

# Control of a Granulation Process Using a Nonlinear MPC Formulation

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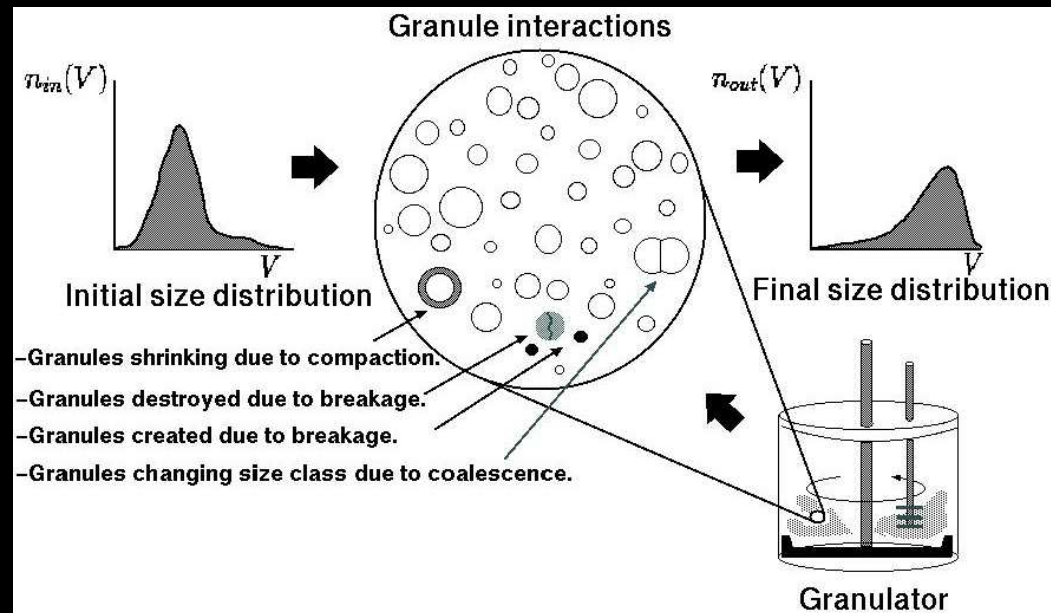
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# Introduction

- Granulation processes require consistent product quality: size uniformity, proper flowability, attrition resistance, etc.
- Product quality indicators are related to:
  - Particle size distribution
  - Bulk density
- Model Predictive Control (MPC) has been used in granulation processes to control these granular quantities.
- What is MPC?
  - Controller formulates an optimization problem, representing the minimization of an objective function.
  - Continuous MPC vs. Batch (Pottman *et al.* 2000)
  - Nonlinear process concerned with PBEs
  - Nonsquare process

# Plant Model

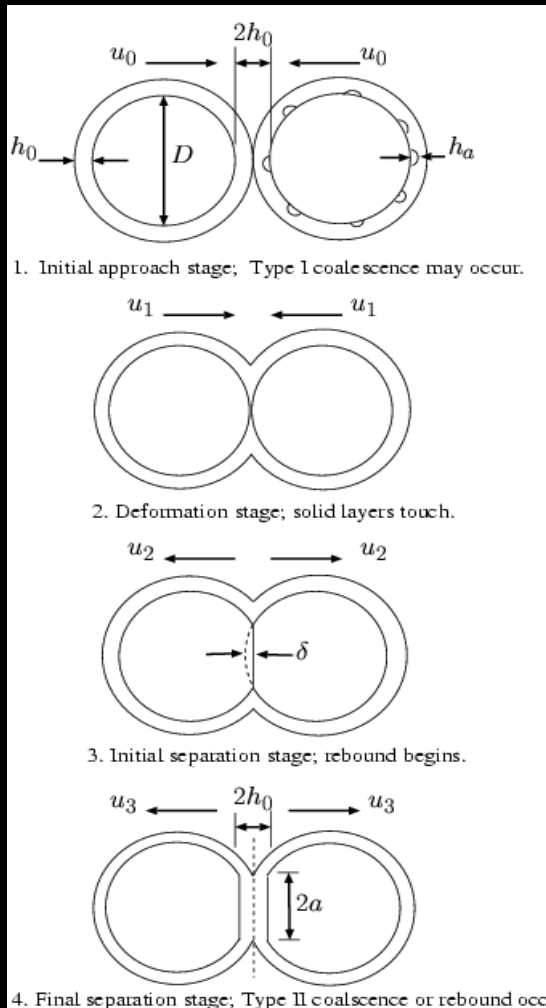
- A high fidelity plant model was derived based on physical models of granular interactions.
- Individual particles are modeled using a DEM approach.
- Due to physical interactions, particles coalesce, consolidate, and break.



