CSCE 350 Spring 2004
HOMEWORK 4
Tuesday 2004-02-24

This is a special homework assignment that allows you to make up 15 of the points lost in the first midterm exam (5 per question). You must turn in your handwritten answer on these two sheets, using the back if necessary. No additional pages will be allowed. You must turn in your work at the beginning of class on Thursday. On an exceptional basis, you may discuss these problems with other students in the class.

1 Correctness of a recursive program–30 points

Prove by induction that the following recursive procedure computes 2^n, for $n \geq 0$. In other words, the following specification is satisfied by the procedure Power:
(* Precondition: $n \geq 0$ *)
p = Power(n);
(* Postcondition: $p = 2^n$ *)

```
procedure Power(n: integer);
var m: integer;
begin
      if n=0 then 1; else if n=1 then 2;
      else if n mod 2 = 0 then
            m := n div 2;
            p := Power(m);
            return(p * p);
      end;
      else (* n is odd *)
          m := n div 2;
          p := Power(m);
          return(2 * p * p);
      end;
end Power;
```

In this (Modula-2 style) pseudocode, mod is the remainder operation, i.e., n mod 2 is zero if and only if n is even, and div is integer division, e.g. 5 div 2 = 2 and 4 div 2 =2.

Hint: Use complete (strong) induction.

2 Complexity of a recursive program–20 points

With reference to the previous question, how many multiplications are made to compute Power(0)? Power(2)? Power(4)? Power(8)? Power(2^k)? What is the complexity of Power(n) as a function of input size n, if multiplication is used as the basic operation and n is used as the measure of input? Give the answer for the case in which $n = 2^k$ only.

3 Correctness of an iterative program–20 points

Give a loop invariant for this loop:

```
while (i <= n) do
  begin
    sum := sum+i;
    i := i+1
  end
```

with precondition $i = 1 \wedge sum = 0 \wedge n \geq 0$ and postcondition $sum = \sum_{j=0}^{n} j$.