Model Predictive Control of a Granulation System Using Soft Output Constraints and Prioritized Control Objectives

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Abstract

A granulation system presented by Pottman *et al.* [2] is used to demonstrate two Model Predictive Control (MPC) control methods. The first method penalizes process output constraint violations using soft constraints in the objective function. It is found that the soft constraints must be much tighter than than the actual constraints for effective control of the granulation system. The soft constraint formulation is presented as a variation of the asymmetric objective function formulation described by Parker *et al.* [1]. The second control method is based on the prioritized objective formulation originally proposed by Tyler and Morari [3]. The prioritized objective method uses optimization constraints involving binary variables to explicitly represent and prioritize control objectives. The formulation presented in this article demonstrates a multi-level objective function which first maximizes the number of objectives satisfied in order of priority, then maximizes the number of total objectives, and finally minimizes the traditional MPC error tracking and move suppression terms. This prioritized objective formulation also allows for delayed implementation of output objective constraints, allowing for relaxation of control objectives.

Keywords: Model Predictive Control, Mixed-Integer Optimization, Granulation System

1 Introduction

Particulate processing presents many challenges for design, optimization, and control. In this work, advanced control methods will be demonstrated for control of a multivariable granulation system. Closed-loop simulation results are presented using a granulation system model originally presented by Pottman *et al.* [2]. The system model is based on an industrial granulation system. In this system, a dry feed is sprayed with liquid containing binding agents. Granules grow in size during processing. The product granules are dried and classified based on particle size. Oversized and undersized particles are recycled to be processed again.

The granulation process exhibits many characteristics common to other particulate processes such as crystallization and emulsion polymerization. Typically, a desired product quality can be inferred from the Particle Size Distribution (PSD) of a process. The ability to manipulate a PSD allows for control of the end product quality, but PSD control can pose a very difficult control problem due to the significant multivariable interacting character of PSD systems. In some situations, values of the measured PSD may be a constrained to a specified acceptable region in order to achieve a desired product quality. Output constraints are found in many different process systems, raising interest in the development and improvement of control methods that can accommodate process limitations.

Model Predictive Control (MPC) can accommodate both multivariable interacting systems and systems with process constraints. Two MPC strategies are proposed for control of the granulation system. The first strategy explores the use of soft output constraints to penalize violation of process quality variable constraints. Enforcing hard output constraints can cause closed-loop instability using MPC formulations, so soft constraints have been suggested as a viable alternative [4]. Soft constraints penalizing measurement limit violation can be used in an attempt to keep a process within specifications. Soft constraints significantly affect the objective function when the process output constraints are violated and avoid the creation of infeasible optimization problems. For the granulation system, one-sided bounds on the 5^{th} and 90^{th} percentile of the product PSD are enforced using

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soft constraints in the MPC formulation. Adequate performance (minimal constraint violation) can be established by tightening the soft output constraints to levels much tighter than the actual constraint values.

Output objectives can also be accommodated using mixed integer approaches. A MPC controller can be developed that explicitly accounts for process output objectives. In most situations, specific control objectives are either satisfied or not satisfied. Discrete (binary) variables can be used to represent the value of output objectives [3]. In real systems, some control objectives are much more important than other objectives. The control objectives can be prioritized for a control system using binary variables. A mixed integer optimization optimization problem can be formulated that:

1. Attempts to meet as many objectives as possible in order of priority
2. Attempts to meet as many additional objectives as possible
3. Minimize the traditional MPC error and move suppression terms

A mixed integer problem must be solved at each sampling time for the optimal control value for the system.

In the granulation control system, three manipulated variables are available. These manipulated variables are three variable flow rates of liquid used in the process. Hard limits on the flow rates due to actuator constraints should be considered in the control calculation. For this system, the measured variables include product bulk density, 5^{th} percentile of the product PSD, and 90^{th} percentile of the product PSD. The process model is poorly conditioned because the particle size distribution measurements cannot be controlled independently. The stated control objectives for this system include [2]:

- Track the bulk density reference
- Keep the PSD measurements within limits
- Minimize control effort if possible

The work in [2] considered a Model Predictive Control (MPC) formulation with output constraints. In the original presentation, additional control problems with relaxed output constraints were developed and solved when output constraints caused the MPC optimization problem to become infeasible.

2 Control Formulation

This section presents the general MPC formulation and the two novel approaches proposed in this work: a formulation using soft constraints and a formulation using prioritized control objectives.

