MULTIPLE MODEL APPROACH FOR CSTR CONTROL

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Abstract: A method for combining multiple local models to describe a nonlinear system. The local model weights are based on the linear interpolation of the current operating point from the closest local model operating points defined as the filtered value of the current process input. This local modeling method can be used to describe systems with changing system gains and dynamics, as well as input multiplicity. A case study for a benchmark CSTR reactor is presented. Local linear models are used to synthesize IMC based PID controllers.

Keywords: Nonlinear Systems, Model Based Control, Process Models, Reactor Control, Weighting Functions

1. INTRODUCTION AND MOTIVATION

Optimization and control of process systems typically requires an accurate process model. Fundamental first principles models can be difficult to develop if the underlying process is not well understood. The resulting fundamental model may also have numerous unknown parameters and severe complexity. At the other extreme, a linear approximation of a system may be easy develop, but such linear models can be insufficient to capture the process system characteristics requisite for optimization and control. In many cases, a methodology that can provide a simple system model while still accurately representing the complex nonlinear behavior of actual process systems is desired. Local modeling is such a method for the development of such system representations.

Many different approaches have been proposed for using local models to approximate nonlinear systems. For a detailed review, see (Murray-Smith and Johansen, 1997). In many cases, the different approaches can be distinguished by the choice of model weights. These weights are based on the current system's location within a partitioned space. Some authors have used exponential functions to characterize the valid space of a local model (Banerjee *et al.*, 1997; Balle *et al.*, 1997; Johansen and Foss, 1993*a*). Fuzzy logic rules have been used to chose model weights for combining local linear models (Rueda, 1996; McGinnity and Irwin, 1997). These applications use fuzzy membership functions to partition the operating space into different model regimes. The basic modeling methods are very similar no matter the approach: determine where the local models are to be used in the operating space and then devise a method to mesh the local models together.

One obvious extension to such modeling work is the development of closed-loop control using local controllers. Work has been presented for control of nonlinear systems using fuzzy logic to choose between local controllers (Chak and Feng, 1994; Logan and Pachter, 1997; Logan and Pachter, 1994). Artificial neural networks have also been used to piece together linear models and controllers in attempts to control nonlinear

nonlinear control on systems that may not be able to use techniques such as Input-Output Linearization (IOL).

We propose a framework where local models can be easily used to approximate a true nonlinear system. Most published results use either exponential functions or fuzzy logic rules for representing model regimes. In the method presented here, individual model weights are based on the distance of the current operating point from the operating points of the nearest local models. This is similar to using fuzzy logic rules with pyramid shaped membership functions. Some other applications do not adequately represent system dynamics as process changes are made. This type of implementation effectively uses discrete switching for jumping between models. Using the proposed method, a range of nonlinear system dynamics are well-modeled. The current operating point is taken as a filtered value of the known system inputs, which realistically slows the model response and prevents discrete jumps from one model to another. This method does not constrain local models to be linear system models. In some cases, low-order nonlinear empirical models can greatly improve model validity, but also be simple to develop. The linear model mixing methodology is used with multiple IMC controllers to provide adequate control in many different situations. This system can also make use of existing models created at different steady-state operating points. Given a system with two standard operating points (for different product grades) and models at these two points, minimal work and modeling effort would be required to blend the models to create a more accurate system model.

2. LOCAL MODELING

A similar treatment of finite state-space modeling may be found in (Johansen and Foss, 1993*b*). A model of general state-space system may be written as:

$$\frac{dx}{dt} = f(x, u, v)$$

$$y = g(x, u, v)$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the input vector, $y \in R^s$ is the output vector, and $v \in R^r$ is the input disturbance vector. The local model M_i may then be described by

$$\frac{dx_i}{dt} = f(x_i, u, v_i)$$

$$y_i = g(x_i, u, v_i)$$

and disturbance vectors of the other models or the actual system being modeled. The operating point ϕ at any time t is a single point in the operating space Φ, which is made up of different operating regimes $\Phi_i \in \Phi$. The operating point ϕ may be written as:

$$\phi(t) = h(y(t), u(t), x(t))$$

Typically, ϕ can be parameterized as a function of the input $u(t)$ or the output $y(t)$. Assume that for each local model there are model validity functions ρ_i that map the entire operating space Φ to $[0, 1]$. The overall model for N different model regimes becomes

$$w_i(\phi(t)) = \frac{\rho_i(\phi(t))}{\sum_{i=1}^{N} \rho_i(\phi(t))}$$

$$\frac{dx_i}{dt} = \sum_{i=1}^{N} f(x_i, u, v_i)$$

$$y = \sum_{i=1}^{N} g(x_i, u, v_i)\, w_i(\phi(t))$$

The weighting function $w_i(\phi)$ is normalized so that at any operating point, the local model weighting functions sum to unity.

3. METHODOLOGY

In process systems where a linear model or controller proves insufficient, it is desirable to have a simple representation that can model or control the nonlinear system. This leads one to consider a multiple model approach. There are many possible routes to take depending upon how much is known about the system.

- For this work, it is assumed that a first-principles model of the system under consideration is not known, although a first-principles model is used for simulation purposes.
- It is also assumed that the system does not exhibit output multiplicity. This implies that there could be input multiplicities in the process, resulting in a process optima with zero-gain.
- One final assumption is that local models may already be available for some operating conditions, while new models can be developed at other operating points.

To best piece together multiple local models, one must devise a method for partitioning the operating space for the system. For input multiplicity systems, it makes sense to partition the space what existing models may be available, the known process characteristics (such as process optima), and the desired system operating range. In the next step for developing a multiple model system, one must assume a method for weighting the independent local models. In order to adequately capture steady-state values, a linear interpolation scheme is used in this work. This means that the model weights are based on the nearest two local models in the SISO case. When operating exactly at the point where the model i was devised, the weight for model i should be equal to 1. When operating halfway between two models each model should be weighed equally, resulting in weights of 0.50. This model weighting scheme removes the problems often associated with the normalization of overlapping basis functions as presented in (Murray-Smith and Johansen, 1997). The tail of a Gaussian model weight can extend well into another model regime, causing a model to become active in the wrong region. Typical model weights for the presented work are shown in Figure 1. In this figure, the models would be established at the operating points 2, 3, 4 and 8 in the input space of the system. This type of weighting has also been extended to a 2x2 MIMO case.

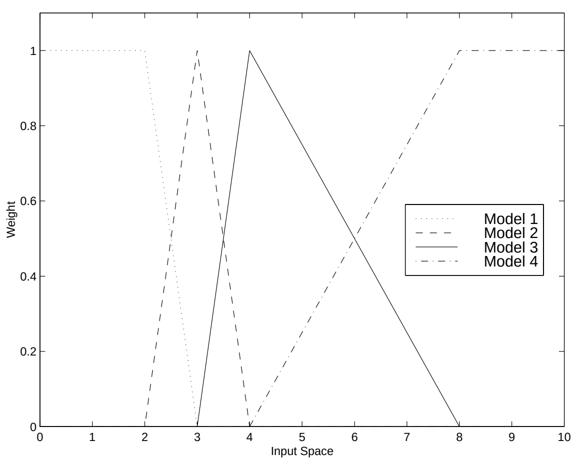


Fig. 1. Model Weighting Function

Using the process inputs to select the correct operating point works well for steady-state analysis. When using this method of model switching, changes in the process inputs will cause the process to jump from one model directly to another. In order to allow for mixing of heterogeneous models, all local models are given absolute process inputs and produce absolute process outputs. As the process input changes from one value to another, the actual system does not respond immediately. This means that the actual state of the process is not moving instantaneously. To cope with this reality, the operating point for model

typical of the system. A second order filter with unity gain is used in this work. This creates a realistically smooth response that is continuously differentiable. See Figure 2 for a schematic. If one considers a linear state-space realization for each model $M_j([A_j, B_j, C_j])$, then the corresponding state-space composite model is given by:

$$
\begin{aligned}
\frac{dx_1}{dt} &= A_1\, x_1 + B_1\, u \\
&\vdots \\
\frac{dx_j}{dt} &= A_j\, x_j + B_j\, u \\
&\vdots \\
\frac{dx_N}{dt} &= A_N\, x_N + B_N\, u \\
\frac{dx_F}{dt} &= A_F\, x_F + B_F\, u
\end{aligned}
$$

$$
\begin{aligned}
y = w_1(x_F)\, C_1\, x_1 + \ldots + w_j(x_F)\, C_j\, x_j + \ldots \\
+ w_N(x_F)\, C_N\, x_N
\end{aligned}
$$

where x_F corresponds to the states of the filter realization. Note that this model is a Wiener structure, and it is piecewise bilinear. Consequently, the rich literature on Wiener and bilinear systems is relevant to this approach.

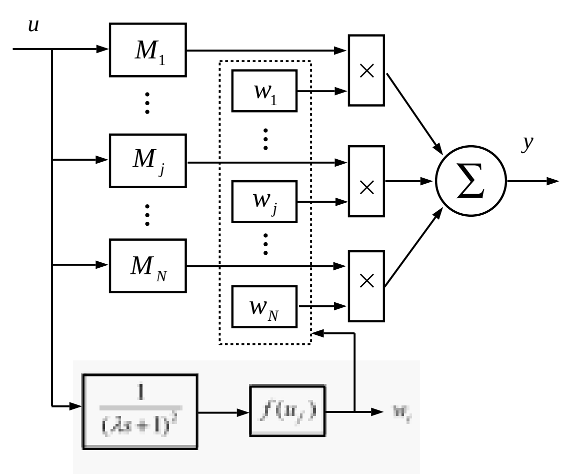


Fig. 2. Model Weighting Schematic

For control purposes, some work has to be done to make use of the model switching techniques. First, a controller must be devised for each local model. For this study, PI controllers were used for all local model controllers. These were tuned using IMC tuning rules and typical process reaction curve parameters. Use of IMC-PI form allows for a single tuning parameter for all controllers. The total system error is given to each controller. The output of each controller is multiplied by the current weights for each model. The total of all

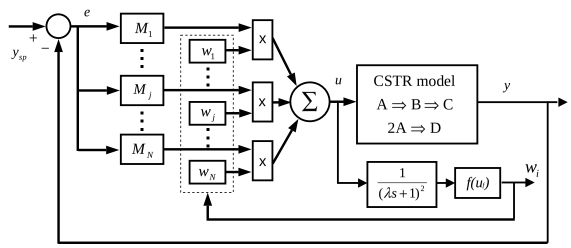


Fig. 3. Closed-loop Schematic

4. CSTR CASE STUDY

For the CSTR case study, we consider the well studied nonlinear benchmark problem of the Van der Vusse scheme (Chen *et al.*, 1995; Engell and Klatt, 1993). This model represents a first-order reaction for $A \Rightarrow B$ with two competing reactions $B \Rightarrow C$ and $2A \Rightarrow D$. All reaction models use temperature dependent Arrhenius reaction rates. The model has four states: concentration of A, concentration of B, reactor temperature, and cooling jacket temperature. With this process, it is desirable to maximize the concentration of product B. This system exhibits many highly nonlinear characteristics which include: input multiplicity, gain sign change, asymmetric response, and transformation from minimum to nonminimum phase behavior. These nonlinear characteristics are most prevalent at the optimal operating point. See Figure 4 for a comparison of dynamic response to a step change in the feed flow rate. From this figure, one can see that inverse response is observed for changing to a low flow rate and not observed at higher flow rates. As seen from the process values at steady-state, the process gain changes from positive to negative, causing input multiplicity. The optimal operating point is at the convergence of changing steady-state gains and inverse to non-inverse dynamic response.

