CSCE 750, Homework 1

This assignment covers material from the lectures on Chapters 1–3 and Appendix A, in preparation for Quiz 1. Page, exercise, and problem numbers refer to the **fourth edition** of the textbook. Numbers in the third edition, if different, are given in brackets. The acronym, "NIT" stands for "not in textbook." Starred exercises and problems are generally more challenging than unstarred ones.

Let $x \in \mathbb{R}$ be any real number. Recall that $\lfloor x \rfloor$ (the *floor* of x) is the greatest integer not exceeding x and that $\lceil x \rceil$ (the *ceiling* of x) is the least integer not less than x. Thus $\lfloor x \rfloor$ and $\lceil x \rceil$ are the unique integers satisfying

$$x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1 .$$

Exercises

Page 25, Exercise 2.1-3 [3rd ed page 22, Ex 2.1-2]: Rewrite the INSERTION-SORT procedure to sort into monotonically decreasing instead of monotonically increasing order.

Page 47, Problem 2-4 [3rd ed pages 41–42]: Inversions

Let A[1:n] be an array of n distinct numbers. If i < j and A[i] > A[j], then the pair (i, j) is called an *inversion* of A.

- **a**. List the five inversions of the array $\langle 2, 3, 8, 6, 1 \rangle$.
- **b**. What array with elements from the set $\{1, 2, ..., n\}$ has the most inversions? How many does it have?
- c. What is the relationship between the running time of insertion sort and the number of inversions in the input array? Justify your answer.
- **d**. Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n \lg n)$ worst-case time. (*Hint:* Modify merge sort.)
- Page 62, Exercise 3.2-1 [3rd ed page 52, Ex 3.1-1]: Let f(n) and g(n) be asymptotically nonnegative functions. Using the basic definition of Θ -notation, prove that $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.
- **NIT.1** Show that for any real constants a and b, where b > 0,

$$(n+a)^b = \Theta(n^b) \; .$$

Note that b need not be an integer, and a need not be positive.

- Page 62, Exercise 3.2-2 [3rd ed page 53, Ex 3.1-3]: Explain why the statement, "The running time of algorithm A is at least $O(n^2)$," is meaningless.
- Page 62, Exercise 3.2-3 [3rd ed page 53, Ex 3.1-4]: Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?
- ***NIT.2** Give an example of two functions f(n) and g(n), both asymptotically positive, for which $f(n) \notin O(g(n))$ and $g(n) \notin O(f(n))$. Prove that your answer is correct. (This shows that the asymptotically positive functions cannot be totally ordered by asymptotic growth rate.) For an additional challenge, find two such functions that are monotonically increasing.
- Page 1144, Exercise A.1-2 [3rd ed page 1149, Ex A.1-1]: Find a simple formula for $\sum_{k=1}^{n} (2k-1)$.
- Page 1144, Exercise A.1-6 [3rd ed page 1149, Ex A.1-3]: Show that $\sum_{k=0}^{\infty} k^2 x^k = x(1+x)/(1-x)^3$ for |x| < 1. (Hint: Start with Equation (A.11) [3rd ed (A.8)], then differentiate both sides.)
- **NIT.3** Evaluate the product $\prod_{k=1}^{n} 2 \cdot 4^k$.
- **Page 1150, Exercise A.2-1 [3rd ed page 1156]:** Show that $\sum_{k=1}^{n} 1/k^2$ is bounded above by a constant.
- Page 1150, Exercise A.2-2 [3rd ed page 1156]: Find an asymptotic upper bound on the summation

$$\sum_{k=0}^{\lfloor \lg n \rfloor} \left\lceil n/2^k \right\rceil \, .$$

Page 1152, Exercises A.2-4 [3rd ed page 1156]: Approximate $\sum_{k=1}^{n} k^3$ with an integral.