CSCE 551
Answers to Midterm Exam I
Monday February 16, 2004

Do all problems, putting your answers in the exam book. There are 70 points total in the exam. For graduate students, 70 points is full credit. For undergrads, 50 points is full credit and the other 20 are extra credit. You have 75 minutes. Please read a question carefully before attempting it. If you have any questions or doubts about what I want, please ask me.

1. (3 points each; 12 points total) For each of the following binary relations over $\{1, 2, 3\}$, say whether or not it is (i) reflexive, (ii) symmetric, (iii) transitive.

   (a) $\{(1,2),(2,3),(3,1)\}$

   (b) $\{(1,3),(1,2),(3,1),(1,1)\}$

   (c) $\{(1,1),(2,2),(3,3)\}$

   (d) $\{\}$

   Answer

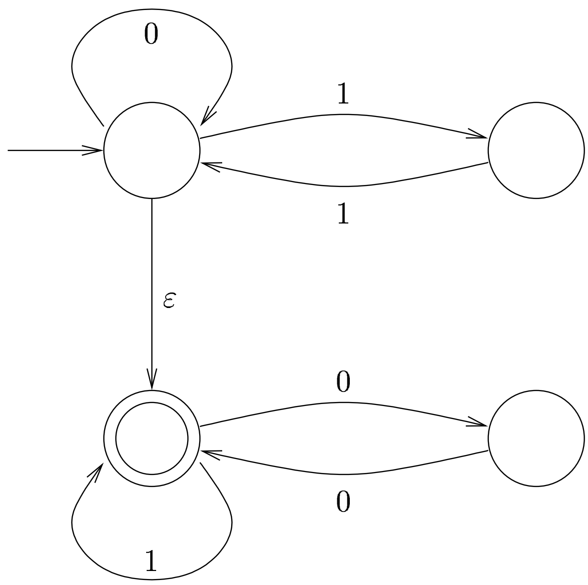
	reflexive	symmetric	transitive
(a)	no	no	no
(b)	no	no	no
(c)	yes	yes	yes
(d)	no	yes	yes

2. (5 points each; 10 points total) Using any method(s) you like (including just plain intuition), draw NFAs recognizing the following languages:

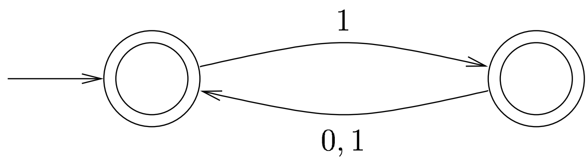
   (a) $(0 \cup 11)^*(00 \cup 1)^*$

   (b) $\{w \in \{0,1\}^* \mid \text{every odd position of } w \text{ is a } 1\}$ (Note that if $w \neq \varepsilon$, the first symbol in w is at position 1.)

   Answer
   There are many correct answers for parts (a) and (b). For (a), there is the usual step-by-step construction of an NFA from a regular expression, which gives an NFA with about 12 to 20 states, depending on how it is done. Here is a much smaller NFA with four states that also works:

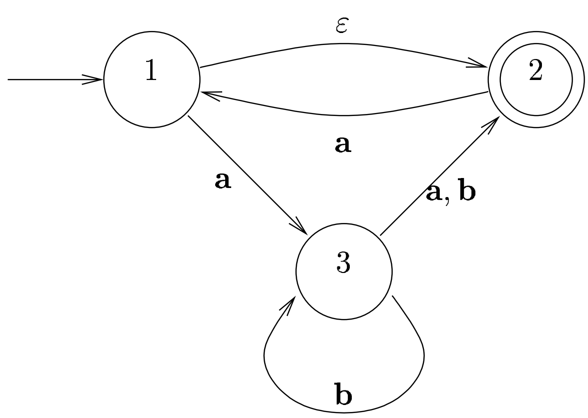


Here is the smallest possible NFA for part (b):



Note that ε is accepted by this NFA. Strictly speaking, every odd position in the string ε is 1!

3. (23 points total) Consider the following NFA (which is also displayed on page 85, Exercise 1.12(b)):



(a) (10 points) Using the construction given in Theorem 1.19 or in class, fill in the rest of the transition table for the equivalent DFA:

state	a	b
$\{\} = q$	q	q
$\{1\} = q_1$	q_3	q
$\{2\} = q_2$		
$\{1,2\} = q_{12}$		
$\{3\} = q_3$		
$\{1,3\} = q_{13}$		
$\{2,3\} = q_{23}$		
$\{1,2,3\} = q_{123}$		

(You don't need to copy the entire table in your exam book, just the blank entries.)

(b) (3 points) In the DFA above, which state is the start state?

(c) (5 points) Which states are accepting?

(d) (5 points) Which states are unreachable from the start state?

Answer

(a) Here is the full transition table:

state	a	b
$\{\} = q$	q	q
$\{1\} = q_1$	q_3	q
$\{2\} = q_2$	q_{12}	q
$\{1,2\} = q_{12}$	q_{123}	q
$\{3\} = q_3$	q_2	q_{23}
$\{1,3\} = q_{13}$	q_{23}	q_{23}
$\{2,3\} = q_{23}$	q_{12}	q_{23}
$\{1,2,3\} = q_{123}$	q_{123}	q_{23}

(b) The start state is q_{12}, the ε-closure of $\{1\}$, that is, $E(\{1\}) = \{1,2\} = q_{12}$.