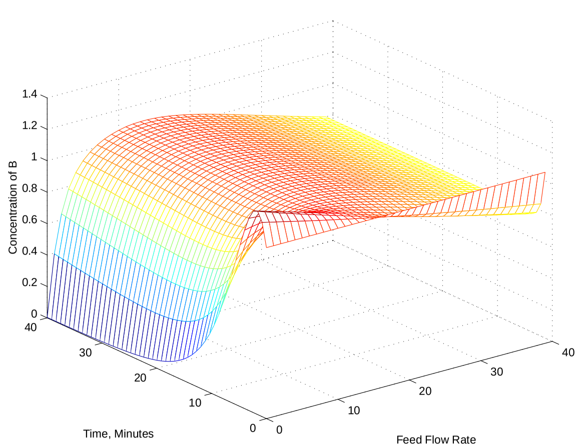


Fig. 4. Response of Model to Step Change at Time=0

A. All simulation results were performed using MATLAB 5.3.

5. CASE STUDY RESULTS

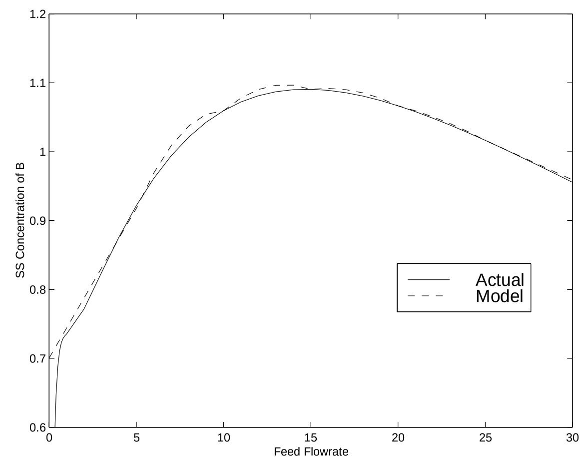


Fig. 5. Comparison of Steady-state Gains

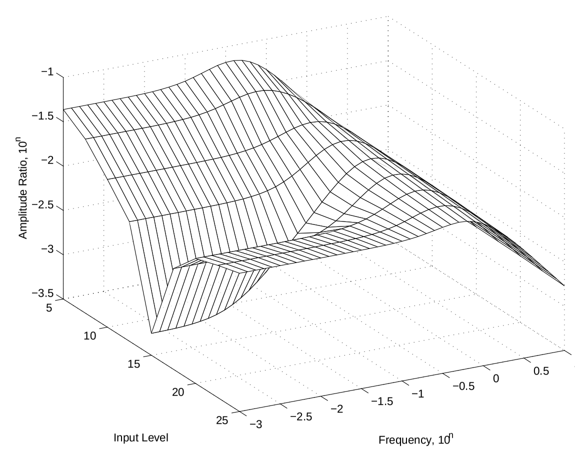


Fig. 6. Amplitude Ratios at Different Operating Points

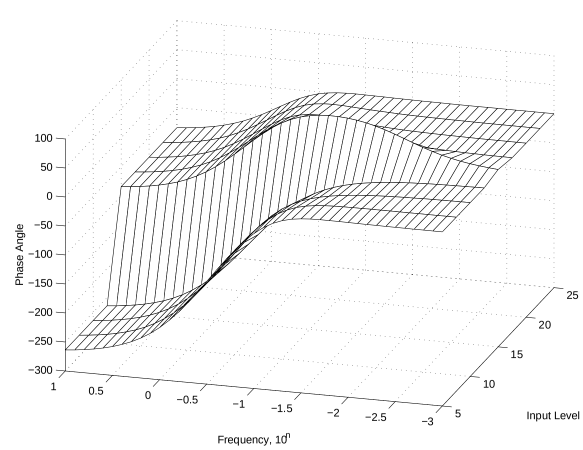
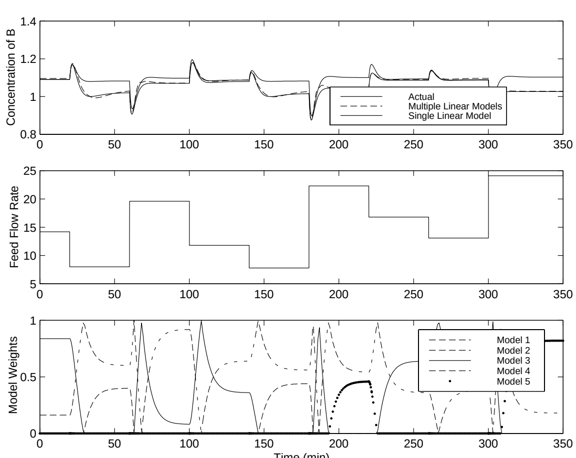