# Plant Model

- *Granule position*: particles are randomly given a fixed, non-overlapping position in 3D space.
- *Granule composition*: Particles begin in a nucleated matrix of solid powder, trapped air, and liquid binder.
- *Granule velocity*: Particles are assigned a 3D velocity from a normal velocity distribution based on the impeller speed.
- *Consolidation*: As granules collide with each other, air is slowly forced out. As air leaves, more binder is pushed to the surface.
- *Coalescence*: As particles collide, they will coalesce or rebound based on criteria defined by Liu *et al.*

# Coalescence of Wet Granules



- Granule growth due to collisions.
- Probability of successful collision is determined by presence of binder layer,  $h$ .
- **Type I coalescence:** viscous dissipation of the binder layer is greater than the kinetic energy of the collision.
- **Type II coalescence:** deformation of collision prevents rebounding of granules.
- **Rebound:** occurs when the kinetic energy of the collision is great enough to prevent coalescence.

# Population Balance Equation Model

- PBE models are used in particulate modeling to calculate size distributions and to determine rate controlling mechanisms.
- PBE's are based on kinetic rate expressions:
  - Birth rate / death rate
  - Growth rate (coalescence) / Attrition and breakage rate
- Batterham *et al.* 1981, Ramkrishna *et al.* 1985, and Hounslow *et al.* 1988 used numerical techniques to solve PBEs.
- Discrete solutions considered particles of different sizes to exist in groups that interacted collectively with particles of different groups.

# Population Balance Equation Model

- Verkoijen *et al.* 2002 proposed a discrete multi-dimensional PBE using volume as the intrinsic parameter. The model tracked evolution of solid, liquid and air volume based on *coalescence* and *consolidation* of granules.
- PBE model: 
$$\frac{dq_{sk}}{dt} = \sum_{j=1}^n \left( \frac{\beta_{j\bar{j}} q_{sj} - \beta_{kj} q_{sk}}{j s_1 N_{tt}} \right) q_j$$
- A coalescence kernel was then created to capture induction behavior, promote granule growth with binder addition and decrease growth due to high impeller speeds.

$$\beta(u, v) = \begin{cases} \beta_0 (\ln(1.5 + \omega))^{2.5} (q_k / q_{k-1}) & \text{for } S \geq S^* \text{ \& } \omega > 0 \\ 0 & \text{for } S < S^* \text{ \& } \omega = 0 \end{cases}$$

# Controller Formulation

- The PBE model of the granulation process can be simplified to the following nonlinear discrete time system:

$$\begin{aligned} x(k+1) &= f(x(k), u_1(k), u_2(k)), \quad k = 0, 1, \dots, n \\ y(k) &= x(k) \end{aligned}$$

$x(k)$ : 5 discrete particle size classes

$u_i(k)$ : inputs impeller speed and binder volume

- MPC control moves are found by minimizing the objective function:

$$\min_{U(k)} \Phi(k) = \sum_{i=k}^{k+p} e_y(i)^T \Gamma_y e_y(i) + \sum_{i=0}^{M-1} \Delta u(i)^T \Gamma_u \Delta u(i)$$

subject to constraints:  $u(i) \in [u_{\min}, u_{\max}]$



# Controller Formulation

- Batch MPC concerns:
  - Batch process follow a prescribed trajectory.
  - Error compares the PBE model to the ref. DEM trajectory.