2.1 General MPC Formulation

In the traditional MPC formulation, a control move is found to minimize the following objective function at each discrete time value k :

$$\min_{\mathbf{u}} \sum_{i=k}^{k+p} ||\Gamma_y \, \mathbf{e}_y(i)||_2^2 + ||\Gamma_u \, \Delta\mathbf{u}(i)||_2^2 \tag{1}$$

where $\mathbf{e}_y(i)$ is the reference/model error vector at time i, $\mathbf{e}_\mathbf{y}(i) = \mathbf{r}(i) - \hat{\mathbf{y}}(i)$. Here, $\mathbf{r}(i)$ is the reference vector at time i for the predicted process measurements $\hat{\mathbf{y}}(i)$. The process input vector is given as $\mathbf{u}(i)$, where $\Delta\mathbf{u}(i)$ is the difference between $\mathbf{u}(i)$ and $\mathbf{u}(i-1)$. The value p is considered the model prediction horizon. The values for the process inputs beyond a point m in the horizon are assumed constant:

$$\mathbf{u}(i+m) = \mathbf{u}(i+m+1) = ... = \mathbf{u}(i+p) \tag{2}$$

The variables Γ_y and Γ_u in Equation 1 are diagonal matrices that can be used to scale and weight process output error and changes in process inputs, respectively. The Γ_y values can express preference for more aggressive control of one measurement over another. The Γ_u values can reduce actuator chatter and suppress aggressive moves in the calculated process inputs.

The formulation takes advantage of a prediction of the future process outputs using a process model. At time k, for a single predicted output o at time i in the prediction, the model value $\hat{y}_o(i|k)$ can be described by the equation:

$$\hat{y}_o(i|k) = \hat{y}_o(k|k-1) + M_o\mathbf{u} + d_o(k) \tag{3}$$

In this relation, M_o is the impulse response matrix relating the input vector $\mathbf{u}$ to $\hat{y}_o$. The model error, or disturbance update term $d_o(k)$, at the current time step relating the actual measurement, $\mathbf{y}(k)$, to the modeled value $\hat{\mathbf{y}}_p(k|k-1)$ is defined for output o as:

$$d_o(k) = y_o(k) - \hat{y}_o(k|k-1) \tag{4}$$

The model error disturbance value for each of the o output measurements can filtered in the formulation to reduce measurement noise effects and introduce robustness into the algorithm. The manipulated input levels may have absolute bounds at each time step or bounds on the rate of change for the input at each time step. The constraints representing the hard limits on the process inputs (and changes in the inputs) may be respectively written as:

$$\underline{\mathbf{u}} \leq \mathbf{u}(i) \leq \overline{\mathbf{u}} \tag{5}$$

$$0 \leq |\Delta \mathbf{u}(i)| \leq \overline{\Delta \mathbf{u}} \tag{6}$$

The MPC formulation results in a constrained quadratic optimization problem. This optimization problem is solved at every time step for the set of control moves $\mathbf{u}$ which minimize the objective function (Equation 1). Only the first control move in the resulting set of control moves is implemented. At the next sampling time, a new measurement is received and the optimization is again performed.

The objective function in standard MPC formulations is the sum of two distinct terms (Equation 1). One term penalizes reference error across a horizon, the other term penalizes aggressive input control moves, effectively providing a tuning parameter for controller aggressiveness. When the process input levels are limited, a constrained Quadratic Programming (QP) problem must be solved at every sampling time.

The general form of the QP optimization problem can be written as:

$$\min \frac{1}{2}\mathbf{x}_c^T H \mathbf{x}_c + \mathbf{c}^T \mathbf{x}_c \tag{7}$$

subject to the constraints:

$$\begin{aligned} a\mathbf{x}_c^T &\leq \mathbf{b} && (8)\\ \underline{\mathbf{x}} \leq \mathbf{x}_c &\leq \overline{\mathbf{x}} && (9) \end{aligned}$$

This optimization problem is solved at every sampling time for an optimal value of $\mathbf{x}_c$, a vector of continuous values.

2.2 Asymmetric Objective (Soft Constraint) Formulation

In this section, the soft constraint formulation is detailed. Consider a SISO problem without input move weights ($\Gamma_u = 0$). The reference error across the prediction horizon at time i, $\mathbf{e}_y(i)$ can be written as:

$$|\mathbf{r}(i) - \hat{\mathbf{y}}(i)| = \mathbf{e}_y(i) \;\; \forall i = k...k+p \tag{10}$$

For the quadratic MPC formulation, coefficients corresponding to $\mathbf{e}_y$ along the diagonal of the matrix H would minimize the sum square of the error (with $\mathbf{c} = 0$). For a l_1 norm formulation, coefficients for $\mathbf{e}_y$ in the vector $\mathbf{c}$ would would force the minimization of the sum absolute error, with $H = 0$. The l_1 formulation results in a constrained Linear Programming (LP) problem.

