

Solution to Quiz 1
 $\lceil x \rceil < x+1$

$$\sum_{k=1}^n \frac{1}{k} \leq \sum_{k=1}^n \left\lceil \frac{1}{k} \right\rceil \leq \sum_{k=1}^n \left(\frac{1}{k} + 1 \right)$$

$$\sum_{k=1}^n \frac{1}{k} \leq \sum_{k=1}^n \frac{1}{k} + \sum_{k=1}^n 1$$

with harmonic series $= O(\lg n) + n$
 $= O(n \lg n + n)$
 $\boxed{\sum_{k=1}^n \frac{1}{k} = O(n \lg n)}$

Analyzing Algos:
 Insertion Sort
 Mergesort

Insert Sort ($A[1..n]$) $n = A.length$
 // $A[i]$ are numbers
 for $j := 2$ to $A.length$
 // $A[1..(j-1)]$ has the same elements as originally, but in ascending order.
 key $= A[j]$
 $i := j-1$
 while $i > 0$ and $A[i] > \text{key}$
 $A[i+1] := A[i]$
 $i := i-1$
 $A[i+1] := \text{key}$

Proving correctness:
 After termination, $A[1..n]$ is sorted
 $\therefore A[1..n]$ contains the same numbers as originally input, but in ascending order.

To do this:

- 1) Show that the loop invariant holds before the 1st iteration of the while loop (base case).
- 2) Show that, if the invariant holds at the start of one iteration, then it must hold at the start of the next (or else, after the loop terminates, if there is no next iteration) [inductive step]
- 3) conclude that the loop invariant holds after the loop terminates [this should tell you something useful].

Runtime: $\Theta(n^2)$ (worst case)
 average case
 $\Theta(n)$ (best case)

MergeSort ($A[p..q]$)
 // Precondition $p \leq q+1$
 // Postcondition $A[p..q]$ contains same numbers as when input, but in ascending order

if $p \geq q$ then	do nothing	time 1
else // $p < q$	$m := \lfloor \frac{p+q}{2} \rfloor$	1
	MergeSort($A[p..m]$)	$T(\frac{n}{2})$
	MergeSort($A[m+1..q]$)	$T(\frac{n}{2})$
	Merge(A, p, m, q)	n
	$n := q-p+1$	

Let $T(n)$ be the (worst-case) time to sort n numbers.
 $T(0) = \Theta(1)$
 $\therefore T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + \Theta(n)$

Recurrence equation
 Enough to determine $T(n)$ asymptotically,
 $T(n) = \Theta(n \lg n)$

Break down the recurrence tree or progress according to "where" the tree is being split.