$$e_y(i) = \sum_j^N (y_j(k+i|k) - y_{sj})$$

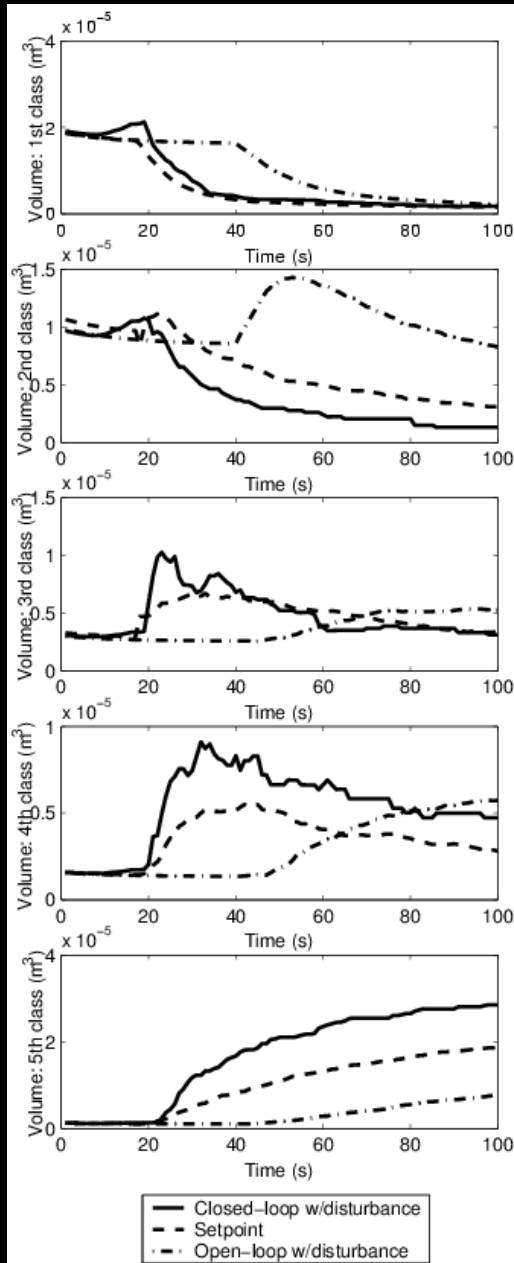
- Trajectory tracking poses problems due to nonlinearity.
- A shrinking horizon approach is use to minimize the objective function over the duration of the batch.
- $U(k)$  is a vector of previous and future input moves.
  - $\Phi(k)$  is minimized through future input moves over the horizon  $m$ .