Equation 10 can be written as:

$$\begin{aligned} \mathbf{r}(i) - \hat{\mathbf{y}}(i) &\leq \mathbf{e}_y(i) \;\; \forall i = k...k+p && (11)\\ -\mathbf{r}(i) + \hat{\mathbf{y}}(i) &\leq \mathbf{e}_y(i) \;\; \forall i = k...k+p && (12) \end{aligned}$$

Equation 11 is an active constraint whenever the error term is positive, and Equation 12 becomes active when the error is negative. The objective function tries to minimize the values for $\mathbf{e}_y(i)$, forcing a reduction in the values for the output error.

For asymmetric objective function weighting, Equations 11 and 12 can be written for a single output o as:

$$\begin{aligned} r_o(i) - \hat{y}_o(i) &\leq \alpha_o^+ e_{yo}(i) \;\; \forall i = k...k+p && (13)\\ -r_o i) + \hat{y}_o(i) &\leq \alpha_o^- e_{yo}(i) \;\; \forall i = k...k+p && (14) \end{aligned}$$

To implement a soft output constraint on a process output o, r_o (the element of $\mathbf{r}$ corresponding to output o) becomes the soft constraint limit, α_o^+ takes a positive value, and α_o^- is 0. This would keep the measurement below the value r_o. Alternatively, α_o^- could take a positive value, and α_o^+ could be 0 for a soft constraint to keep the measurement above the value r_o. This implies that setpoint tracking is not needed for the output measurement under consideration. If both setpoint tracking and soft constraints are required for a measurement, a new process model with the measurement expressed twice can be used (once for enforcing soft constraints and once for reference tracking).

2.3 Ordered Objective Formulation

For the prioritized objective control formulation, a control move must be found at every sampling time that minimizes the following objective function:

$$\min_{\mathbf{u},\mathbf{o},\mathbf{p}} \; -P_1\mathbf{p} - P_2\mathbf{o} + \sum_{i=k}^{k+p} ||\Gamma_y\left(\mathbf{r}(i) - \hat{\mathbf{y}}_p(i)\right)||_1 + ||\Gamma_u \, \Delta\mathbf{u}(i)||_1 \tag{15}$$

$$p_j, \, o_j \in \{0, 1\} \tag{16}$$

Here, the binary variables p_j and o_j (elements of vectors $\mathbf{p}$ and $\mathbf{o}$, respectively) are used to represent the status of objectives met in order of priority and satisfied objective for the control problem. Vectors $\mathbf{o}$ and $\mathbf{p}$ each contain N_j binary variable elements if there are N_j prioritized objectives. Note that the reference-error and $\Delta\mathbf{u}$ terms are expressed using the l_1 norm rather than a quadratic objective, making this problem a Mixed Integer Linear Programming Problem (MILP).

Consider the case without prioritized objectives (no priority variables, p_j). Constraints can be written such that when a control objective is met, the corresponding o_j binary variable is allowed to take a value of 1. If a prioritized objective cannot be met (due to process time delays, input level saturation, or other reasons) the corresponding o_j binary variable is forced to take a value of 0. P_1 represents the objective function weights for o_j. This value is chosen as a large number. When this number is large enough, as many control objectives as possible will be attempted before the MPC formulation minimizes the terms representing reference tracking error and move penalty.

There are many types of propositional logic constraints that can be written using this formulation. The following objective constraint is written such that a predicted model output ($\hat{y}_o$) stays within a given range ($\pm B_j$) of the reference value, r_o.

$$|r_o(i) - \hat{y}_o(i)| < B_j \;\; \forall i = k...k+p \tag{17}$$

This constraint is implemented using the following two constraints in the MPC formulation:

$$\begin{aligned} r_o(i) - \hat{y}_o(i) - B_j &\leq M_w(1 - o_j) \;\; \forall i = k...k+p & (18) \\ -r_o(i) + \hat{y}_o(i) - B_j &\leq M_w(1 - o_j) \;\; \forall i = k...k+p & (19) \end{aligned}$$

M_w is chosen as a large number. Whenever the objective expressed is Equation 17 can be met, the left-hand-side of Equations 18 and 19 will be negative. This implies that the right-hand-side of Equations 18 and 19 can take on a value of 0 and the binary variable o_j can take a value of 1. If the constraint cannot be met, one of these two constraints will have a positive left-hand-side. To keep the constraint from being violated, the large value of M_w will keep the right-hand-side of the constraint positive and o_j will not be able to switch to a value of 1.

