Introduction to Advanced Control

- Discrete Time Systems
- Programming (Excel and Matlab)
- Selected Math Topics
  - Linear Algebra and Norms
  - Numerical Optimization
  - Singular Value Decomposition
- Model Predictive Control
- Discrete Value Systems (Programmable Logic Control)
- Parameter and State Estimation
- Nonlinear Systems

Discrete Time Systems

- All computer controlled data acquisition systems produce and use sampled data.
- The measured values occur as discrete points in time.
- Usually samples are taken at uniform intervals:
  - Sampling Time Δt, Sampling Frequency, ω
- Values exist at all points in time, just available at discrete points in time.
- Convention, use k instead of t
  - $t = k \ \Delta t$

Sampling Time Example

- First order time delay system response to step of size A:

$$
\begin{aligned}
y(t) &= 0 & t < \alpha \\
&= KA(1 - e^{(\frac{t-\alpha}{\tau})}) & t > \alpha
\end{aligned}
$$

- Sketch y(t) and y(k) for Δt=10, 1, K=2, A=1, τ=5, α=1

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Background: Dynamic Modeling

Generalized Mass Action (GMA) Systems (Voit 2000)

- Mass balance on each species
- Power law rate equations, $r = a\, x_1^{b1}\, x_2^{b2}\, x_3^{b3}$

Simple Ordinary Differential Equation model form capturing qualitative / quantitative behavior:

$$\frac{dx_1}{dt} = a_{11}x_1^{b11}x_2^{b12} + a_{12}x_1^{b13}x_3^{b14} - a_{13}x_1^{b15}x_4^{b15}x_5^{b16}$$

$$\frac{dx_2}{dt} = a_{21}x_2^{b21}x_3^{b22} + a_{22}x_2^{b23}x_4^{b24} - a_{21}x_1^{b11}x_2^{b12}$$

$$\vdots$$

$$\frac{dx_n}{dt} = f(x,a,b)$$

a are rate constants, *b* are kinetic orders

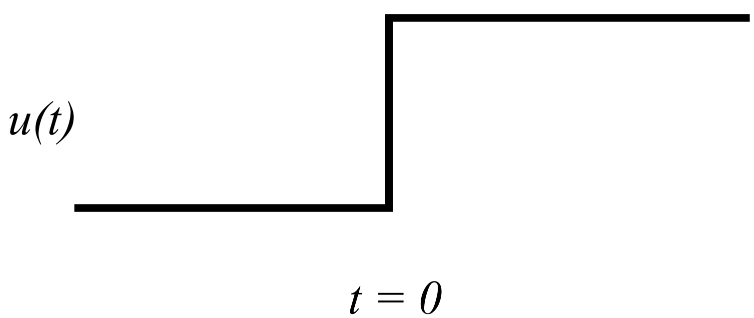
Outline

- Last Time: Discrete Time Systems
  - Sampled data, Δt sample time
  - Integer values for time, $k=0, 1, 2, 3, \ldots$
- Today: SISO Discrete Time Models
  - Step response coefficients
  - Impulse response coefficients
  - Auto Regressive and Moving Average Models
  - Excel modeling
  - Matlab modeling

Continuous Time Step and Impulse

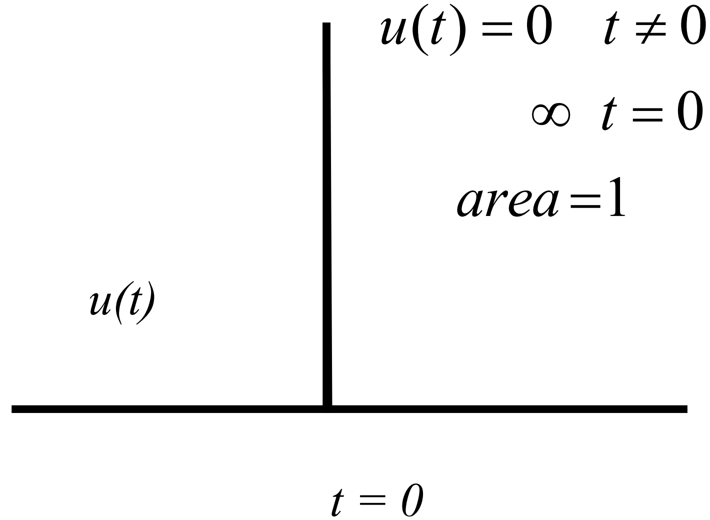
Continuous time ideal step and impulse: *H(t)* δ*(t)*

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$\frac{d}{dx} \Rightarrow$

$\Leftarrow \int$



$$u(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$area = 1$$

Usually forcing function for open-loop modeling, *u(t)*

Step: change valve position

Impulse: instantaneous "dump" into system

Discrete Time Step and Impulse

Discrete time ideal step and impulse

$$u(k) = 0 \quad k < 0$$
$$1 \quad k \geq 0$$

$$u(k) = 0 \quad t \neq 0$$
$$1 \quad t = 0$$

x x x x x x x x x x

u(k)

