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Note Title 10/21/2009

18.2 [T]

{Precondition} S {Postcondition}

If the precondition holds at the beginning of the program, then the postcondition holds at the end of the program.

This computes the product of x and y (x * y) by repeated sums.

$\{y \geq 0\}$ $z = 0;$ ...... od $\{z = x * y\}$

(a) Why $\{y \geq 0 \wedge \underbrace{0 = x\,(y-y)}\}$ is equivalent to $\{y \geq 0\}$?

true, because any number multiplied by ϕ is ϕ and $y-y$ is zero for any value y.

The condition $\{y \geq 0 \wedge 0 = x(y-y)\}$ may replace $\{y \geq 0\}$ by precondition strenghtening (precondition consequence rule).

(b) $\{y \geq 0 \wedge 0 = x(y-y)\}$

$z = 0;$ $\leftarrow \{R\}$

$n = y;$

assignment rule

& sequence rule

$\{y \geq 0 \wedge z = x(y-n) \wedge n \geq 0\}$

(c) + (d)

Let y, n, z, x stand for their respective values before the execution of the loop body.
Let y', n', z', x' stand for the values of y, n, z, x, resp., after execution of the loop body.

So: $y' = y$, $z' = z + x$, $n' = n - 1$, $x' = x$

In short, you need to show

$I \wedge T \Rightarrow I$

$$\{y \geq 0 \wedge n \geq 0 \wedge z = x(y-n) \wedge n > 0\} \Rightarrow$$

$$\{\underbrace{y' \geq 0}_{\substack{\equiv \\ y \geq 0}} \wedge \underbrace{n' \geq 0}_{n-1 \geq 0 \;\equiv\; n \geq 1 \;\equiv\; (\text{b/c } n \text{ is integer}) \;\equiv\; n > 0} \wedge \underbrace{z' = x'(y'-n')}\}$$

$$z + x = x(y-(n-1)) = x(y-n) + x$$

$$\equiv \qquad\qquad = x(y-n+1) = x(y-n) + x \cdot 1$$

$$z = x(y-n)$$

(e) In short, $I \wedge \neg T \Rightarrow Post$

$$\{ y \geq 0 \wedge n \geq 0 \wedge z = x(y-n) \wedge \neg(n > 0) \} \Rightarrow z = x * y$$

$$n = 0$$

$$z = x(y-0) \equiv z = x * y$$