CSCE 790, Spring 2020, Homework 1 (due 2/5/2020)

Reading

KLM stands for Kaye, Laflamme, & Mosca.

- KLM, Chapter 1
- KLM, Chapter 2, Sections 2.1–2.2
- Course notes, first three lectures
- Recall how we defined the inner product over $\mathbb{C}^n$: For vectors $u = (u_1, \ldots, u_n)^\mathsf{T}$ and $v = (v_1, \ldots, v_n)^\mathsf{T}$ in $\mathbb{C}^n$,[1] the inner product is defined as

$$\langle u, v \rangle := \sum_{k=1}^{n} u_k^* v_k \ \in \ \mathbb{C} \, .$$

Exercises

- Course notes, Exercises 2.1, 2.4, 2.5, 2.6, 2.8
- Verify the defining properties of the inner product over $\mathbb{C}^n$: for all $u, v, w \in \mathbb{C}^n$ and $\alpha \in \mathbb{C}$,
  1. $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$ and $\langle u, \alpha v \rangle = \alpha \langle u, v \rangle$. (Linearity in the second argument)
  2. $\langle u, v \rangle = \langle v, u \rangle^*$. (Conjugate symmetry)
  3. $\langle u, u \rangle \geq 0$ with equality holding if and only if $u = 0$ (i.e., iff u is the zero vector).[2] (Positive definiteness)

[1] The T superscript means "transpose." This is because u and v are really column vectors, and displaying them as such in the running text takes up too much room.

[2] If we every compare a complex number z with a real constant, e.g., $z \geq 0$, then this tacetly implies that z itself is real.

- Show that any orthogonal set of nonzero vectors is linearly independent. [Hint: Let v be any linear combination of such vectors, and consider $\langle v, v \rangle$. You'll need the fact that $\langle \cdot, \cdot \rangle$ is positive definite.]

- Extra credit: KLM Exercise 2.3.1, Course notes, Exercises 2.10, 3.3

Note: The notations used in the book and in my course notes are pretty similar, but beware of the following two discrepancies:

1. I denote the adjoint of an operator A by A^*, which is the general convention in pure math. The KLM book uses $A^\dagger$ to denote the adjoint of A, which is used more in the physics community. (Discrepancies like this are inevitable in such a multidisciplinary subject.)

2. I don't introduce Dirac notation until later, whereas KLM uses it early on.