Similar constraints can be written for one-sided bounds on the reference-model error. In cases where the reference error is only limited in one direction, only one of Equations 18 or 19 will be needed. Additionally, objectives bounding the input moves or bounding the change in the input moves can be expressed. Objectives placing limits on inputs could be expressed as hard constraints. In some situations, soft limits on u or Δu may be desirable. For the purposes of prioritized objectives, input level objectives can be written as control objectives on fictitious model outputs which correspond to values of system input levels.

Output bound constraints can be written for all values in the prediction horizon, from the current time k to the last value in the prediction horizon, time $k + p$. A relaxed objective can also be written that only enforces the controller objective constraint for a portion of the prediction horizon. For example, to allow violation of the output objective for the first two prediction steps, the following objective can be used:

$$|r_m(i) - \hat{y}_m(i)| < B_j \;\; \forall i = k+2...k+p \tag{20}$$

This would relax in the first two prediction steps the objective constraint which keeps the outputs close to the reference value. This constraint can be implemented by allowing B_j in Equations 18 and 19 to take an extremely large value for the appropriate values of i in the formulation. A graphical representation of reference bound constraints is shown in Figure 1.

The first two terms in the MPC objective function presented in Equation 15 contain terms for vectors of binary variables $\mathbf{p}$ and $\mathbf{o}$. Rather than formulating the optimization problem to meet as many objectives as possible (maximize the sum of o_j), priorities can be established on control objectives. The binary variables p_j can be used to force objectives to be met in order of their importance (priority). If objective j cannot be met, then the variable for the corresponding priority j must be 0. Priority objective j must be met before priority objective $j + 1$. This is expressed mathematically as:

$$\begin{aligned} p_1 &\leq o_1 \\ p_2 &\leq o_2 \\ &\vdots \\ p_j &\leq o_j \\ &\vdots \\ p_{Nj} &\leq o_{Nj} \end{aligned} \tag{21}$$

The constraints for enforcing priority of objectives are given as:

$$
\begin{array}{ccc}
p_2 & \leq & p_1 \\
p_3 & \leq & p_2 \\
\vdots & & \vdots \\
p_{j+1} & \leq & p_j \\
\vdots & & \vdots \\
p_{N_j-1} & \leq & p_{N_j}
\end{array} \tag{22}
$$

This indicates that for every prioritized objective, there will be two binary variables in the formulation. One is used to express if the objective can be met in the current optimization problem, and one is used to express if the objective can be met AND all higher priority objectives have been accomplished.

The binary variables for objectives and the binary variables for the priority status of objectives both influence the objective function as seen in Equation 17. Values for P_1 and P_2 can be chosen such that the maximum number of objectives are met in order of priority, then as many objectives as possible are met, finally the reference error and move weighting terms are minimized. This means that $P_1 \gg P_2$ and P_2 is much greater than the typical values of the error tracking / move suppression terms in the objective function. Figure 2 graphically shows the relative importance in the objective function for priorities, objectives, and error tracking / move suppression.

Consider a simple example. Suppose that objective 3 cannot be achieved in a control problem due to a time delay in the process (but objectives 1 and 2 can be achieved). This means that $o_3 = 0$ and therefore $p_3 = 0$. As a result, p_4, p_5, and all other lower priority variables must take values of 0. With appropriate values for P_1 and P_2, objectives with lower priority than objective 3 will be attempted, as long as fulfilling these low priority objectives does not interfere with the status of control objectives 1 and 2. Additionally, setpoint tracking will be fulfilled due to the terms for reference error in the objective function.

Control objectives are usually determined by the physical requirements of the process. Typically, for a chemical process, safety constraints must accomplished with highest priority. Additional constraints can be used to minimize long term damage to process equipment. Finally, economic constraints can be used to optimize the process. If reference tracking is highly desirable, multiple discrete output objectives can be written for a single reference such that the controller tracks the reference. High priority objectives would state relaxed tracking for the output, while lower priority objectives would ensure tight tracking. Even if priorities or objectives are set up erroneously (contradictory objectives, mis-ordered objectives) this formulation will reduce to the standard MPC problem.

