

HW 1 due Friday, January 23

Conversion from base b to base B

General technique based on the definition of positional notation for numbers. The conversion formula is the same as the representation formula,

$$N = \sum_{i=-m}^{n-1} c_i b^i , \text{ where the number to be converted is}$$

$$c_{n-1} \dots c_1 c_0 \overset{\text{radix or fractional point}}{\underset{\uparrow}{c_{-1}}} c_{-2} \dots c_{-m}$$

Express the digits and the original base in the new (target) base B

Ex, convert 140_{10} into binary. So: $b=10$, $B=2$,

$$c_2 = 1, c_1 = 4, c_0 = 0, n=3, m=0$$

Express the digits in binary:

$$c_1 = 10_2 \quad c_2 = 1_2 \quad c_0 = 0_2$$

Express the base in binary:

$$b = 10_{10} = 1010_2$$

$$N = c_0 \cdot b^0 + c_1 b^1 + c_2 b^2 = 0 + 100_2 (1010_2) + 1 (1010_2)^1 =$$

$$(1010)_2 = \overrightarrow{1010}$$

$$1010 = 10_{10}$$

$$\times 1010 = 10_{10}$$

$$\overbrace{1} \quad \overbrace{0} \quad \overbrace{1} \quad \overbrace{0}$$

$$\overbrace{10} \quad \overbrace{10} = \overrightarrow{0}$$

$$1010$$

$$\overbrace{1010} \quad \overbrace{1010} = ?$$

$$\overbrace{110010D_2} = 100_{10}$$

$$64 \quad 32 \quad 4$$

$$1010 = 10_{10}$$

$$\times \overbrace{100} = 4_{10}$$

$$\overbrace{10} \quad \overbrace{10} = 0$$

$$\overbrace{101000} = ?$$

$$32 + 8 = 40_{10}$$

$$\overbrace{40} = 40_{10}$$

$$+ 101000$$

$$+ 1100100$$

$$\overbrace{100011000} = ?$$

$$128 + 8 + 4 = 140$$

$$\checkmark$$

$$= 140$$

More efficient conversion algorithms.

Converting an integer in base 10 to base 2 by repeated division (page 9 of your text).

$$N = (a_n a_{n-1} \dots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \dots + a_2 R^2 + a_1 R + a_0$$

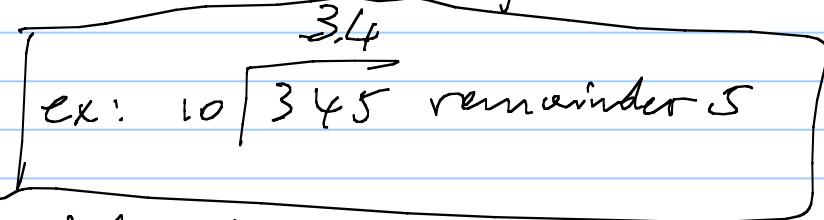
Divide by R

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \dots + a_2 R + a_1 = Q, \text{ remainder of } a_0$$

$$\frac{Q}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \dots + a_2 = Q_2$$

\vdots

$$\frac{Q_1}{R} = \text{remainder of } a_1$$



Keep this process until Q is 0.

The remainders are the digits of the number in base R

Ex. Convert 752_{10} to binary

$$2 \overline{)752}$$

$$2 \overline{)376} \quad \text{rem } 0 = q_0$$

$$2 \overline{)188} \quad \text{rem } 0 = q_1$$

$$2 \overline{)94} \quad \text{rem } 0 = q_2$$

$$2 \overline{)47} \quad \text{rem } 0 = q_3$$

$$2 \overline{)23} \quad \text{rem } 1 = q_4$$

$$2 \overline{)11} \quad \text{rem } 1 = q_5$$

$$2 \overline{)5} \quad \text{rem } 1 = q_6$$

$$2 \overline{)2} \quad \text{rem } 1 = q_7$$

$$2 \overline{)1} \quad \text{rem } 0 = q_8$$

$$0 \quad \text{rem } 1 = q_9$$

$$\begin{array}{r} 101110000 \\ \hline 512 + 128 + 64 + 32 + 16 = 752 \\ \hline 128 \\ 64 \\ 32 \\ 16 \\ \hline 752_{10} \end{array}$$

$$2 \overline{)752} \quad \text{rem } 0$$

Quiz 2 assigned