$$U(k) = [u_1, u_2, \dots, u_{rp}, u(k), u(k+1), \dots, u(n)]$$

# Results

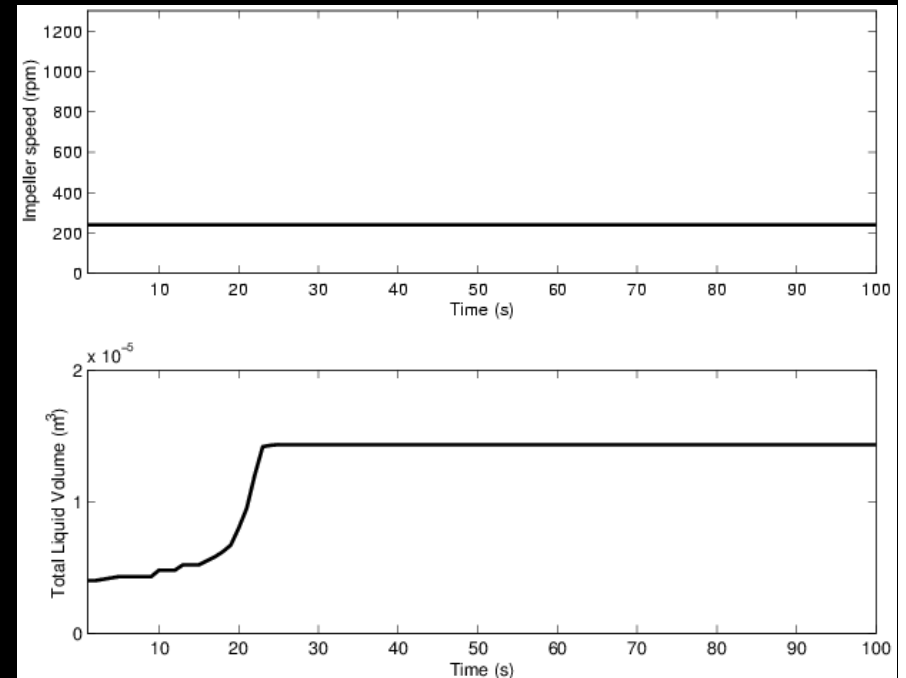
- The following parameters were chosen for the controller:  
 $\Gamma_{u1} = 0$ , and  $\Gamma_{u2} = 0$ , and  $m=2$ .
- The desired trajectory used was an average of 20 open-loop DEM batch results.
- Using the PBE model, a batch trajectory was calculated and compared to the desired DEM averaged trajectory.
- Several different trials were examined:
  - Trial 1: Identical initial PSD for open and closed loop, with the desired trajectory including *more initial binder*.
  - Trial 2: Identical initial PSD for open and closed loop with the desired trajectory including *less initial binder*.

# Results: Trial1

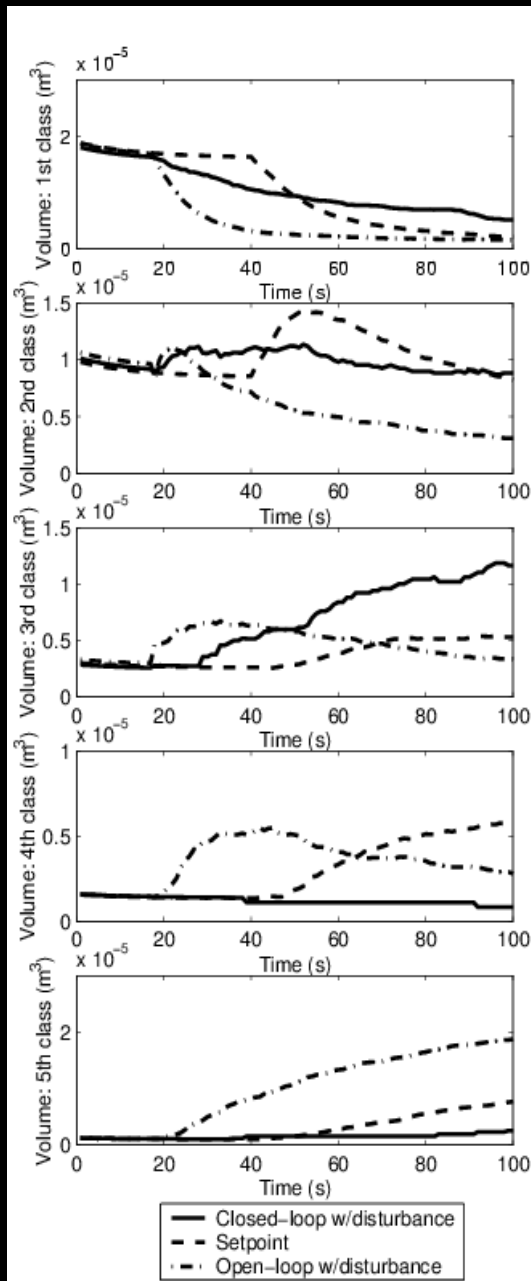


A move horizon less than 2 produced inadequate results.

