Poly reduction from CNF-SAT to IS:

Given any cnf fmla
$\varphi = C_1 \wedge \cdots \wedge C_k$

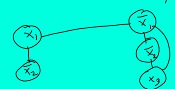
Construct in ptime a pair (G, m) where G is a graph and m is a natural number such that φ is satisfiable iff G has an i.s. of size m.

Given $\varphi = C_1 \wedge \cdots \wedge C_k$ cnf, assume wlog no two literals of the same var appear in the same clause:
- consolidate two identical literals in the same clause into one
- remove any clause containing contradictory literals, i.e., x and $\bar{x}$ for some x.

Construction
- set $m := k$ (# of clauses)
- for G:
  - vertex of G for each occurrence of a literal in φ

Ex: $\varphi = (x_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee x_3)$

- two types of edges:
  - put an edge connecting all literals in the same clause ("internal edges")
  - for every pair of contradictory literals, put an edge connecting them. ("external edges")



Claim: φ is satisfiable iff G has an indep set of size k.

($\Longrightarrow$): Assume φ sat. Let $\vec{a}$ be any satisfying assignment to φ.
Choose one literal from each clause made true (1) by $\vec{a}$.
Let C be the resulting set.
Then $|C| = k$ and C is an indep set:
- no internal edges in C
- no external edges in C:

can't both be made true by $\vec{a}$, so C can't contain both vertices.

$\therefore C$ is an i.s.

($\Longleftarrow$) Assume C is an indep subset of $G.V$ of size k.
WTS: φ is satisfiable.
1. C has at most one vertex in each clause (b/c internal edges)

$\therefore C$ has exactly one vertex in each clause (only k clauses)

2. C cannot have contradictory literals (b/c external edges)

Consider the following truth assignment: For every variable x in φ:
1. If $x \in C$, set $x := 1$
2. If $\bar{x} \in C$, set $x := 0$
3. if neither is in C, set $x := 1$ (doesn't matter).

Every vertex in C is made true, $\therefore$ every clause of φ is satisfied, $\therefore$ this assignment satisfies φ. ▨

CLIQUE:
Instance: A graph G and number m
Question: Does G have a clique of size m?

$IS \leq_p$ CLIQUE
Given any graph G, let G^c be its complement
$\forall u \neq v \in G.V$, u is adjacent to v in G iff u is not adjacent to v in G^c (G & G^c share the same vertex set).

Given (G, m) instance of IS, output (G^c, m).
Obviously polynomial time.
G has an i.s. of size m iff G^c has a clique of size m. (same set!) //

$IS \leq_p VC$

Idea:

C is a vertex cover of G

iff $G.V - C$ is an i.s.