CSCE 551/MATH 562, Homework 5

For the following, you can assume that all devices use the binary alphabet $\Sigma := \{0, 1\}$ for input/output/printing. For these problems, you may assume that every string can be interpreted as the encoding of a Turing machine.

1. Let
   $$L := \{\langle M \rangle \mid 00 \in L(M) \text{ and } 11 \notin L(M)\} .$$
   Show that $A_{\mathsf{TM}} \leq_m L$ and that $\overline{A_{\mathsf{TM}}} \leq_m L$. Describe the two m-reductions directly, without appealing to Rice's theorem.

2. Let
   $$L := \{\langle M \rangle \mid 00 \in L(M) \text{ or } 11 \notin L(M)\} .$$
   Show that $A_{\mathsf{TM}} \leq_m L$ and that $\overline{A_{\mathsf{TM}}} \leq_m L$. Describe the two m-reductions directly, without appealing to Rice's theorem.

3. Recall that a string w is a *palindrome* if $w = w^{\mathcal{R}}$. Let $\mathit{PALINDROMES} := \{w \mid w \text{ is a palindrome}\}$, and let
   $$L := \{\langle M \rangle \mid L(M) = \mathit{PALINDROMES}\} .$$
   Show that $A_{\mathsf{TM}} \leq_m L$ and that $\overline{A_{\mathsf{TM}}} \leq_m L$. Describe the two m-reductions directly, without appealing to Rice's theorem.

4. Let
   $$L := \{\langle M \rangle \mid M \text{ is a DFA and } 0^*1^* \subseteq L(M)\} .$$
   Show that $L \in \mathbf{P}$ by giving a polynomial-time decision procedure for L. [Hint: DFA minimization, the product and complement constructions, and deciding whether two given DFAs are isomorphic (i.e., the same DFA up to state re-labeling) can all be done in polynomial time.]