CSCE 551
Midterm Exam II
Thursday April 7, 2005

This test is open book, open notes, but no electronic devices. Do all problems, putting your answers in the sheets provided. There are 100 points total in the exam. For graduate students, 90 points constitute full credit, with 10 points extra credit. For undergrads, 70 points constitute full credit and the other 30 are extra credit. You have 75 minutes. Please read a question *carefully* before attempting it. If you have any questions or doubts about what is expected, please ask me.

1. (25 points) Let

$$L = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } L(R) \cap L(S) = \emptyset\}.$$

    Show that L is decidable by giving a high-level decision procedure for L.

2. (25 points) Recall that ε is the empty string. Let

$$L = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ rejects } \varepsilon\}.$$

    Show that L is undecidable by giving a mapping reduction from A_{TM} to L.

3. (20 points) Let

$$L = \{A \mid A \text{ is an NFA and } L(A) \text{ is infinite}\}.$$

    Show that L is decidable in polynomial time (that is, in the class **P**) by giving a polynomial-time decision procedure for L.

4. (30 points total)

    (a) (20 points) A function $f : \Sigma^* \to \Sigma^*$ is *honest* if there is a natural number k such that for all $x \in \Sigma^*$,
$$|x| \leq |f(x)|^k.$$

    That is, f is honest if it does not map really long strings to really short strings. Suppose that f is polynomial-time computable and honest. Show that the range of f is in **NP**, where the range of f is defined as $\{f(x) \mid x \in \Sigma^*\}$.

    (b) (10 points) Describe a polynomial-time computable, honest function whose range is equal to SAT.