3 Granulation Control System Application

A granulation system can be used to convert a particulate feed into larger sized particles. In this study, linear models of a granulation system are taken from [2]. The model has three inputs and three outputs. The three inputs to the system are the flow rates to three nozzles that spray a liquid mixture containing binding agents onto the granules. The model was presented such that units for the model steady states, inputs, outputs, and constraints are taken as dimensionless. Steady state flow rates for the three inputs are 175, 175, and 245, respectively. The three outputs are bulk density of the product slurry, the particle size of particles in the 5^{th} percentile in the product, and particle size of particles in the 90^{th} percentile in the product. Steady states for the outputs are 40, 400, and 1620, respectively. All input and output values are reported in scaled dimensionless units. The model for the input-output channel for output i and input j is:

$$
G_{i,j}(s) = \frac{K_{i,j} e^{-\theta j s}}{\tau_{i,j} s + 1} \tag{23}
$$

For the model used in the MPC formulation, the following gain and time constraint parameters are used:

$$
\hat{K}_{i,j} = \begin{bmatrix} 0.20 & 0.58 & 0.35 \\ 0.25 & 1.10 & 1.30 \\ 0.30 & 0.70 & 1.20 \end{bmatrix} \quad \hat{\tau}_{i,j} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix} \tag{24}
$$

For this model, the time delays are considered equal in all channels: $\theta_1 = \theta_2 = \theta_3 = 3$.

For the purposes of the closed-loop simulations, the actual model is perturbed to produce an uncertain "virtual plant". The perturbed process parameters are:

$$
K_{i,j} = \begin{bmatrix} 0.20 & 0.58 & 0.35 \\ 0.25 & 1.10 & 1.30 \\ 0.30 & 0.70 & 1.20 \end{bmatrix} \quad \tau_{i,j} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix} \tag{25}
$$

In the model-mismatch case, the output delays become $\theta_1 = 2$, $\theta_2 = 3$, and $\theta_3 = 4$.

3.1 Standard MPC Results

Initially, the standard MPC formulation was applied to the granulation system. Figure 3 shows closed-loop simulation results for reference tracking and disturbance rejection. The three controlled process measurements are shown above the three manipulated process inputs. Filtered measurement references are shown with dash-dot lines, and constraints on inputs and outputs are shown with solid lines. A sampling time of one minute is used. For the simulation run, a +50 reference change in the bulk density is made at time $t = 5\,min$. Step disturbances of -35 and $+50$ are introduced into the bulk density measurement at times $t = 60\,min$ and $t = 110\,min$. The MPC controller was developed using a move horizon $m = 2$ and a prediction horizon $p = 40$. Measurement error weights of $\Gamma_y = [10\,0.1\,0.1]$ and move suppression weights of $\Gamma_u = [10\,10\,10]$ were used. The mismatch simulation model was used to simulate plant-model error. Output noise with a variance $\sigma^2 = 0.05$ was added to the output measurement signal. Input level constraints for the feed flow rates are plotted, as well as the quality constraints for the PSD measurement. Note that there are significant violations of the quality constraints using this controller configuration.

3.2 Soft Output Constraint Results

The asymmetric objective function can be seen as a general case of soft output constraints. The asymmetric objective can be used to enforce a soft constraint value by setting one of the α values to 0 and the other to a positive value. The reference for the process output now becomes the soft constraint value. No control action will be taken when the process output is on the acceptable side of the reference. When the process output moves across the output constraint (reference) and into the constrained region, the objective function will be affected and control action will be taken to correct the situation.

Figure 4 shows a closed-loop simulation result using the same reference changes and disturbances illustrated in Figure 3. In Figure 4, soft constraints are used to penalize violation of PSD limits. The soft constraints are set to values equal to the limits for the measurements. No reference value is used for the 5^{th} and 90^{th} percentile measurements. Note that the PSD limits are still violated using this control framework.

Figure 5 shows a similar closed-loop simulation, again using the same reference changes and disturbances illustrated in Figure 3. The soft constraint values are set to levels much tighter than the actual limits for the measurements. Again, no reference value is used for the 5^{th} and 90^{th} percentile measurements. In this case, the amount of constraint violation is reduced using soft output constraints in the formulation.

3.3 Ordered Objective Results

The control objectives are developed from [2]. Tracking of the slurry density is the most important objective. The particle size constraints are lower in priority, but strong control action is required if these values exceed their limits. Over-sized and under-sized particles must be recycled and processed again so reprocessing is economically undesirable. Table 1 shows the list of prioritized control objectives used to formulate the controller for this problem.

