A Mixed Integer Moving Horizon Formulation for Prioritized Objective Inferential Control of a Bioprocess System

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Presentation Outline

- Motivation
- Mixed Integer Formulation
  - ➢Multiple objectives of equal priority
  - ➢Application for inferential control
- Case Study
  - ➢Fermentation Pathway System
- Summary

Motivation

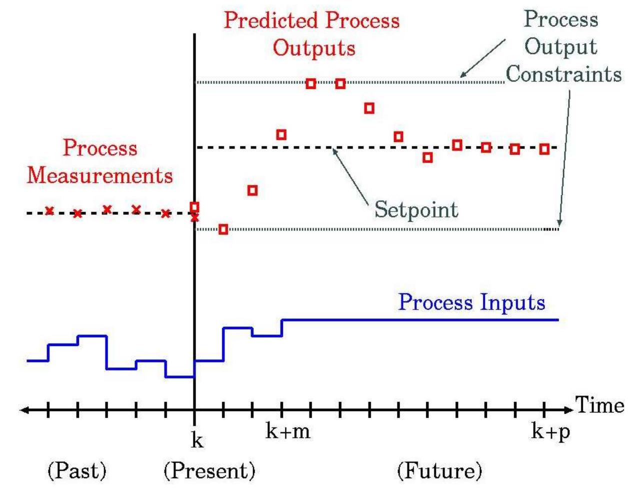
- Many process states may be unmeasured.
  - Too expensive, too time consuming, or impossible to measure
- Control of unmeasured states may be desired.
  - Ex. Various metabolites in a fermentation reactor.
- Model Predictive Control (MPC) methods can handle a broad class of systems.
  - Adequate dynamic model is needed to model states
  - Closed-loop results are often difficult to predict with traditional MPC tuning.
  - Prioritized objective MPC formulation offers a well-defined ordered list of control objectives. (Tyler and Morari, 1998, Bemporad and Morari, 1998)

Model Predictive Control (MPC)

(Garcia, Prett, and Morari, 1989)

- Model Predictive Control is an advanced control algorithm that handles:
  - Multivariable interacting systems
  - Soft constraints on both inputs and outputs
  - Measured disturbances
  - Process time delays and difficult dynamics
  - Known future setpoint changes (reference transitions)
- Limitations
  - Requires an explicit model of the process
  - Need to solve a constrained optimization problem online

1. Using process data, formulates an optimization problem.
2. Solves the problem online and implements the optimal control move.
3. Waits for new data at next time step.



The Optimization Problem

- Objective function:

$$\Phi = \sum_{i=1}^{p} \Gamma_e e(i) + \sum_{i=1}^{m} \Gamma_u \Delta u(i)$$

LP formulation

where:

Setpoint error (e): $e(i) = |r(i) - y(i)|$

Input movement (Δu): $\Delta u(i) = |u(i) - u(i-1)|$

Γ_e is a vector of weights (penalties) with entries corresponding to each error term.

Γ_u is a vector of weights with entries corresponding to each input movement term.

- The optimization problem:

$$\min_{\{u(i), \dots, u(m)\}} \Phi$$

subject to:

output, error (e), and input movement (Δu) constraints

*Only continuous variables → standard LP (1-norm, ∞-norm)

QP (2-norm)

State-Space Approach

- A linear state-space model of the system was obtained:

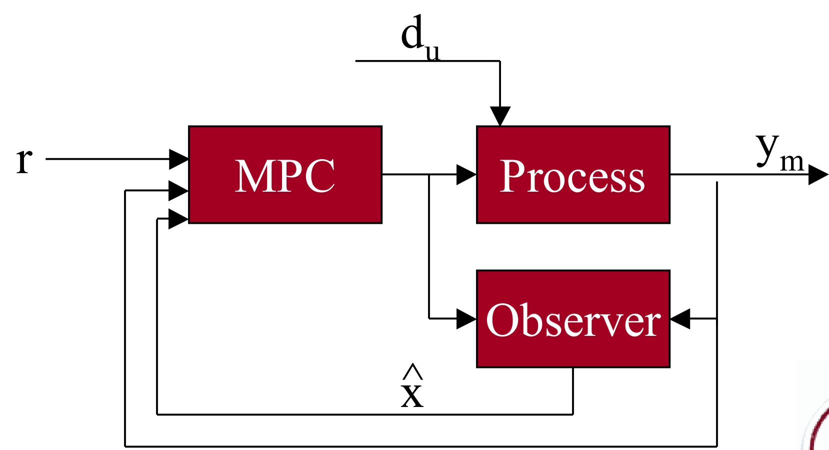
$$x(k+1) = Ax(k) + B_u u(k)$$

$$y(k) = Cx(k) + Du(k) + \textit{update}$$

> ➢Model explicitly defines all states, enabling constraints to be placed on unmeasured states just as they are placed on process outputs.

- State estimates for both measured and unmeasured states are found using an external state estimation routine such as a Luenberger Observer.

r = reference (setpoint)
y_m = measured output
d_u = unmeasured disturbance
$\hat{x}$ = state estimate



Mixed Integer Formulation to Prioritize Objectives

(Tyler and Morari, 1998, Gatzke and Doyle, 2001)

[redacted] Discretizing and prioritizing objectives leads to explicitly defined control objects avoiding difficulties associated with MPC tuning.

[redacted] Increase in the computational complexity of the optimization problem.

(more constraints, and new binary variables)

Additional objective function terms: penalize not meeting discrete objectives and not doing so in order.

$$\Phi = \Gamma_O O + \Gamma_P P + \sum_{i=1}^{p} \Gamma_e e(i) + \sum_{i=1}^{m} \Gamma_u \Delta u(i)$$

Additional constraints:

$$e(i) \le N_j(1 - O_j) + B_e \qquad \forall i = 1..p$$

Discretize Continuous Control Objectives

$$O_i \le P_i \qquad \forall i = 1..N_o$$

Ensure objective is met before priority flag is satisfied.

$$P_{i+1} \le P_i \qquad \forall i = 1..N_{p-1}$$

Ensures objectives are met in order of priority.

Binary Variables: LP $\Rightarrow$ MILP

QP $\Rightarrow$ MIQP

Multiple Objectives of Equal Priority

(Grossmann, 1991)

$$O_i \leq P_i$$

$$\forall i = 1..N_o$$

Ensures objective is met before priority flag is satisfied.

$$N_o = N_p$$

\# of Objectives

\# of Priorities

- Each discrete objective is assigned its own priority flag.

- This does not have to be the case.

Extension: Objectives of Equal Priority

[redacted] (N_o=4,N_p=3)

Both O_1 and O_2 are of priority 1.

