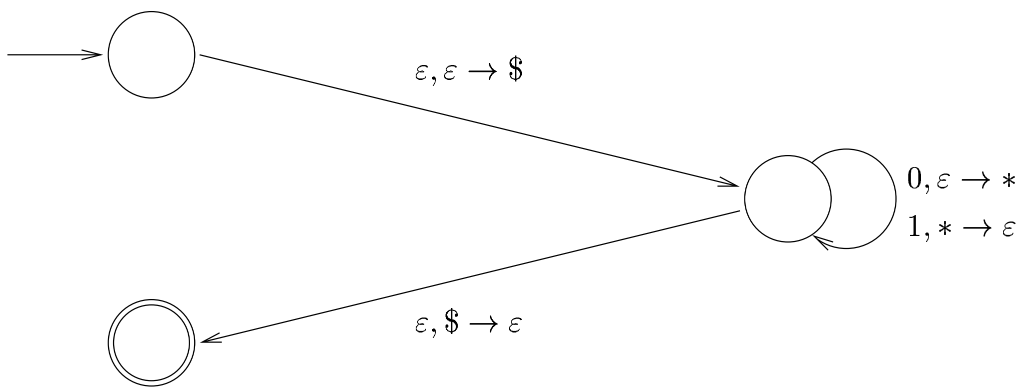
CSCE 551
Midterm Exam
Wednesday April 12, 2004

Do all problems, putting your answers in the exam book. There are 60 points total in the exam proper, plus 15 extra credit. You have 75 minutes.

For all problems, assume that the input alphabet $\Sigma = \{0, 1\}$ unless otherwise specified.

1. (10 points) Consider the following PDA M:



Circle the input strings below that are accepted by M:

000111   111000   00101101   0010 ϵ

2. (10 points total plus 15 points extra credit) The following context-free grammar G expresses the language of all nonpalindromes.

$$\begin{aligned} S &\rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0T1 \mid 1T0 \\ T &\rightarrow \epsilon \mid 0T \mid 1T \end{aligned}$$

   (a) (5 points) Show that this grammar is ambiguous by giving two different parse trees yielding the same string.

   (b) (5 points) The grammar can be made into an equivalent unambiguous grammar simply by removing two productions. Which two?

   (c) (15 points EXTRA CREDIT) Using any method you like, describe a PDA that recognizes this language. Your PDA should be presented as a state diagram, not a table. You need not specify the stack alphabet explicitly.

3. (15 points) The Pumping Lemma for CFLs can be used to show that the language

$$L = \{w\#x \mid x \text{ is a suffix of } w \text{ and } w, x \in \{0,1\}^*\}$$

is not context-free. Fill in the missing parts (bracketed) of the following proof:

> Suppose L is a CFL. Let p be given by the Pumping Lemma for CFLs. Let $s \in L$ be the string [give a value for s]. The Pumping Lemma decomposes s into $xuyvz$ satisfying the usual properties. There are three cases: (i) if x contains the # then let $i =$ []; (ii) if uyv contains the # then let $i =$ []; (iii) if z contains the # then let $i =$ []. Then $xu^iyv^iz \notin L$ because [explain each case]. Thus, L cannot be a CFL.

4. (15 points) A standard (1-tape, deterministic) TM M decides membership in the language

$$\{0^i1^j \mid 0 \le i \le j\}$$

by repeatedly erasing the first and last nonblank symbols on its tape. Give a formal description for M, using as few states as you can and tape alphabet $\{0, 1, \text{\textvisiblespace}\}$. You may give either a transition table or state diagram.

5. (10 points) Give an implementation-level description of a TM M that decides the language

$$\{x\#w \mid x \text{ is a substring of } w \text{ and } w, x \in \{0,1\}^*\}.$$

M can use multiple tapes, and any tape head can stay stationary during a computation step. Say how many tapes you are using.