CSCE 551
Midterm Exam I
Monday February 16, 2004

Do all problems, putting your answers in the exam book. There are 70 points total in the exam. For graduate students, 70 points is full credit. For undergrads, 50 points is full credit and the other 20 are extra credit. You have 75 minutes. Please read a question carefully before attempting it. If you have any questions or doubts about what I want, please ask me.

1. (3 points each; 12 points total) For each of the following binary relations over $\{1, 2, 3\}$, say whether or not it is (i) reflexive, (ii) symmetric, (iii) transitive.

   (a) $\{(1, 2), (2, 3), (3, 1)\}$

   (b) $\{(1, 3), (1, 2), (3, 1), (1, 1)\}$

   (c) $\{(1, 1), (2, 2), (3, 3)\}$

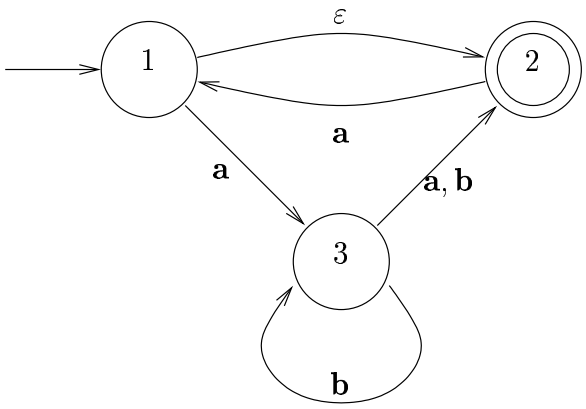
   (d) $\{\}$

2. (5 points each; 10 points total) Using any method(s) you like (including just plain intuition), draw NFAs recognizing the following languages:

   (a) $(0 \cup 11)^*(00 \cup 1)^*$

   (b) $\{w \in \{0, 1\}^* \mid$ every odd position of w is a 1$\}$ (Note that if $w \neq \varepsilon$, the first symbol in w is at position 1.)

3. (23 points total) Consider the following NFA (which is also displayed on page 85, Exercise 1.12(b)):



   (a) (10 points) Using the construction given in Theorem 1.19 or in class, fill in the

rest of the transition table for the equivalent DFA:

state	**a**	**b**
$\{\} = q$	q	q
$\{1\} = q_1$	q_3	q
$\{2\} = q_2$		
$\{1,2\} = q_{12}$		
$\{3\} = q_3$		
$\{1,3\} = q_{13}$		
$\{2,3\} = q_{23}$		
$\{1,2,3\} = q_{123}$		

(You don't need to copy the entire table in your exam book, just the blank entries.)

(b) (3 points) In the DFA above, which state is the start state?

(c) (5 points) Which states are accepting?

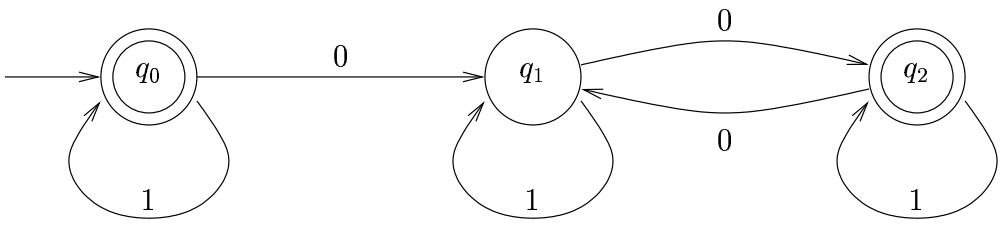
(d) (5 points) Which states are unreachable from the start state?

4. (20 points total)

(a) (10 points) Using any method you like (including just plain intuition), draw a five-state DFA recognizing the language L consisting of all strings w over $\{0,1\}$ such that $w \neq \varepsilon$ and the first symbol of w occurs an odd number of times in w.

(b) (10 points) For the language L of part (a), above, give five strings $w_1, \ldots, w_5$ over $\{0,1\}$ such that the languages $L_{w_1}, \ldots, L_{w_5}$ are all different (which implies that the DFA you constructed above is minimal). [You may wish—but are not required—to show how they are different by giving "test strings," e.g., a string that is in one language but not another. If you give $w_1, \ldots, w_5$ correctly, however, you need no further explanation.]

5. (5 points) Consider the following DFA:



Using any method you like (including just plain intuition), list any and all pairs of distinct but equivalent (i.e., indistinguishable) states.