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Note Title 2010-10-25

Some FP examples

{fact (eq0 → %1 ; * @ [id, fact @ sub1])}

{sub1 - @ [id, %1]} subtract 1

{eq0 = @ [id, %0]} equality to zero

composition

sub1 : 3 ⇒ - @ [id, %1] : 3 ⇒

construction

$$- : \langle id : 3,\ \%1 : 3 \rangle \Rightarrow - : \langle 3, 1 \rangle \Rightarrow 2$$

$$eq0 : 2 \Rightarrow = @ [id, \%0] : 2 \Rightarrow$$

$$= : \langle id : 2,\ \%0 : 2 \rangle \Rightarrow = : \langle 2, 0 \rangle \Rightarrow false$$

$$fact\ 2 \Rightarrow (eq0 \rightarrow \%1 ;\ * @ [id, fact @ sub1]) : 2 \Rightarrow$$

$$\Rightarrow * @ [id, fact @ sub1] : 2 \Rightarrow$$

$$* : \langle id : 2,\ fact @ sub1 : 2 \rangle \Rightarrow$$

Composition $f \circ g$ is defined as follows;
for all x, $f \circ g : x = f(g(x))$

$\Rightarrow * : \langle 2, \; fact : 1 \rangle \Rightarrow$

$\Rightarrow * : \langle 2, \; \overbrace{(eq0 \rightarrow \bar{1} \; ; \; * \circ [id, \; fact \circ sub1])}^{fact} : 1 \rangle \Rightarrow$

$\Rightarrow * : \langle 2, \; * \circ [id, \; fact \circ sub1] : 1 \rangle \Rightarrow$

$\Rightarrow * : \langle 2, \; * : \langle 1, \; fact : 0 \rangle \rangle \Rightarrow$

$\Rightarrow * : \langle 2, * : \langle 1, (eq\emptyset \rightarrow \bar{1} ; * \circ [id, fact \circ sub1]) : 0 \rangle \rangle$

$\Rightarrow * : \langle 2, * : \langle 1, 1 \rangle \rangle \Rightarrow * : \langle 2, 1 \rangle \Rightarrow 2$

Matrix Multiplication

in program unm
$M \times N = P$ M of size $a \times b$ $\underbrace{r \times s}$ below
N of size $b \times c$ $s \times ?$
P of size $a \times c$

$$p_{i,j} = \sum_{k=1}^{b} m_{ik} \times n_{kj}$$

We represent matrices in row-major format, so

$m = \langle m_1, \ldots, m_r \rangle$, $m_i = \langle m_{i1}, \ldots m_{is} \rangle$

$\{mm \quad (\&\& \; ip) \circ (\& \; distl) \circ [1, trans \circ 2]\}$

$[id, trans \circ 2] : \langle m, n \rangle \equiv (\text{yields}) \equiv \langle m, n' \rangle$, where

$n' = trans : n$

$distr : \langle m, n' \rangle \equiv \overbrace{\langle \langle m_1, n' \rangle, \ldots, \langle m_i, n' \rangle, \ldots, \langle m_r, n' \rangle \rangle}^{A}$

$\& \; distl : A \equiv \langle p_1, \ldots, p_r \rangle$, where

$p_i = distl : \langle m_i, n' \rangle = \langle \langle m_i, n_1' \rangle, \ldots, \langle m_i, n_s' \rangle \rangle$

for $i = 1..r$ and n_j' is the jth column of n (the jth row of n')

So, p_i represents "row i of m times the columns of n"

Finally, $\ell\ell$ ip $(= \ell(\ell \text{ ip}))$ applies ℓ ip to each

p_i, which in turn means that ip is applied to

each "row by column" pair $m_i\, n_j$.

Ex.: $mm : \langle\langle\langle 1\ 2\rangle\ \langle 1\ 2\rangle\rangle\ \langle\langle 1\ 3\rangle\ \langle 1\ 3\rangle\rangle\rangle$

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1+2 & 3+6 \\ 1+2 & 3+6 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 3 & 9 \end{bmatrix}$$

$[1, trans \circ 2] : \langle\langle\langle 1\ 2\rangle\ \langle 1\ 2\rangle\rangle\ \langle\langle 1\ 3\rangle\ \langle 1\ 3\rangle\rangle\rangle =$

$$= \overbrace{\langle\langle\langle 1\ 2\rangle\ \langle 1\ 2\rangle\rangle\ \underbrace{\langle\langle 1\ 1\rangle\ \langle 3\ 3\rangle\rangle}_{\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix},\ \text{transpose of } \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}}\rangle}^{A}$$

$$distr : A = \overbrace{\langle\langle\underbrace{\langle 1\ 2\rangle}_{\text{row 1 of } m}\ \underbrace{\langle\langle 1\ 1\rangle\ \langle 3\ 3\rangle\rangle}_{\text{transpose of } n}\rangle\ \langle\underbrace{\langle 1\ 2\rangle}_{\text{row 2 of } m}\ \langle\langle 1\ 1\rangle\ \langle 3\ 3\rangle\rangle\rangle\rangle}^{B}$$

$$\& \, distl \; B = \langle distl : \langle\langle 1 \; 2\rangle \langle\langle 1 \; 1\rangle \langle 3 \; 3\rangle\rangle\rangle$$

$$distl : \langle\langle 1 \; 2\rangle \langle\langle 1 \; 1\rangle \langle 3 \; 3\rangle\rangle\rangle \rangle =$$

$$= \langle \langle\langle \overbrace{\langle 1 \quad 2\rangle}^{\text{row 1}} \overbrace{\langle 1 \quad 1\rangle}^{\text{col 1}}\rangle \langle \overbrace{\langle 1 \quad 2\rangle}^{\text{row 1}} \overbrace{\langle 3 \quad 3\rangle}^{\text{col 2}}\rangle\rangle$$

$$\langle\langle \underbrace{\langle 1 \quad 2\rangle}_{\text{row 2}} \underbrace{\langle 1 \quad 1\rangle}_{\text{col 1}}\rangle \langle \underbrace{\langle 1 \quad 2\rangle}_{\text{row 2}} \underbrace{\langle 3 \quad 3\rangle}_{\text{col 2}}\rangle\rangle\rangle \rangle \quad C$$

$$\& \, \& ip : C = \langle \& ip : \langle\langle\langle 1 \quad 2\rangle \langle 1 \quad 1\rangle\rangle \langle\langle 1 \quad 2\rangle \langle 3 \quad 3\rangle\rangle\rangle$$

$$\& ip : \langle\langle\langle 1 \quad 2\rangle \langle 1 \quad 1\rangle\rangle \langle\langle 1 \quad 2\rangle \langle 3 \quad 3\rangle\rangle\rangle \rangle =$$

$$\equiv \langle\langle ip : \langle\langle 1 \;\; 2\rangle \langle 1 \;\; 1\rangle\rangle ,\; ip : \langle\langle 1 \;\; 2\rangle \langle 3 \;\; 3\rangle\rangle \rangle$$
$$\langle ip : \langle\langle 1 \;\; 2\rangle \langle 1 \;\; 1\rangle\rangle ,\; ip : \langle\langle 1 \;\; 2\rangle \langle 3 \;\; 3\rangle\rangle\rangle \rangle \equiv$$

$$\equiv \langle\langle 3 \quad 9\rangle$$
$$\langle 3 \quad 9\rangle \rangle$$