(c) The accepting states are all those that contain 2, that is, $q_2, q_{12}, q_{23}, q_{123}$.

(d) The states q_1, q_2, q_3, q_{13} are the ones not reachable from the start state. (I gave you full credit for this part if your answer was consistent with your previous answers about the transition table and start state, even if those previous answers had errors.)

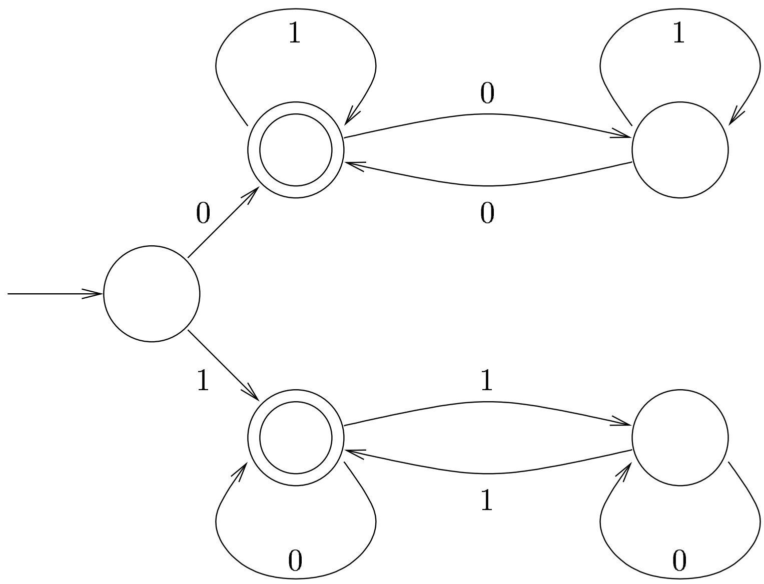
4. (20 points total)

(a) (10 points) Using any method you like (including just plain intuition), draw a five-state DFA recognizing the language L consisting of all strings w over $\{0, 1\}$ such that $w \neq \varepsilon$ and the first symbol of w occurs an odd number of times in w.

(b) (10 points) For the language L of part (a), above, give five strings $w_1, \ldots, w_5$ over $\{0, 1\}$ such that the languages $L_{w_1}, \ldots, L_{w_5}$ are all different (which implies that the DFA you constructed above is minimal). [You may wish—but are not required—to show how they are different by giving "test strings," e.g., a string that is in one language but not another. If you give $w_1, \ldots, w_5$ correctly, however, you need no further explanation.]

Answer

(a) Here is the (unique) five-state DFA:



(b) Here are possibilities for the five strings w_1, w_2, w_3, w_4, w_5. They correspond to ways of getting from the start state of the DFA to each of its five states.

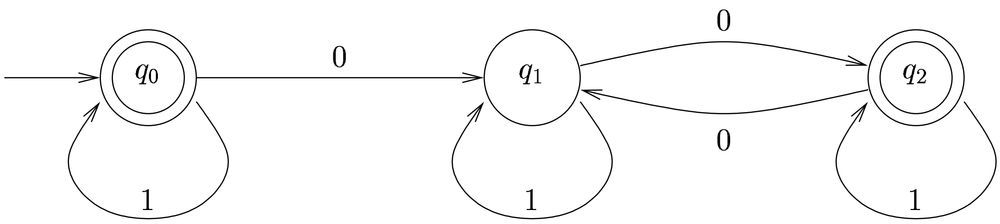
- $w_1 = \varepsilon$
- $w_2 = 0$ (or any other string starting with 0, with an odd number of 0s and any number of 1s)
- $w_3 = 00$ (or any other string starting with 0, with an even number of 0s and any number of 1s)
- $w_4 = 1$ (or any other string starting with 1, with an odd number of 1s and any number of 0s)
- $w_5 = 11$ (or any other string starting with 1, with an even number of 1s and any number of 0s)

(Note that ε *must* be one of the five strings, since no other string will leave us in the start state.) We can use three test strings—ε, 0, and 1—to show how the languages $L_{w_1}, \ldots, L_{w_5}$ differ:

	L_ε	L_0	L_{00}	L_1	L_{11}
ε	$\notin$	$\in$	$\notin$	$\in$	$\notin$
0	$\in$	$\notin$	$\in$	$\in$	$\notin$
1	$\in$	$\in$	$\notin$	$\notin$	$\in$

No two columns are identical, so the languages are all different.

5. (5 points) Consider the following DFA:



Using any method you like (including just plain intuition), list any and all pairs of distinct but equivalent (i.e., indistinguishable) states.

Answer
(q_0, q_2) are equivalent, and this is the only pair. The other two pairs, (q_0, q_1) and (q_1, q_2), are both distinguished by ε. The states q_0 and q_2 are not distinguished by ε, and when running the table-filling procedure, $T[q_0, q_2]$ is left blank, because $T[\delta(q_0, 0), \delta(q_2, 0)] = T[q_1, q_1]$, which is a diagonal entry and so is always left blank, and $T[\delta(q_0, 1), \delta(q_2, 1)] = T[q_0, q_2]$, which is blank.