For the closed-loop MPC runs, the tuning parameters are chosen as: $m = 2$, $p = 40$, $\Gamma_y = [10\,0.1\,0.1]$, $\Gamma_u = [10\,10\,10]$, measurement update filter $= 1$, with an output noise variance of $\sigma^2 = 0.05$ added to each output. Recall that Figure 3 shows a closed-loop run without using soft constraints or the integer formulation.

The closed-loop results using the prioritized objective MPC formulation are shown in Figure 6. This closed-loop run consists of a setpoint change in the slurry density, a -35 bias step change change in the slurry density measurement, and a +50 bias step change in the slurry density measurement. The particle size constraints are shown in Figure 6 for outputs 2 and 3. Note that excursions across process outputs constraints are limited using the MILP MPC formulation.

Figure 7 shows the number of objectives met in order of priority and the total number of objectives met as a function of time. It can be seen that many control objectives are initially infeasible when significant reference changes or disturbances affect the process. As the process returns back to normal operating conditions, more of the control objectives can be met. Figure 7 also shows that in some cases, few control objectives can be accomplished in order of priority, but some of the additional objectives are satisfied. Figure 8 shows the values for all of the nine objectives as a function of time. As expected, high priority objectives are violated less often than low priority objectives. This formulation shows that graceful relaxation of control objectives when output constraints cannot be achieved.

4 Summary

In this article, two extensions to traditional MPC controllers were presented for control of a granulation system: soft output constraints and prioritized control objectives.

Some systems may have constraints on process outputs. A MPC formulation using asymmetric error weights was used to enforce soft constraints on process measurements. Using the granulation system, it was shown that the soft constraint values must be much tighter than the actual limits in order to avoid constraint violation.

Finally, mixed integer methods were used to develop control strategies based on prioritized objectives. Binary variables were used to express the the feasibility of output objectives and the feasibility of meeting output objectives in prioritized order. Proper objective function weighting of the priority and objective binary variables results in a controller that can meet as many objectives as possible in order of priority, then meet as many of the remaining additional objectives as possible, then minimize tracking error and control changes. Again, a mixed integer optimization problem must be solved at each sampling time to calculate the control move. The presented control methods rely upon the real-time solution of computationally demanding constrained optimization problems.

Acknowledgments

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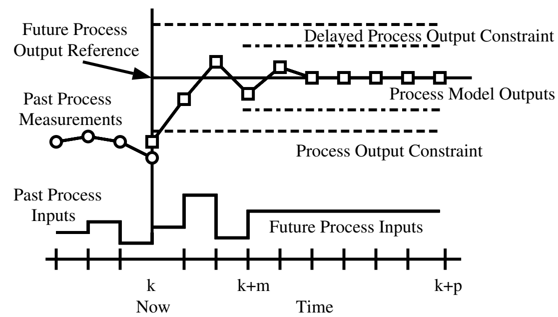

Figure 1: General diagram of Model Predictive Control with output constraints. The constraints are used to keep the predicted output values close to the reference value.

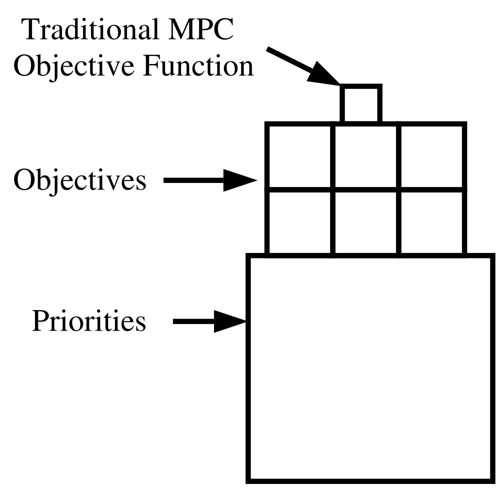

Figure 2: Relative contributions to objective function for the prioritized objective MPC formulation showing the large weight on priorities, moderate weight on objectives, and small weight for the traditional MPC objective function terms.

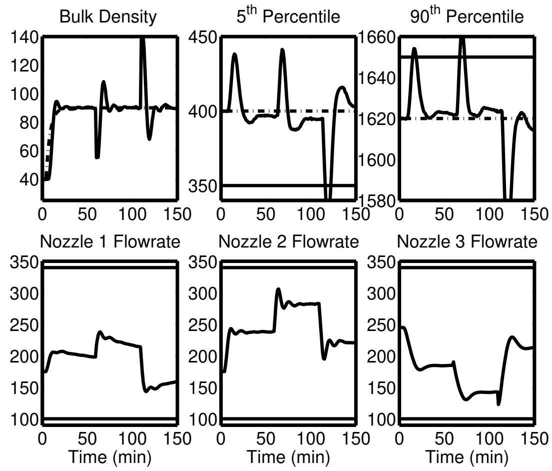

Figure 3: Closed-loop results for the granulation system using the normal QP MPC formulation. MPC tuning parameters are $m = 2$, $p = 40$, $\Gamma_y = [10\,0.1\,0.1]$, $\Gamma_u = [10\,10\,10]$, with an output noise variance $\sigma^2 = 0.05$. Filtered output reference levels (dash-dot), output constraint values (solid), and input constraint levels (solid) are plotted.