x x x x x x x x x x

k = 0

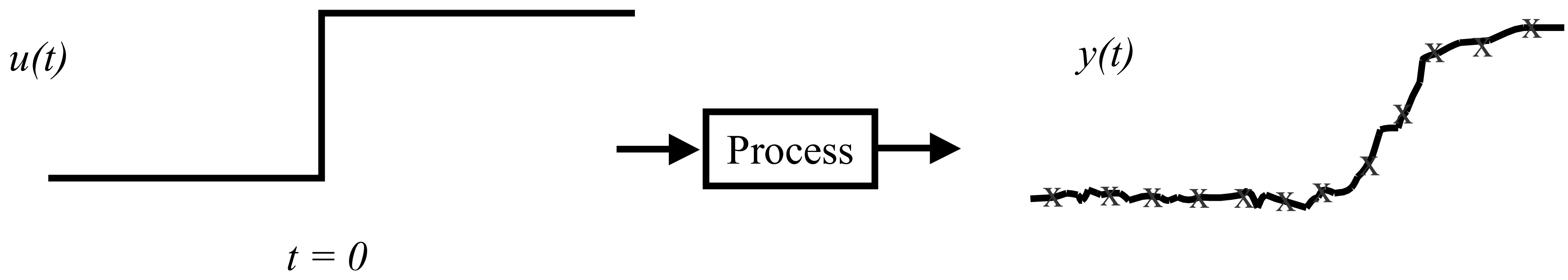
x

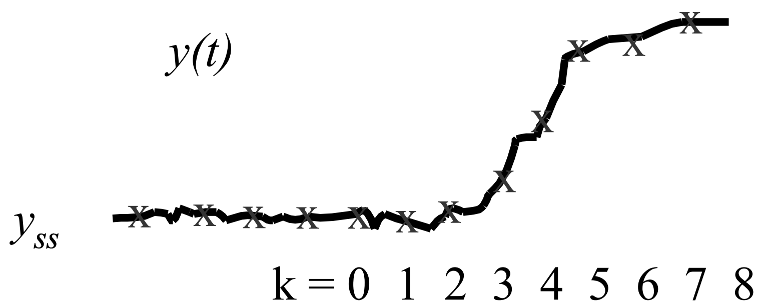
u(k)

x x x x x x x x x x x x x x x x

k = 0

Discrete Time Step Response





$$s_i = \frac{y(i) - y_{ss}}{\Delta u}$$

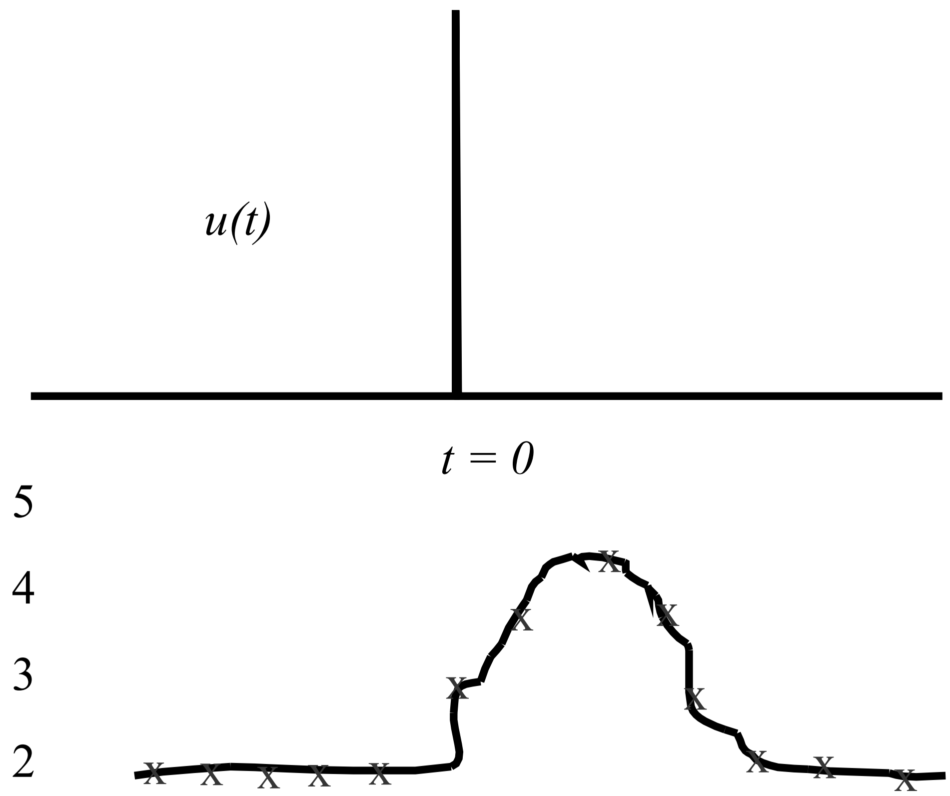
Discrete Time Step Response Coefficients

k	*u(k)*	*y(k)*	*si*
-3	1	10.00	*yss = 10*
-2	1	10.00	
-1	1	10.00	
0	3	10.00	0.00
1	3	16.32	3.16
2	3	18.65	4.32
3	3	19.50	4.75
4	3	19.82	4.91
5	3	19.93	4.97
6	3	19.98	4.99
7	3	19.99	5.00
8	3	20.00	5.00
9	3	20.00	5.00
10	3	20.00	5.00
11	3	20.00	5.00

s_0 is important, direct change

Why have more than 7 coefficients?

Discrete Time Impulse Response Coefficients



$$b_i = \frac{y(i) - y_{ss}}{imp\ size}$$

$$b_0 = 1$$
$$b_1 = 2$$
$$b_2 = 3$$
$$b_3 = 2$$
$$b_4 = 1$$
$$b_5 = 0$$
$$b_6 = 0$$
$$b_7 = 0$$

Relationship Between Step and Impulse Response Coefficients

Impulse from step:

$$b_i = s_i - s_{i-1}$$

Step from impulse:

$$s_i = \sum_{j=0}^{j=i} b_j$$

Think of step

as many impulses!