$$O_1 \leq P_1$$
$$O_2 \leq P_1$$
$$O_3 \leq P_2$$
$$O_4 \leq P_3$$

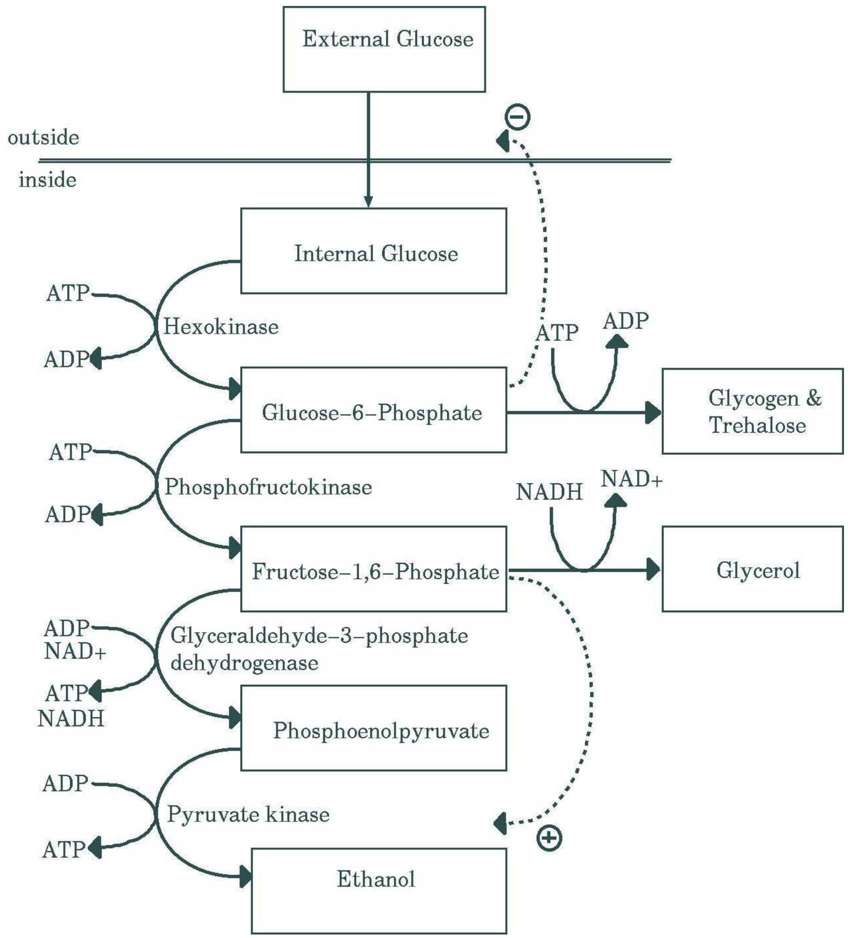
Same Priority

- Both O_1 and O_2 must be met for P_1 flag to be satisfied.
- Can be extended to handle any logical clause using propositional logic.
- Ensure objectives are met in order as before:

$$P_{i+1} \leq P_i$$

$$\forall i = 1..N_{p-1}$$

Anaerobic Fermentation Pathway of *Saccharomyces cerevisiae*



(adapted from Curto et al. 1995)

X_1 = Internal glucose
X_2 = Glucose-6-phosphate
X_3 = Fructose-1,6-diphosphate
X_4 = Phosphoenol pyruvate
X_5 = ATP
} Dynamic States

X_6 = Glucose uptake } Input

X_7 = Hexokinase/glucokinase
X_8 = Phosphofructokinase
X_9 = Glyceraldehyde-3-phosphate dehydrogenase
X_{10} = Pyruvate kinase
} Assumed Constant

X_{11} = Glycogen and trehalose production
X_{12} = Glycerol production
} Disturbances

X_{13} = ATPase
X_{14} = NADH/NAD+ ratio
} Assumed Constant

Modeling the Complex Metabolic Pathway

- By representing each individual metabolic reaction as a power-law term, the resulting model will be a Generalized Mass Action (GMA) model.

(Cascante et al., 1995, Curto et al., 1995 and Sorribas et al., 1995)

- The 5 state GMA model:

Internal glucose

$$\dot{X}_1 = 0.8122\, X_2^{-0.2344} X_6 - 2.8632\, X_1^{0.7464} X_5^{0.0243} X_7$$

Glucose-6-phosphate

$$\dot{X}_2 = 2.8632\, X_1^{0.7464} X_5^{0.0243} X_7 - 0.5232\, X_2^{0.7318} X_5^{-0.3941} X_8 - 0.0009\, X_2^{8.6107} X_{11}$$

Fructose-1,6-diphosphate

$$\dot{X}_3 = 0.5232\, X_2^{0.7318} X_5^{-0.3941} X_8 - 0.011\, X_3^{0.6159} X_5^{0.1308} X_9 X_{14}^{-0.6088} - 0.04725\, X_3^{0.05} X_4^{0.533} X_5^{-0.0822} X_{12}$$

Phophoenol pyruvate

$$\dot{X}_4 = 0.022\, X_3^{0.6159} X_5^{0.1308} X_9 X_{14}^{-0.6088} - 0.0945\, X_3^{0.05} X_4^{0.533} X_5^{-0.0822} X_{10}$$

