Some Pumping Lemma examples

Turing Machines

To prove a lang. L is not pumpable:

For any $p > 0$, find an $s \in L$ with $|s| \geq p$ such that, given any x, y, z (strings) such that

$$s = xyz$$
$$|xy| \leq p$$
$$|y| > 0$$

find an $i \geq 0$ such that

$$xy^iz \notin L.$$

[$i := 1$ never works]

Examples

$L := \{0^m 1^n : 0 \leq m \leq n\}$

Given $p > 0$, let

$s := 0^p 1^p$

Given x, y, z, // know that $y = 0^k$
// some $k > 0$

let $i := 2$.

// Then $xy^2z = 0^{p+k} 1^p$

$xyyz$

$s = xyz$

$\notin L$

// because $p + k > p$. ☐

$i := 0$ doesn't work;

$xz = 0^{p-k} 1^p \in L$

$s = 0^{p-1} 1^p$ doesn't work:

$x := 0^{p-1}$ $s = xyz$

$y := 1$ $|xy| = p$

$z := 1^{p-1}$ $|y| = 1$

$i := 0$: $xz = 0^{p-1} 1^{p-1} \in L$

$i \geq 2$: $xy^iz = 0^{p-1} 1^{p-1+i} \in L$

$L = \{0^n 1^n : 0 \leq n \leq m\}$

Given $p > 0$,

let $s := 0^p 1^{p-1}$ doesn't work:

$x = 0^{p-1}$

$y = 0$

$z = 1^{p-1}$

$i := 0$: $xz = 0^{p-1} 1^{p-1} \in L$

$i \geq 2$: $xy^iz = 0^{p-1+i} 1^{p-1} \in L$

$s := 0^p 1^p$

Given x, y, z,

let $i := 0$. Then

$xz = 0^{p-k} 1^p$ [$y = 0^k$, $k > 0$]

$\in L$ because $p - k < p$ //

$L := \{w \in \{0,1\}^* : w$ has a 0 somewhere in its 1st half (not including the middle char if $|w|$ is odd)$\}$

Given $p > 0$,

let $s := 1^p 0^{p+2}$

given x, y, z with

$s = xyz$
$|xy| \leq p$
$|y| > 0$, // $y = 1^k$ some $k > 0$

let $i := 2$ (or more).

Then $xy^2z = xyyz = 1^{p+k} 0^{p+2}$

$\notin L$

(any $k > 0$) //

Pumping Lemma + closure properties

Prop: $L := \{0^m 1^n : m \neq n\}$ is not regular.

Proof: Suppose L is regular.

Then $\overline{L}$ is regular.

But then, $\overline{L} \cap 0^*1^*$ is regular (closure under intersection)

But $\overline{L} \cap 0^*1^*$

$= \{0^m 1^n : m = n\}$

$= \{0^n 1^n : n \geq 0\}$

not pumpable (proved last lec), thus

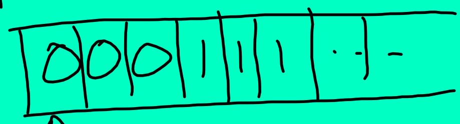
$\overline{L} \cap 0^*1^*$ is not regular by the Pumping Lemma. ↯

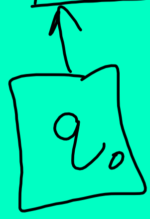
Thus L is not regular.

Turing Machines

$(L := \{0^n 1^n : n \geq 0\})$

Input



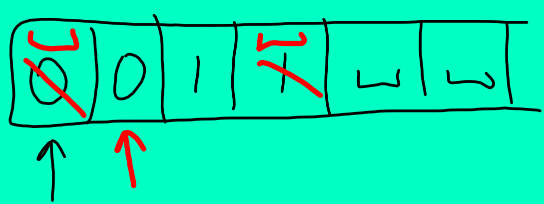


Transitions depend only on 2 things:

- symbol being scanned
- current state

3 actions:

1) change state (or not)
2) change scanned cell contents (or not)
3) move one cell left or right



⊔ = blank