CSCE 551

In-person final exam for J60 section is not required, but remote proctoring is necessary.

https://cse.sc.edu/~fenner/csce551

All materials found here.

Part 1: Automata

Part 2: Computability

Part 3: Comput. Complexity

Recommend: Sipser Chap. 0

Alphabets, strings, operations

Def: An alphabet is any nonempty finite set.

If Σ is an alphabet, call the elements of Σ symbols (letters, characters)

Ex: $\Sigma := \{0, 1\}$ binary alph.

$\Sigma := \{0, \triangle, ☆\}$

$\Sigma := \{0, \ldots, n-1\}$ $(n > 0)$
n-ary alphabet

$\Sigma := \{0\}$ unary alphabet

$\Sigma := \{a, b, c\}$
symbols treated literally as shown.

Def: Let Σ be an alphabet. A string over Σ is any finite sequence of zero or more symbols from Σ.

If x is a string, then use $|x|$ to denote the length of x.

Ex: $\Sigma = \{a, b, c\}$

$|abac| = 4$

$abac \neq baac$

Suppose Σ has 10 symbols. How many strings over Σ are there of length 3?

Ans: $10^3 = 1000$

Generally: $|\Sigma| = n$
$\Longrightarrow \forall k$ there are n^k many strings of length k.

$k=0$: ε denotes the empty string (lower-case epsilon), unique string of length 0 (over any alphabet)

[ε is never a symbol in any alphabet]

$|\varepsilon| = 0$

Concatenation:

Def: let x, y be strings (over some alphabet).

The concatenation of x and y (or: x followed by y) is the string of length $|x| + |y|$ consisting of ~~the~~ x followed immediately by y. (denoted xy)

Ex: $x = aba$
$y = bc$

$xy = ababc$
$yx = bcaba$.

Concatenation is associative:

Fact: $(xy)z = x(yz)$ $\Big\} = xyz$

for all strings x, y, z.

$\therefore$ We can drop parentheses when concatenating 3 or more strings

Notice: For any string x,
$x\varepsilon = \varepsilon x = x$.

$|xy| = |x| + |y|$

Me: Natural numbers
$\mathbb{N} := \{0, 1, 2, \ldots\}$ ← yes!

Book: $\mathcal{N} := \{1, 2, 3, \ldots\}$
↑ no

The length of a string is always a natural number.

Basis for string induction:

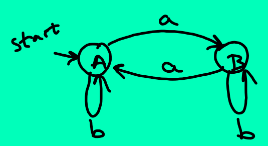
Every string x is either
- the empty string ε, or
- the concatenation ya of a unique string y $(|y| = |x| - 1)$ and a unique symbol $a \in \Sigma$.

[these cases are mutually exclusive]

[won't distinguish between symbols of Σ and strings of length 1.]

We take all inputs & outputs of computations to be strings.

$\Sigma = \{a, b\}$



Input string: abbbaaba

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

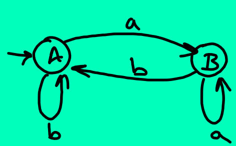
A B B B B A B B (A)

a a b a

A B A A (B)

tells you whether there are an even or odd # of a's
(A) (B
in the input string.

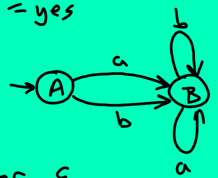
Ex:



Answers the question, "does the input string have last symbol a?"

final A = no
B = yes

Ex:



A for ε

B for any other string.