CSCE 750, Homework 2

This assignment covers material from the lectures on Sections 4.3–4.5, in preparation for Quiz 2. NIT means "not in textbook."

Exercises

Page 94, Exercise 4.3-1(a,b,c) [Can substitute 3rd ed. Ex. 4.3-1, 4.3-2, 4.3-3]: Use the substitution method to show that each of the following recurrences defined on the reals has the asymptotic solution specified:

- ***a***. $T(n) = T(n-1) + n$ has solution $T(n) = O(n^2)$.
- ***b***. $T(n) = T(n/2) + \Theta(1)$ has solution $T(n) = O(\lg n)$.
- ***c***. $T(n) = 2T(n/2) + n$ has solution $T(n) = \Theta(n \lg n)$.

NIT1 Show that the solution to $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \lg n)$. [3rd ed. Ex. 4.3-6]

NIT2 Use a recursion tree to determine good asymptotic upper bounds for $T(n)$ satisfying the following recurrences:

- ***a***. $T(n) = 3T(n/2) + n$.
- ***b***. $T(n) = T(n/2) + n^2$.
- ***c***. $T(n) = 2T(n-1) + 1$.

Use the substitution method to verify your answer in each case.

NIT3 Argue that the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + cn$, where $c > 0$ is a constant, is $\Omega(n \lg n)$ by appealing to a recursion tree.

NIT4 Draw the recursion tree for $T(n) = 4T(n/2) + cn$, where $c > 0$ is a constant, and provide a asymptotically tight bound on its solution. Verify your bound by the substitution method. ("Asymptotically tight" means an upper bound and a lower bound that are big-Θ of each other, i.e., asymptotically the same.)

Page 106, Exercise 4.5-1(a,b,d,e) [3rd ed. Ex. 4.5-1(a,b,c,d)] Use the master method to give tight asymptotic bounds for the following recurrences.

- ***a***. $T(n) = 2T(n/4) + 1$.
- ***b***. $T(n) = 2T(n/4) + \sqrt{n}$.
- ***d***. $T(n) = 2T(n/4) + n$.

e. $T(n) = 2T(n/4) + n^2$.

Page 106, Exercise 4.5-3 [same in the 3rd ed.]: Use the master method to show that the solution to the binary-search recurrence $T(n) = T(n/2) + \Theta(1)$ is $T(n) = \Theta(\lg n)$. (See Exercise 2.3-6 for a description of binary search.)

NIT5 Can the master method given in class (and in the 3rd edition of the textbook) be applied to the recurrence $T(n) = 4T(n/2) + n^2 \lg n$? Why or why not? Give asymptotically tight bounds for this recurrence.

Page 119, Problem 4-1(a,b,c,e,f,g,h) [Can substitute 3rd ed. Prob. 4-1]: *Recurrence examples*

Give asymptotically tight upper and lower bounds for $T(n)$ in each of the following algorithmic recurrences. Justify your answers.

a. $T(n) = 2T(n/2) + n^3$.

b. $T(n) = T(8n/11) + n$.

c. $T(n) = 16T(n/4) + n^2$.

e. $T(n) = 8T(n/3) + n^2$.

f. $T(n) = 7T(n/2) + n^2 \lg n$.

g. $T(n) = 2T(n/4) + \sqrt{n}$.

h. $T(n) = T(n - 2) + n^2$.