Fig. 7. Phase Angle at Different Operating Points

For this system, five local models were used.

at of these operating points using a Jacobian linearization of the process model. Other types of models or methods for model development could be used. The filter time constant for adjusting model weights was chosen as 4 minutes. This value is typical of the open-loop process system. A steady state comparison of the actual model and the multiple model is given in Figure 5. This shows that at steady-state there is very little modeling error. At the linearization points, the modeling error is zero. The greatest amount of error is found away from the linearization points, between the selected modeling points. In Figures 6 and 7, the frequency dependent amplitude ratios and phase angles of the linearized nonlinear system model are shown for various operating points. The input levels displayed include the five input values as well as operating points between the five operating points. The amplitude ratio displays the expected gain approaching zero near the optimal operating point. The phase angle plots show the inverse response models rolling off to -270 and the non-inverse response models rolling off to -90 as expected.

From Figure 8, one can see that the multiple local models overlay the actual models in most dynamic simulation cases. The nonlinear model fits both the dynamics and steady-state values of the actual process. A comparison with a single linear model (modeled close to the optima, $u = 15\,h^{-1}$) shows that both the steady-state values as well as the dynamic response of the linear model were inaccurate. The multiple linear model correctly predict inverse and non-inverse response, depending upon the input levels and recent operating conditions. The multiple model approach does exhibit a slightly under-damped behavior in some cases where the actual model does not. This can be attributed to the fact that the multiple model approach does not switch to new models quickly enough in some situations.



Results from closed-loop simulations also appear promising (see Figure 9). Here, various step changes in the process setpoint are given to the system. The control system successfully tracks the setpoint at many different levels. One of the linear controller responses (from model 5, $u = 25\,h^{-1}$) demonstrating linear controller limitations is shown for comparison in Figure 9. The nonlinear system is more aggressive than a single linear controller. When given an unobtainable setpoint, the nonlinear controller finds a steady-state near the optimal value. The linear controller destabilizes when faced with an unreachable reference. The nonlinear controller is winding up error and will take some time to track a new feasible reference value. The manipulated input does not constantly remain on one side of the process optima in this example, managing to cross the optima and make use of all the system models. In all simulation cases, input level constraints of 0 and 60 were enforced.

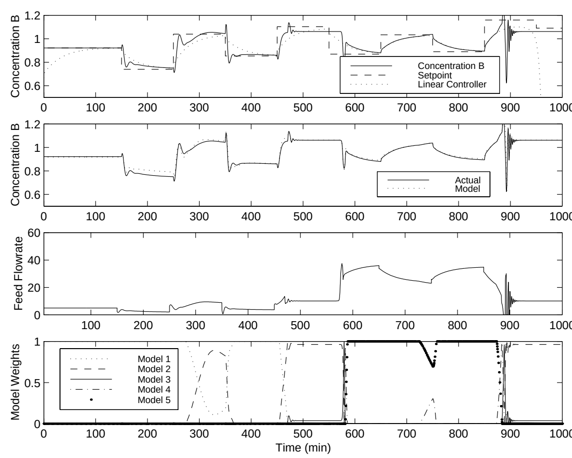


Fig. 9. Closed-loop Response for Setpoint Changes

Disturbance rejection was also tested. In this example, the cooling water flowrate dropped from 10.5 to 5.0 at time=100. As one can see in Figure 10, the nonlinear control system moves to reject this disturbance. This disturbance results in a new system with an optimal value for the concentration of B below the system setpoint. The multiple model control system moves to correct as much as possible. If the disturbance is persistant, the integral action for the controller eventually causes problems in the system since the reference is now unreachable.