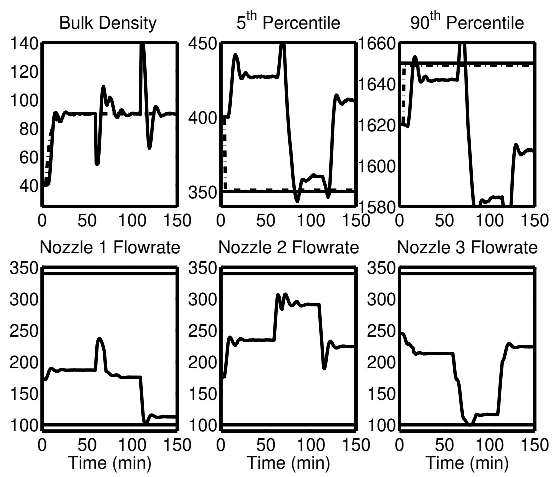

Figure 4: Closed-loop results for the granulation system using soft constraints for 5^{th} and 90^{th} percentile measurements. Output constraint levels (dash-dot) in this case are equal to the actual measurement limits. QP MPC tuning parameters are $m = 2$, $p = 40$, $\Gamma_y = [10]$, $\Gamma_u = [10\,10\,10]$, with an output noise variance $\sigma^2 = 0.05$. Soft constraint limits are shown for 5^{th} and 90^{th} percentile measurements. Constraint violation was weighted as $\alpha = [10\,10]$. Filtered bulk density reference level (dash-dot), output constraint values (solid), and input constraint levels (solid) are plotted.

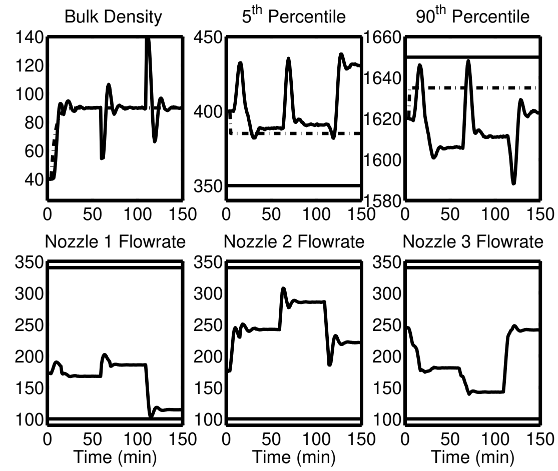

Figure 5: Closed-loop results for the granulation system using soft constraints for 5^{th} and 90^{th} percentile measurements. Output constraint levels (dash-dot) in this case are tighter than the actual measurement limits. QP MPC tuning parameters are $m = 2$, $p = 40$, $\Gamma_y = [10]$, $\Gamma_u = [10\,10\,10]$, with an output noise variance $\sigma^2 = 0.05$. Soft constraint limits are shown for 5^{th} and 90^{th} percentile measurements. Constraint violation was weighted as $\alpha = [10\,10]$. Filtered bulk density reference level (dash-dot), output constraint values (solid), and input constraint levels (solid) are plotted.

1. $\lvert r_1 - y_1 \rvert \leq 5$
2. $\lvert r_1 - y_1 \rvert \leq 3$
3. $\lvert r_1 - y_1 \rvert \leq 2$
4. $y_3 \leq 1650$
5. $y_3 < 1635$
6. $y_2 > 350$
7. $y_2 > 370$
8. $\lvert r_1 - y_1 \rvert \leq 1$
9. $\lvert r_1 - y_1 \rvert \leq 0.5$

Table 1: The nine objectives for the 3x3 control problem. y_1 is the bulk density of the product slurry, y_2 is the particle size of particles in the 5^{th} percentile in the product, and y_3 is the particle size of particles in the 90^{th} percentile in the product. Enforcement of all objectives are delayed by 1 prediction step, except for the loose one-sided objectives (4 and 6) which are delayed by three prediction steps.

