CSCE 531 2020-01-16

Example of loop invariant for a program fragment to sum the integers from 1 to n

```
i := 1;
sum := 0;
while i ≤ n do
    sum := sum + i;
    i := i + 1
od;
```

while B do S od

P =

Q =

$P: i = 1 \wedge sum = 0$

$Q: sum = \sum_{j=1}^{n} j$

$I: sum = \sum_{j=1}^{i-1} j \wedge i \leq n+1$

$I \wedge \neg B \Rightarrow Q$ ✓

$sum = \sum_{j=1}^{i-1} j \wedge i \leq n+1 \wedge i \geq n+1 \Rightarrow sum = \sum_{j=1}^{n} j$

$sum = \sum_{j=1}^{i-1} j \wedge i = n+1$

$sum = \sum_{j=1}^{n+1-1} j = \sum_{j=1}^{n} j$

$P \Rightarrow I$ ✓

$i = 1 \wedge sum = 0 \Rightarrow sum = \sum_{j=1}^{i-1} j$ (the upper index is greater than the lower one when P holds)

$sum = 0$

$I \wedge B \{S_L\} I$ If $I \wedge B$ hold, then I holds after executing S_L.

I use primes $(', '')$ to indicate the value of a variable after executing the body of the loop.

$$sum = \sum_{j=0}^{i-1} \wedge\ i \leq n+1 \wedge i \leq n \Rightarrow sum' = \sum_{j=1}^{i'-1} \wedge\ i' \leq n+1$$

B

I

I after executing S_L

$$sum = \sum_{j=0}^{i-1} \wedge\ i \leq n$$

$sum' = sum + i$

$i' = i+1$

$$sum + i = \sum_{j=1}^{i+1-1} \wedge\ i+1 \leq n+1$$

✓

(rewrite)

<start> => <letter> => b

<start> => <start><letter> => <letter><letter> =>

(1) (2)

=> b<letter> => bc

Two derivations. The end of a derivation is called a string of the sentence. An intermediate string is called a sentential form.

The second derivation is a left-most derivation.

<start> => <start><letter> => <start>c => <letter>c =>

=> bc , (A rightmost derivation)

<start> => <start><letter> => <letter><letter> => <letter>c =>

$\Rightarrow b\ c$ (neither leftmost nor rightmost)