Adenosine triphosphate (ATP)

$$\dot{X}_5 = 0.022\, X_3^{0.6159} X_5^{0.1308} X_9 X_{14}^{-0.6088} + 0.0945\, X_3^{0.05} X_4^{0.533} X_5^{-0.0822} X_{10} - 2.8632\, X_1^{0.7464} X_5^{0.0243} X_7$$
$$- 0.0009\, X_2^{8.6107} X_{11} - 0.5232\, X_2^{0.7318} X_5^{-0.3941} X_8 - X_5 X_{13}$$

The Simplified System

[redacted]

Manipulated Variable (u) = X_6 (Rate of External Glucose Uptake)

Process States (x) = X_1, X_2, X_3, X_4, X_5 (X_3 = Fructose-1,6-diphosphate conc.)

Process Output (y) = X_5 (ATP concentration)

Disturbances (d) = X_{11}, X_{12} (Polysaccharide/glycerol production rates)

[redacted]

u_{ss} = 19.7 mM/min

X_{3ss}= 9.1969 mM

y_{ss} = 1.1247 mM

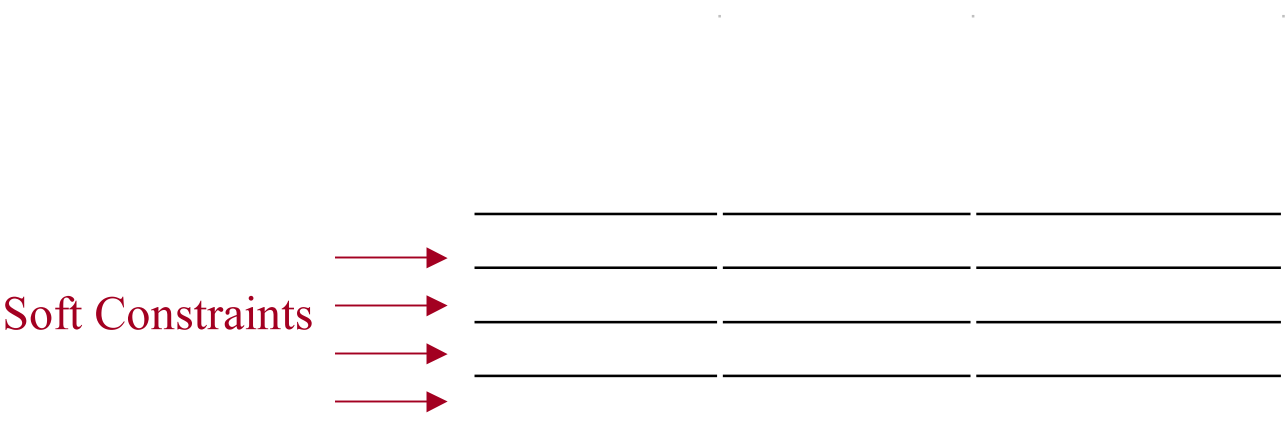
d_{ss} = 14.31 mM/min and 203mM/min

SIMO Process (single-input multi-output) system.

Two variables (X_3 and y) are to be controlled by manipulating a single input (X_6).

Closed-Loop Performance Test

- At t = 5 min, the setpoint of ATP concentration (y_{sp}) was stepped from 1.1247 mM to 1.2 mM.
- The concentrations of ATP (y) and fructose-1,6-phosphate (X_3) are bounded above and below at numerous levels.
- At t = 25, a disturbance is introduced by stepping the rate of glycerol production (d_2) from 203 mM/min to 0 mM/min.