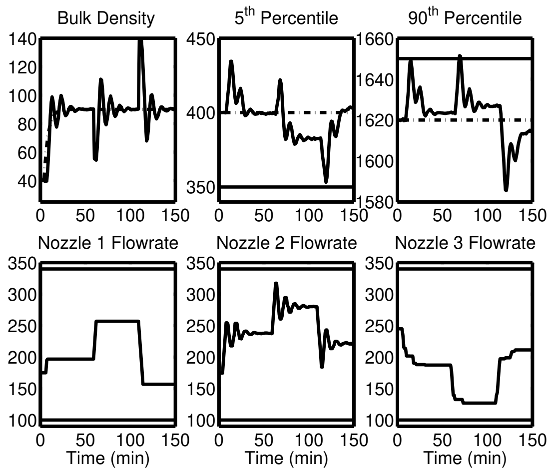

Figure 6: Closed-loop results for the granulation system using prioritized objective control. The prioritized list of objectives from Table 1 are used in the formulation. The MILP MPC tuning parameters are $m = 2, p = 40, \Gamma_y = [10\,0.1\,0.1], \Gamma_u = [10\,10\,10]$, with an output noise variance $\sigma^2 = 0.05$. Filtered output reference levels (dash-dot), output constraint values (solid), and input constraint levels (solid) are plotted.

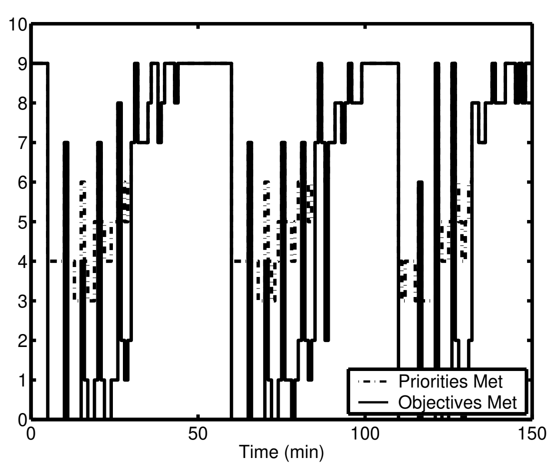

Figure 7: Plot showing the number of control objectives met in order (priorities) and the total number of objective met at every sampling time for the granulation control problem.

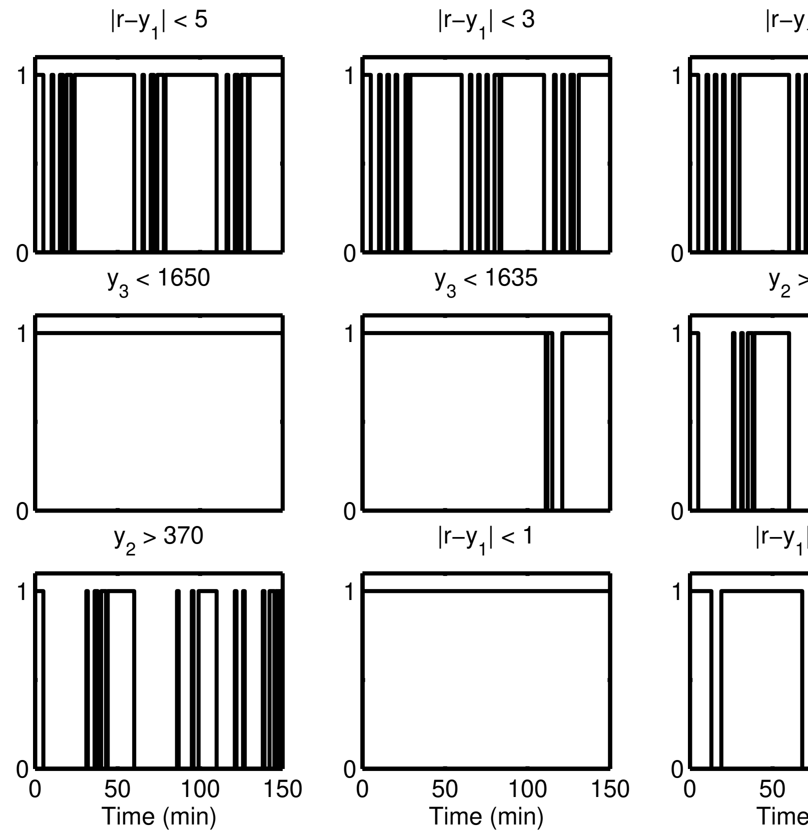

Figure 8: Plots of the values for the separate control objectives in the granulation control problem.