6. CONCLUSIONS

In this paper, we have presented a method for using multiple local models to both approximate and control nonlinear systems. This method uses model weighting functions that are based on the

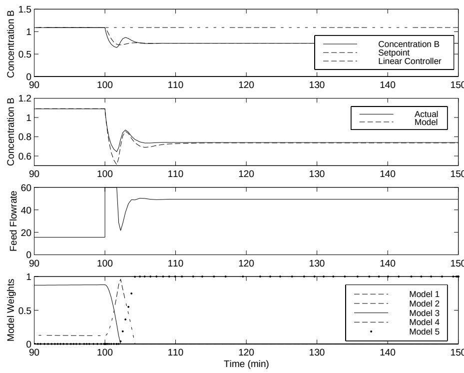


Fig. 10. Closed-loop Response for Disturbance Rejection

models may be used, while few additional models must be identified. Both the dynamic and steady-state characteristics of the nonlinear system may be approximated within this framework. The local models are also not required to be linear in form. The multiple local model controller performs set-point tracking and disturbance rejection.

7. ACKNOWLEDGMENTS

The authors would like to acknowledge funding from the Office of Naval Research (Young Investigator Award N00014-96-1-0695).

8. APPENDIX A- VAN DER VUSSE CSTR EQUATIONS AND PARAMETERS

The state-space model employed in this study is taken from (Chen *et al.*, 1995).

$$
\begin{aligned}
\frac{dC_A}{dt} &= \frac{\dot{V}}{V_R}(C_{AO} - C_A) - k_1(v)C_A - k_3(v)C_A^2 \\
\frac{dC_B}{dt} &= -\frac{\dot{V}}{V_R}C_B + k_1(v)C_A - k_2(v)C_B \\
\frac{dv}{dt} &= \frac{\dot{V}}{V_R}(v_O - v) + \frac{k_w A_R}{\rho C_p V_R}(v - v_K) \\
&- \frac{1}{\rho C_p}\left(k_1 C_A \Delta H_{RAB} + k_2 C_B \Delta H_{RBC} + k_3 C_A^2 \Delta H_{RAD}\right) \\
\frac{dv_K}{dt} &= \frac{1}{m_K C_{PK}}\left(F_K C_{PK}(\nu_{ko} - \nu_K) + k_w A_R (v - v_K)\right) \\
k_i(\nu) &= k_{io} e^{\left(\frac{E_i}{v + 273.15}\right)}
\end{aligned}
$$

The model parameters are detailed below.

$k_1o = 1.287 \cdot 10^{12} h^{-1}$	$E_1 = -9758.3K$
$k_2o = 1.287 \cdot 10^{12} h^{-1}$	$E_2 = -9758.3K$
$k_3o = 9.043 \cdot 10^{9} \frac{1}{mol\ A\ h}$	$E_3 = -8560K$
$\Delta H_{RAB} = 4.2 \frac{kJ}{mol\ A}$	$\rho = 0.9342 \cdot 10^{-4} \frac{kg}{l}$
$\Delta H_{RBC} = -11 \frac{kJ}{mol\ A}$	$C_P = 3.01 \frac{kJ}{kg\ K}$
$\Delta H_{RAD} = -41.85 \frac{kJ}{mol\ A}$	$C_{PK} = 2.0 \frac{kJ}{kg\ K}$
$k_w = 4032 \frac{kJ}{h\ m^2\ K}$	$m_k = 5.0\ kg$
$A_R = 0.215\ m^2$	$V_R = 0.1\ m^3$

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