IS $\leq_p$ VC (cont.)

Fact: For any graph G and any set $C \subseteq G.V$, C is an indep set iff $G.V - C$ is a vertex cover.

$\leq_p$:
Given a graph G and number k ((G,k) is an instance of IS). Return $(G, |G.V| - k)$
Clearly polynomial time.
G has an indep set of size $\geq k$ iff G has a vertex cover of size $\leq |G.V| - k$ by the Fact.

Search reduces to decision
Suppose CLIQUE $\in P$
Then can use the ptime algo for CLIQUE to find a max size clique in a graph in ptime:

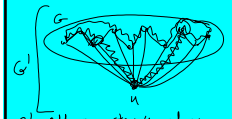
```
MaxClique(G)
// G is a graph
// returns a clique C ⊆ G.V
// of maximum size.
// uses algo for CLIQUE as
// a subroutine
Find the max size of a clique first:
k := |G.V|
while G does not have a clique of size k:
    k := k - 1
// k is the max size of any
// clique in G
while there is a vertex u ∈ G.V such that G - u has a clique of size k:
    G := G - u        (remove u from G & all its incident edges)
// |G.V| = k and so
// G.V is a k-clique of
// the original graph
return G.V
```

HAM CIRCUIT (HC):
Instance: A graph G (≥ 3 vertices)
Q: Is there a simple cycle in G that runs thru each vx exactly once.

HAMPATH (HP): (≥ 3 vertices)
Instance: Graph G
Q: Is there a simple path in G that touches every vx exactly once? (not a cycle in this case!)

Fact: Both are NP-complete.

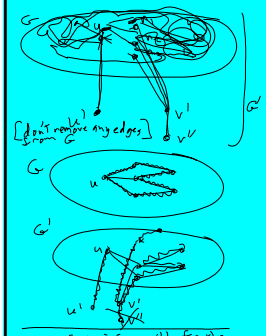
HP $\leq_p$ HC:
WTS: Given a graph G, find a graph G' in ptime such that G has a H.P. iff G' has a H.C.:



G': Add new vertex u, edges connecting u to every vertex of G.

G has a H.P. iff G' has a H.C.

HC $\leq_p$ HP
G G'



List of replica possible for the final:

- Sums: geometric, arith, harmonic
- Asymptotic definitions O, Ω, Θ, o, ω
- Recurrences: subst method, tree method, Master thm
- Order Stats / selection
- Randomized algos, expected runtime
- BSTs, variants (treaps), rotations,
- Augmented data structs (low prob)
- Hash tables (low prob)
- Amortized analysis (potential method)
- Dynamic Programming (definitely)
- (Mergeable) heaps: (med/low prob) binary, binomial, Fibonacci
- Disjoint set systems (Union-Find)
- BFS, DFS (fairly likely)
- MSTs (Kruskal, Prim) (high prob)
- Shortest Paths (single source — Dijkstra) (high prob)
- P, NP, NP-completeness
  - polynomial reductions
  - NP-hardness
  - search reduces to decision