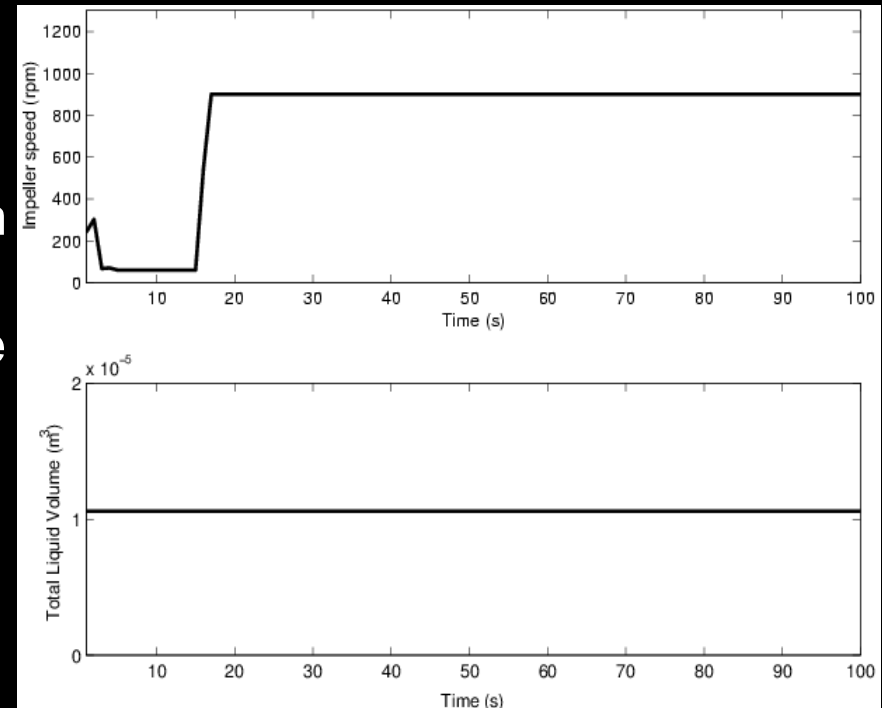
Binder addition was only input move used.



# Results: Trial 2



- Test 2 used and open loop trajectory with initially too much binder present.
- Only one degree of freedom: length of induction time could not be altered.
- Increasing impeller speed to saturation was only input move.



# Results: Error Comparison Trial 1

| Size Class     | Open loop error<br>( $m^3 \times 10^9$ ) | Closed loop<br>error ( $m^3 \times 10^9$ ) |
|----------------|------------------------------------------|--------------------------------------------|
| 1              | 3.583                                    | 0.7031                                     |
| 2              | 2.992                                    | 0.6744                                     |
| 3              | 0.5012                                   | 0.1486                                     |
| 4              | 0.5035                                   | 0.8644                                     |
| 5              | 6.984                                    | 5.482                                      |
| $\Sigma$ error | 14.56                                    | 7.877                                      |

# Results: Error Comparison Trial 2

| Size Class     | Open loop error<br>( $m^3 \times 10^9$ ) | Closed loop<br>error ( $m^3 \times 10^9$ ) |
|----------------|------------------------------------------|--------------------------------------------|
| 1              | 3.583                                    | 0.9214                                     |
| 2              | 2.992                                    | 0.3673                                     |
| 3              | 0.5012                                   | 1.413                                      |
| 4              | 0.5035                                   | 0.6619                                     |
| 5              | 6.984                                    | 0.5835                                     |
| $\Sigma$ error | 14.56                                    | 3.947                                      |

# Conclusions

- A batch Nonlinear MPC controller has been formulated which follows a batch trajectory for the PSD's of 5 particle size classes in a simulated granulation process.
- The DEM effectively modeled the plant high shear granulator.
- A new coalescence kernel was derived to take account for changes in binder volume and impeller speed when using a PBE model.
- The NMPC controller minimized the error of 5 size classes over the open-loop trajectory for two given circumstances, each with only one degree of freedom.

# Future Work

- Model Validation
- Real-time considerations



# Acknowledgements

- University of South Carolina