CSCE 551
Midterm Exam I
Tuesday February 15, 2005

This test is open book, open notes, but no electronic devices. Do all problems, putting your answers in the sheets provided, except for Problem 4, which you should answer on this sheet. There are 100 points total in the exam. For graduate students, 100 points constitute full credit. For undergrads, 75 points constitute full credit and the other 25 are extra credit. You have 75 minutes. Please read a question *carefully* before attempting it. If you have any questions or doubts about what is expected, please ask the proctor.

1. (3 points each; 15 points total) For each of the following identities involving regular expressions built from r and s, either say that the identity is true (for all regular expressions r and s), or give a counterexample (a specific instance of r and of s that makes the identity false).

   (a) $(rs)^* = r^*s^*$
   Answer: counterexample: $r = 0$ and $s = 1$.

   (b) $(r \cup s)^* = r^* \cup s^*$
   Answer: counterexample: $r = 0$ and $s = 1$.

   (c) $(r \cup \varepsilon)^* = r^*$
   Answer: true.

   (d) $(r \cup \varepsilon)(s \cup \varepsilon) = rs \cup \varepsilon$
   Answer: counterexample: $r = 0$ and $s = 1$.

   (e) $(r^*s)^* = (rs^*)^*$
   Answer: true.

2. (20 points total) Recall that a string x is a *subsequence* of a string y (written $x \preceq y$) if x results from removing zero or more symbols from y and joining together the remaining symbols in order. (Note that a subsequence may not necessarily be a substring.) Let

$$L_1 = \{w \in \{a, b, c\} \mid ab \preceq w\},$$

and let

$$L_2 = \{a, b, c\}^* - L_1 = \{w \in \{a, b, c\} \mid ab \not\preceq w\}.$$

   (a) (10 points) Using any method you like, give a three-state DFA recognizing L_1 (either a transition graph or a transition table).
   Answer: Let the DFA be $\langle\{q_0, q_1, q_2\}, \{a, b, c\}, \delta, q_0, \{q_2\}\rangle$, where δ is given by the following table:

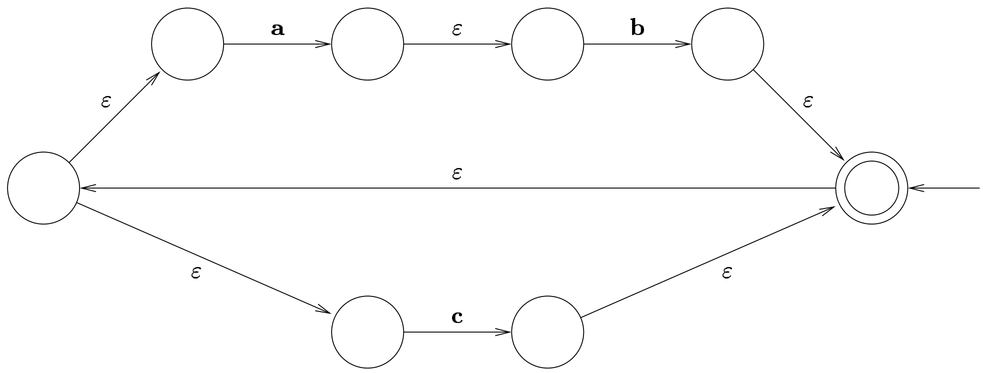
	a	**b**	**c**
q_0	q_1	q_0	q_0
q_1	q_1	q_2	q_1
q_2	q_2	q_2	q_2

   (b) (10 points) Using any method you like, give a regular expression for L_2. Your regular expression should not be unreasonably complex.
   Answer: $(b \cup c)^*(a \cup c)^*$

3. (10 points) Using the method described in class or in the proof of Lemma 1.29 in the textbook, convert the regular expression

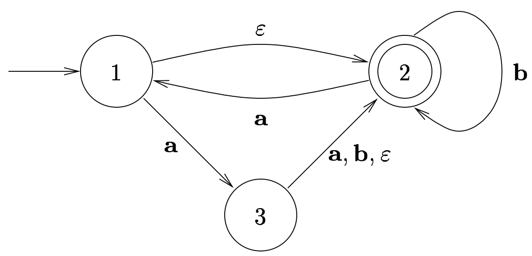
$$(ab \cup c)^*$$

into an equivalent NFA.
Answer: Using the method given in class:



There are other solutions.

4. (25 points total) Consider the following NFA:



(a) (10 points) Using the construction given in Theorem 1.19 or in class, fill in the rest of the transition table for the equivalent DFA:
Answer:

state	**a**	**b**
$\{\} = q$	q	q
$\{1\} = q_1$	q_{23}	q
$\{2\} = q_2$	q_{12}	q_2
$\{1,2\} = q_{12}$	q_{123}	q_2
$\{3\} = q_3$	q_2	q_2
$\{1,3\} = q_{13}$	q_{23}	q_2
$\{2,3\} = q_{23}$	q_{12}	q_2
$\{1,2,3\} = q_{123}$	q_{123}	q_2

(b) (4 points) In the DFA above, which state is the start state?
Answer: q_{12}

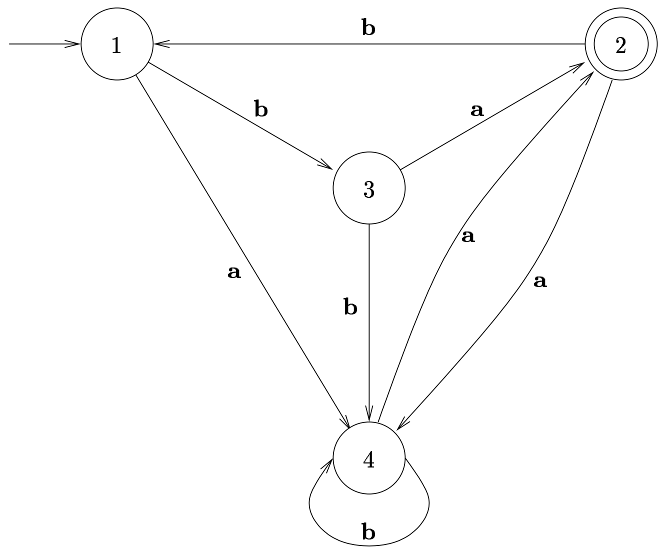
(c) (5 points) Which states are accepting?
Answer: $q_2, q_{12}, q_{23}, q_{123}$

(d) (6 points) Which states are unreachable from the start state?
Answer: $q, q_1, q_3, q_{13}, q_{23}$

5. (15 points) Let $L = \{xx \mid x \in \{0,1\}^*\}$. Fill in the missing parts of the following proof by the Pumping Lemma that L is not regular:
Answer:

Let $p > 0$ be arbitrary.
Define $s = 0^p 1 0^p 1$. Clearly, $s \in L$.
Let x, y, z be any strings such that $s = xyz$, $|xy| \le p$, and $|y| > 0$.
Then since $|xy| \le p$ and $|y| > 0$, y contains one or more leading 0's. Let $i = 0$. We have $xy^0z = xz = 0^r 1 0^p 1$ where $r < p$, and so this string is clearly not in L. Thus by the Pumping Lemma, L is not regular.

6. (15 points total) Consider the following DFA:



(a) (10 points) Use the table filling method described in class to find all distinguishable pairs of states. You only need to give the final contents of the table T. Only show the proper upper triangle of T, that is, $T[i, j]$ for $1 \leq i < j \leq 4$.
Answer:

T	1	2	3	4
1		X	X	X
2			X	X
3				
4				

(b) (5 points) Draw the equivalent minimum DFA.
Answer: