CSCE 790-003, Fall 2021, Homework 2
due Thursday September 30, 2021

Reading

Reading my course notes is primary, and reading KLM is secondary. Reading my course notes will likely be helpful in filling in some background, and I will draw some exercises from the notes (KLM seems to have a dearth of exercises).

- KLM, Chapter 3, Sections 3.1 and 3.2
- Course notes, Lectures 4, 5, 6, 7

Written Exercises

1. For Hilbert space $\mathcal{H}$ of dimension n, the set $\mathcal{L}(\mathcal{H})$ of linear maps from $\mathcal{H}$ to $\mathcal{H}$ (alternatively, $n \times n$ matrices) is also clearly a Hilbert space if we interpret a matrix as just a vector whose entries are arranged in an $n \times n$ array instead of a single column. That is, for matrices $A, B \in \mathcal{L}(\mathcal{H})$, we define

$$\langle A, B \rangle := \sum_{i=1}^{n} \sum_{j=1}^{n} [A]_{i,j}^{*} [B]_{i,j} ,$$

   where $*$ here means complex conjugation (of scalars). Show that $\langle A, B \rangle = \mathrm{Tr}(A^* B)$, where here, $*$ means the adjoint (conjugate transpose) of a matrix.

2. Let $A \in \mathcal{L}(\mathcal{H})$ and $B \in \mathcal{L}(\mathcal{H})$ be positive operators on a Hilbert space $\mathcal{H}$. Show that if $\sqrt{A}\sqrt{B} = 0$, then $AB = 0$. (This is the missing step in the proof of the result I gave on 2/10/2020 that if $\langle A, B \rangle = 0$ then $AB = 0$.)

3. Do Course Notes Exercises 5.3, 5.4, 5.6, 5.8, 5.12, 5.13, 7.2, 7.3.

Note: Here are some more notational discrepancies between my lecture/notes and the book:

1. If E is some expression representing a matrix, then I use $[E]_{ij}$ to denote the (i, j)th entry of E. This is useful particularly when E is an expression of any complexity, because it does not require coming up with a new letter to represent the matrix.

2. I've seen KLM use the same symbol for a ket label and for a coefficient in the same expression, e.g., $\sum_{i=1}^{n} T_i|T_i\rangle$. This is not ambiguous, because the Dirac notation makes it clear what purpose each occurrence is for. However, to avoid confusion, I will keep ket labels distinct from coefficients to the extent possible.