Traditional MPC Results

[redacted]

$m = 2$

$p = 30$

$$\left.\begin{aligned}\Gamma_{x_3} &= 1000\\ \Gamma_y &= 100\end{aligned}\right\}\Gamma_e$$

$\Gamma_{\Delta u} = 50$

[redacted]

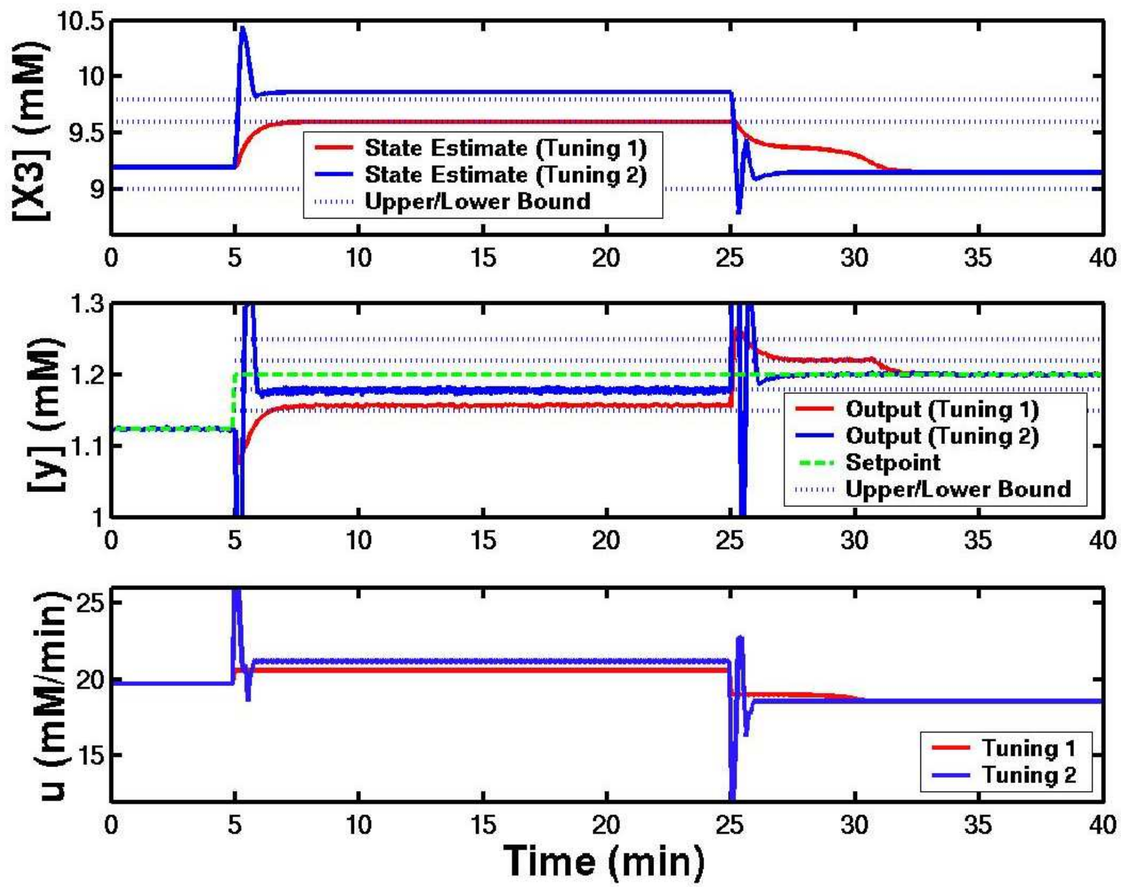
$m = 2$

$p = 30$

$\Gamma_{x_3} = 100$

$\Gamma_y = 1000$

$\Gamma_{\Delta u} = 50$



Sample Time: 6 secs

LP: (424 Continuous variables, 0 binary variables)

