CSCE 551/MATH 562, Homework 4

For the following, you can assume that all devices use the binary alphabet $\Sigma := \{0, 1\}$ for input or printing and that all strings and languages are over $\{0, 1\}$.

1. Show that every enumerable language is enumerated by an enumerator that never prints the same string twice.

2. Show that every enumerable language L is enumerated by an enumerator that prints each string in L infinitely many times.

3. Let E be an enumerator such that

   - $L(E)$ is infinite, and
   - E prints strings in length-monotone order (that is, for any strings w and x, if E prints w then later prints x, then it must be that $|w| \leq |x|$).

   Show that $L(E)$ is decidable by giving a decision procedure for $L(E)$ (high-level description only). [Note that there are only finitely many strings of any given length.]

4. Suppose L is a Turing-recognizable language that contains exactly one string of every length. Show that L is decidable.

5. Let $L := \{\langle M \rangle : M \text{ is a TM and } |L(M)| \geq 17\}$.

   (a) Show that L is Turing-recognizable.

   (b) Show that L is undecidable.

   (c) Given (a) and (b), what can you conclude about $\overline{L}$?

6. Define the language

   $$R_{\mathsf{TM}} := \{\langle M, w \rangle : M \text{ is a TM, } w \text{ a string, and } M \text{ rejects } w\} .$$

   Show that R_{TM} is Turing-recognizable and undecidable.

7. Let $f : \Sigma^* \to \Sigma^*$ be a function such that, for every TM M and string w, $f(\langle M, w \rangle) = \langle t \rangle$ where t is a natural number such that, if M accepts w, it does so in $\leq t$ steps. (We make no assertions about t if M does not accept w.) Show that f is not computable.