the form $(1 \to A)$ in F.

2. <u>while</u> there is a subformulas G in F of the form $(A_1 \wedge \cdots \wedge A_n \to B)$ or of the form $(A_1 \wedge \cdots \wedge A_n \to 0)$, $n \geq 1$, where $A_1, \ldots, A_n$ are already marked (and B is not yet marked)

<u>do</u>

<u>if</u> G is of the first form

<u>then</u> mark every occurrence of B

<u>else</u> output "unsatisfiable" and halt

3. Output "satisfiable" and halt. (The satisfying assignment is given by: A_i is true $[\, \mathcal{Q}(A_i)=1 \,]$ iff A_i has a mark.)

$\square$

a. This procedure is sound & complete (proof similar to that of the bottom-up procedure for definite clauses)

b. Since "unsatisfiable" is output only for integrity constraints, every definite clause KB is satisfiable

c. Sat. for Horn clauses is linear in the # of clauses.

[I hope I did not make a mistake.]

d. The model obtained by this procedure is the minimal one. [Follows from the detailed proof: completeness part.]

e. KB $\begin{cases} \neg a \\ b \leftarrow a \end{cases}$ This has minimal model: empty (no true atoms)

(a = false, b = true is also a model, but non minimal)

Every Horn KB with no facts is satisfiable