LP solver from the IBM OSL Library : Solution time ~2.5 seconds

Prioritized Objective MPC

Discrete 6 5
Discrete 7 6

Move horizon (m) = 2

Prediction horizon (p) = 30

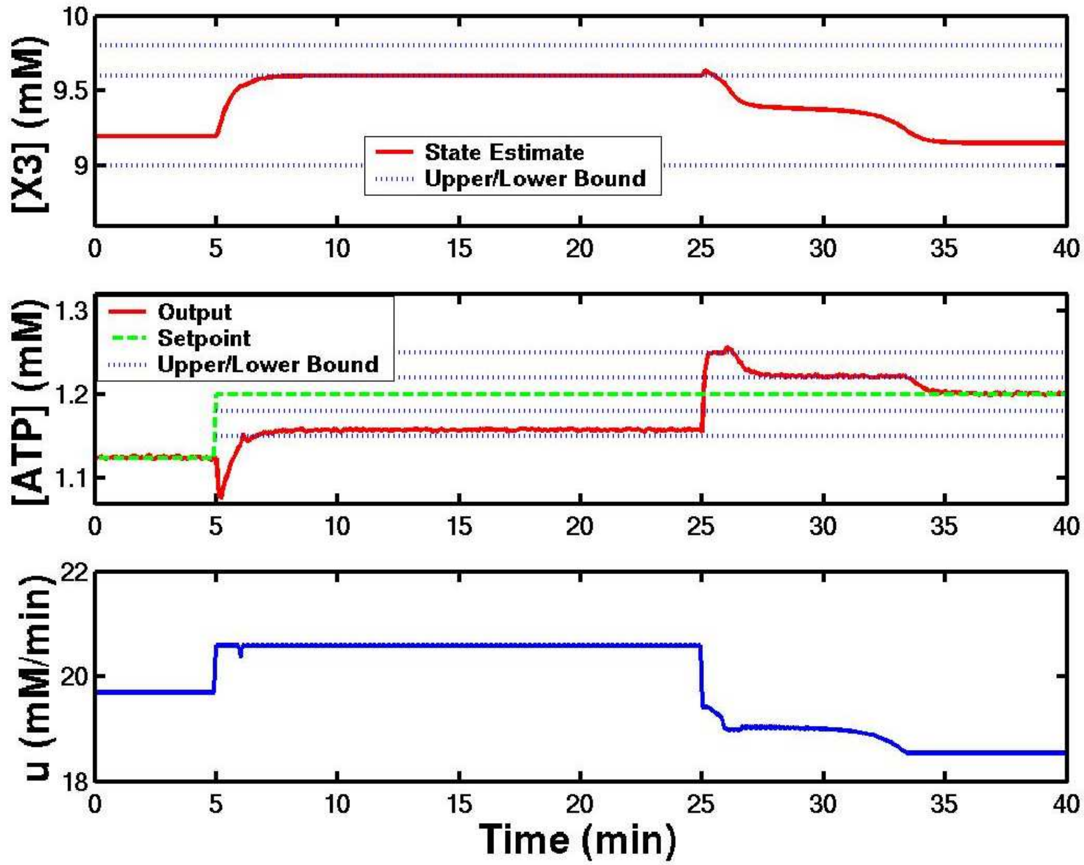
Elements of Γ_e = 100

Elements of $\Gamma_{\Delta u}$ = 50

Elements of Γ_O = -1,000

Elements of Γ_P = -20,000

Prioritized Objective MPC Results



m=2, p = 30

$\Gamma_e = 100$

$\Gamma_{\Delta u} = 50$

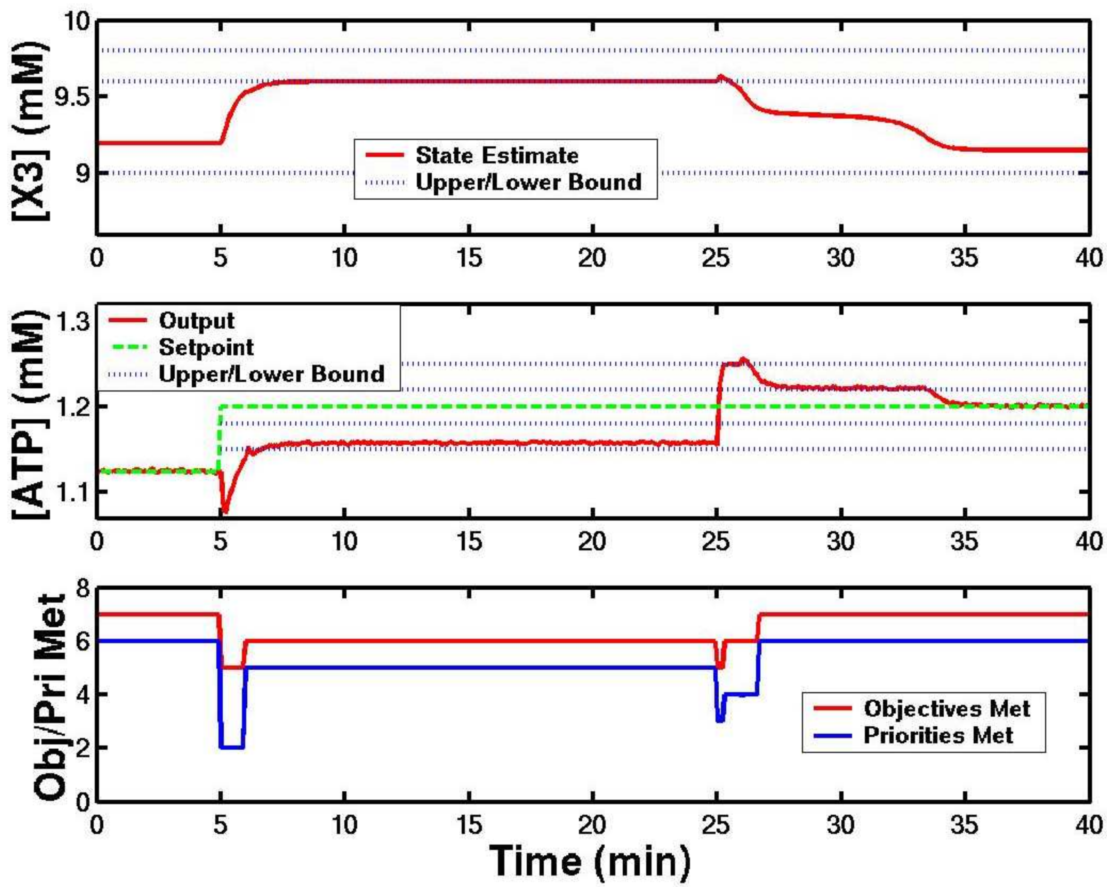
$\Gamma_O = -1{,}000$

$\Gamma_P = -20{,}000$

Sample Time: 6 secs

MILP: (424 Continuous variables, 13 binary variables)
MILP solver from the IBM OSL Library: Solution time ~5 seconds

Prioritized Objective MPC Results



m=2, p = 30

$\Gamma_e = 100$

$\Gamma_{\Delta u} = 50$

$\Gamma_O = -1{,}000$

$\Gamma_P = -20{,}000$

Sample Time: 6 secs

MILP: (424 Continuous variables, 13 binary variables)
MILP solver from the IBM OSL Library: Solution time ~5 seconds

Future Work

- Nonlinear dynamic systems
  - Use of a nonlinear model in a similar Nonlinear MPC formulation.
  - Find the global solution to the nonconvex optimization problem associated with the nonlinear MPC.

- Large scale applications
  - Address computational issues associated with bigger problems for real-time control applications. Find more efficient solution methods to solve larger problems faster.

- Robustness analysis
  - Attempt to characterize the robustness of the controller for different levels of model uncertainty. (Linear model describing a nonlinear process.)

Summary

- State-Space model explicitly defines all process states.
  - Using the model and current state estimates from the observer, the states can be predicted over the horizon (p).
    - Constraints on the state trajectories can be incorporated into the optimization problem for inferential control of unmeasured states.
- Using propositional logic, objectives can be prioritized.
  - Extended to objectives of equal priority.
- Prioritized Objective MPC shows improvement over traditional MPC.
- Biological processes, such as the Fermentation Pathway system presented, could benefit from this MPC formulation.

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