n	n	$n/2$	$n/4$	$n/8$	$n/16$	$n/32$	$n/64$	$n/128$	$n/256$	$n/512$	$n/1024$	$n/2048$	$n/4096$	$n/8192$	$n/16384$	$n/32768$	$n/65536$	$n/131072$	$n/262144$	$n/524288$	$n/1048576$	$n/2097152$	$n/4194304$	$n/8388608$	$n/16777216$	$n/33554432$	$n/67108864$	$n/134217728$	$n/268435456$	$n/536870912$	$n/1073741824$	$n/2147483648$	$n/4294967296$	$n/8589934592$	$n/17179869184$	$n/34359738368$	$n/68719476736$	$n/137438953472$	$n/274877906944$	$n/549755813888$	$n/1099511627776$	$n/2199023255552$	$n/4398046511104$	$n/8796093022208$	$n/17592186044416$	$n/35184372088832$	$n/70368744177664$	$n/140737488355328$	$n/281474976710656$	$n/562949953421312$	$n/1125899906842624$	$n/2251799813685248$	$n/4503599627370496$	$n/9007199254740992$	$n/18014398509481984$	$n/36028797018963968$	$n/72057594037927936$	$n/144115188075855872$	$n/288230376151711744$	$n/576460752303423488$	$n/1152921504606846976$	$n/2305843009213693952$	$n/4611686018427387904$	$n/9223372036854775808$	$n/18446744073709551616$	$n/36893488147419103232$	$n/73786976294838206464$	$n/147573952589676412928$	$n/295147905179352825856$	$n/590295810358705651712$	$n/1180591620717411303424$	$n/2361183241434822606848$	$n/4722366482869645213696$	$n/9444732965739290427392$	$n/18889465931478580854784$	$n/37778931862957161709568$	$n/75557863725914323419136$	$n/151115727451828646838272$	$n/302231454903657293676544$	$n/604462909807314587353088$	$n/1208925819614629174706176$	$n/2417851639229258349412352$	$n/4835703278458516698824704$	$n/9671406556917033397649408$	$n/19342813113834066795298816$	$n/38685626227668133590597632$	$n/77371252455336267181195264$	$n/154742504910672534362390528$	$n/309485009821345068724781056$	$n/618970019642690137449562112$	$n/1237940039285380274899124224$	$n/2475880078570760549798248448$	$n/4951760157141521099596496896$	$n/9903520314283042199192993792$	$n/19807040628566084398385987584$	$n/39614081257132168796771975168$	$n/79228162514264337593543950336$	$n/158456325028528675187087900672$	$n/316912650057057350374175801344$	$n/633825300114114700748351602688$	$n/1267650600228229401496703205376$	$n/2535301200456458802993406410752$	$n/5070602400912917605986812821504$	$n/10141204801825835211973625643008$	$n/20282409603651670423947251286016$	$n/40564819207303340847894502572032$	$n/81129638414606681695789005144064$	$n/162259276829213363391578010288128$	$n/324518553658426726783156020576256$	$n/649037107316853453566312041152512$	$n/1298074214633706907132624082305024$	$n/2596148429267413814265248164610048$	$n/5192296858534827628530496329220096$	$n/10384593717069655257060992658440192$	$n/20769187434139310514121985316880384$	$n/41538374868278621028243970633760768$	$n/83076749736557242056487941267521536$	$n/166153499473114484112975882535043072$	$n/332306998946228968225951765070086144$	$n/664613997892457936451903530140172288$	$n/1329227995784915872903807060280344576$	$n/2658455991569831745807614120560689152$	$n/5316911983139663491615228241121378304$	$n/10633823966279326983230456482242756608$	$n/21267647932558653966460912964485513216$	$n/42535295865117307932921825928971026432$	$n/85070591730234615865843651857942052864$	$n/170141183460469231731687303715884105728$	$n/340282366920938463463374607431768211456$	$n/680564733841876926926749214863536422912$	$n/1361129467683753853853498429727072845824$	$n/2722258935367507707706996859454145691648$	$n/5444517870735015415413993718908291383296$	$n/10889035741470030830827987437816582766592$	$n/21778071482940061661655974875633165533184$	$n/43556142965880123323311949751266331066368$	$n/87112285931760246646623899502532662132736$	$n/174224571863520493293247799005065324265472$	$n/348449143727040986586495598010130648530944$	$n/696898287454081973172991196020261297061888$	$n/1393796574908163946345982392040522594123776$	$n/2787593149816327892691964784081045188247552$	$n/5575186299632655785383929568162090376495104$	$n/11150372599265311570767859136324180752990208$	$n/22300745198530623141535718272648361505980416$	$n/44601490397061246283071436545296723011960832$	$n/89202980794122492566142873090593446023921664$	$n/178405961588244985132285746181186892047843328$	$n/356811923176489970264571492362373784095686656$	$n/713623846352979940529142984724747568191373312$	$n/1427247692705959881058285969449495136382746624$	$n/2854495385411919762116571938898990272765493248$	$n/5708990770823839524233143877797980545530986496$	$n/11417981541647679048466287755595961091061972992$	$n/22835963083295358096932575511191922182123945984$	$n/45671926166590716193865151022383844364247891968$	$n/91343852333181432387730302044767688728495783936$	$n/182687704666362864775460604089535377456991567872$	$n/365375409332725729550921208179070754913983135744$	$n/730750818665451459101842416358141509827966271488$	$n/1461501637330902918203684832716283019655932542976$	$n/2923003274661805836407369665432566039311865085952$	$n/5846006549323611672814739330865132078623730171904$	$n/11692013098647223345629478661730264157247460343808$	$n/23384026197294446691258957323460528314494920687616$	$n/46768052394588893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