CSCE 330 Fall 2006

QUIZ 9

Assigned Thussday, 06-10-12

Give a loop invariant for this program fragment :

```
x := 2;
i := 1;
(*What is the precondition here?*)
while (i <= n) do
  begin
    x := x*x;
    i := i+1
  end
```

with precondition $n \geq 1$ and postcondition $x = 2^{2^n}$.
Answer: $x = 2^{2^{i-1}} \wedge i \leq n + 1$.
Also answer the following questions.

1. What is the precondition before the loop? **Answer:** $x = 2 \wedge i = 1 \wedge n \geq 1$
2. Your invariant should consist of the conjuction of two formulas. One of them should be very similar to the condition of the while loop. Explain why that formula is needed. **Answer:** To insure that $i = n + 1$ (rather than just $i > n$) when the loop is exited.
3. Show that the precondition at the line with asterisks implies the invariant. **Answer:** In short: (1) Since $i = 1$ and $x = 2$, then $x = 2^{2^{i-1}}$. (2) Since $i = 1$ and $n \geq 1$, then $i \leq n + 1$.
4. Show that the invariant together with the negation of the loop implies the postcondition. **Answer:** In short: (1) Since $i \leq n + 1$ and $i > n$ then $i = n + 1$. (2) Since $x = 2^{2^{i-1}}$ and $i = n + 1$, then $x = 2^{2^n}$.
5. Let x be the value of the variable `x` before executing the body of the loop and x' be the value of the variable `x` after executing the body of the loop. Write an equation that relates x and x'. **Answer:** $x' = x * x$.
6. Let i be the value of the variable `i` before executing the body of the loop and i' be the value of the variable `i` after executing the body of the loop. Write an equation that relates i and i'. **Answer:** $i' = i + 1$.