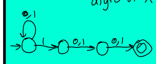
NFAs

Recall: NFA recognizing

$L := \{x \in \{0,1\}^* : \text{3rd to last digit of } x \text{ is } 1\}$



Ex:

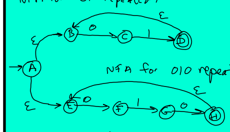
$L = \{x \in \{0,1\}^* : x \text{ is } 01 \text{ repeated 1 or more times or } x \text{ is } 010 \text{ repeated 1 or more times}\}$

<u>ε-transitions</u> (<u>ε-moves</u>)

$q \xrightarrow{\varepsilon} r$ {don't consume an input symbol moving from q to r

are allowed.

NFA for 01 repeated:



<u>Def</u>: A <u>nondeterministic</u> <u>finite</u> <u>automaton</u> (<u>NFA</u>) is a 5-tuple $\langle Q, \Sigma, \delta, q_0, F \rangle$ where Q, Σ, q_0, F are as with a DFA and

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \longrightarrow 2^Q$$

[$2^Q := P(Q) =$ <u>powerset</u> of Q = set of all subsets of Q]

Tabular form for last example:

	0	1	ε
→ A	∅	∅	{B,E}
B	{C}	∅	∅
C	∅	D	∅
*D	∅	∅	B
E	F	∅	∅
F	∅	G	∅
G	H	∅	∅
*H	∅	∅	E

Shorthand: Σ alphabet, let $\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$

so $\delta : Q \times \Sigma_\varepsilon \longrightarrow 2^Q$

<u>Def</u>: Given NFA $A = \langle Q, \Sigma, \delta, q_0, F \rangle$ and an input string $w \in \Sigma^*$, a <u>computation</u> (<u>comput. trace</u>; " <u>path</u>) of A on input w is a sequence $s_0, s_1, \ldots, s_n \in Q$ for some $n \geq 0$ such that there exist $w_1, w_2, \ldots, w_n \in \Sigma_\varepsilon$ such that:

1) $w = w_1 w_2 \cdots w_n$,
2) $s_0 = q_0$, and
3) $\forall i,\ 1 \leq i \leq n$,

$$s_i \in \delta(s_{i-1}, w_i).$$

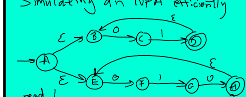
Say that the computation <u>ends</u> in s_n.

A <u>accepts</u> w if there exists a computation of A on input w that ends in an accepting state (member of F)

$L(A) := \{x \in \Sigma^* : A \text{ accepts } x\}$

A <u>recognizes</u> $L(A)$.

Simulating an NFA efficiently



$w = 010100$

read so far ↓	possible states
ε	A, B, E
0	C, F
1	D, G, B
0	H, E, C
1	D, B
0	C
0	— (∅)

$w = 01$

ε	A B E
0	C F
1	D G B

Sets of states construction to convert an NFA into an equiv. DFA:

Let $A := \langle Q, \Sigma, \delta, q_0, F \rangle$ be an NFA we build a DFA

$$D := \langle 2^Q, \Sigma, \Delta, Q_0, \mathcal{F} \rangle$$

where

$Q_0 := \varepsilon\text{-cl}(\{q_0\})$,

[$\varepsilon\text{-cl}(S)$:= set of states to get to, following 0 or more ε-moves from any state in S]

S: set of states

$\mathcal{F} := \{S \subseteq Q : S \cap F \neq \emptyset\}$,

and for every $S \in 2^Q$ ($S \subseteq Q$) and $a \in \Sigma$, define

$$\Delta(S, a) := \varepsilon\text{-cl}\left(\bigcup_{q \in S} \delta(q, a)\right)$$

$\Delta(S, a)$ = set of states reachable from states in S reading symbol a

<u>Def</u>: Given $S \subseteq Q$, say that S is <u>ε-closed</u> if $\delta(q, \varepsilon) \subseteq S$ for all $q \in S$.

Given any set $T \subseteq Q$,

$$\varepsilon\text{-cl}(T) := \bigcap \{S \subseteq Q : T \subseteq S \text{ and } S \text{ is } \varepsilon\text{-closed}\}$$

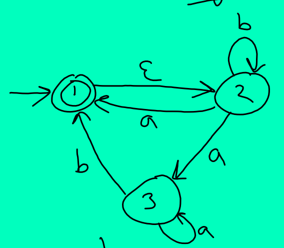
= least ε-closed superset of S.

$\varepsilon\text{cl}(S)$:
- $C := S$
- while $\exists q \in C$ such that $\delta(q, \varepsilon) \not\subseteq C$:
  - $C := C \cup \delta(q, \varepsilon)$
- return C

<u>Theorem</u>: Given NFA A as above, the DFA D constructed above satisfies $L(D) = L(A)$

(A & D are <u>equivalent</u>)

<u>Ex</u>:

A: 

$\Sigma = \{a, b\}$

D: