Graduate Credit Exercises
due Friday April 25, 2025 at 11:30pm EDT

These exercises are required for those taking this class for graduate credit and those taking it for Honors College credit. Everyone else is of course welcome to try their hand at these, but I will not grade them.

Each numbered question has either no stars, one star ($\star$), or two stars ($\star\star$) following the question's number. The number of stars indicates my own estimate of a question's difficulty. For full credit, everyone must submit answers to both of the unstarred questions. Beyond that, you can either **take the standard route:** do your choice of any **TWO** of the single-starred questions; or **shoot the moon:** do only **ONE** of the double-starred questions. In other words, a double-starred question essentially counts as two single-starred questions.

To get credit for a problem, you must answer it reasonably correctly. No partial credit will be given.

You may discuss these questions and possible answers with other people, e.g., your classmates or me, but you must at least (1) write up your answers yourself, (2) properly cite any written or online resources you consulted (including AI), and (3) list any people with whom you discussed or shared answers. *NOTE: Failing to do this violates the class policy on plagiarism and constitutes a violation of the Carolina Honor Code.*

A instance of the Editing Problem. Recall the Editing Problem defined in lecture:

$$\begin{aligned}\mathrm{EP} := \{\langle\langle\Sigma, E\rangle, w\rangle \mid {} & \langle\Sigma, E\rangle \text{ is an editing system, } w \text{ is a string over } \Sigma, \\ & \text{and there exists a finite sequence of edits using } E \\ & \text{starting with } w \text{ and ending with } \varepsilon\}\ .\end{aligned}$$

1. Let $\Sigma := \{0, 1\}$, let $E := \{(10, 011), (10, 001), (000011111, \varepsilon)\}$, and let $w := 110$. Show that $\langle\langle\Sigma, R\rangle, w\rangle$ is in EP by giving an explicit editing sequence starting with w and ending with ε.

Restrictions of the Editing Problem. There are several ways to restrict the instances of the Editing Problem, some leading to decidable problems and some not. For example, in our proof that $A_{\mathsf{TM}} \leq_{\mathrm{m}} \mathrm{EP}$, some of the pairs (x, y) we included in the set E of allowed edits had the property that $|x| < |y|$ (this was only for the purpose of padding the ID with blanks on the right). This inclusion is crucial.

2. Define the *length-nonincreasing editing problem* $\mathrm{EP}_{\geq}$ to be the same as EP except that all allowed edits $(x, y) \in E$ satisfy $|x| \geq |y|$. Show that $\mathrm{EP}_{\geq}$ is decidable. (In fact, $\mathrm{EP}_{\geq}$ is PSPACE-complete, but you do not need to show this.) [To be clear, you are *not* trying to decide whether an input $\langle\langle\Sigma, E\rangle, w\rangle$ satisfies the length-nonincreasing condition above (that's too easy); you should *assume* that the input does satisfy the length-nonincreasing condition and decide whether the string w edits to ε.]

3. ($\star$) Define the *length-decreasing editing problem* $\mathrm{EP}_{>}$ to be the same as EP except that all allowed edits $(x, y) \in E$ satisfy $|x| > |y|$. Show that $\mathrm{EP}_{>} \in \mathsf{NP}$.

4. ($\star\star$) Show that $\mathrm{EP}_{>}$ is NP-complete.

5. ($\star$) For any positive integer n, define the *length-n restricted editing problem* EP_n to be EP restricted to inputs $\langle\langle\Sigma, E\rangle, w\rangle$ where every allowed edit $(x, y) \in E$ satisfies $|x| \leq n$. In the reduction from A_{TM} to EP given in class, the instance of EP we constructed was actually an instance of EP_3. (The first coordinate of each pair in E was of length ≤ 3, the length-3 strings being used to handle left head movements of the TM.) That means that we actually showed that $A_{\mathsf{TM}} \leq_{\mathrm{m}} \mathrm{EP}_3$, and thus EP_3 is undecidable.

   Show that $\mathrm{EP}_3 \leq_{\mathrm{m}} \mathrm{EP}_2$, and thus EP_2 is also undecidable.

6. ($\star$) Show that EP_1 is decidable in polynomial time (i.e., $\mathrm{EP}_1 \in \mathsf{P}$).

7. ($\star$) Define the *binary editing problem* $\mathrm{EP}_{\mathit{bin}}$ to be EP restricted to the binary alphabet. Show that $\mathrm{EP}_{\mathit{bin}}$ is undecidable.

8. ($\star\star$) (This refers to the previous problem.) Show that EP restricted to the unary alphabet $\Sigma = \{0\}$ is in P.

9. ($\star\star$) Consider the Padded Editing Problem PEP:

   $$\mathrm{PEP} = \{\langle E, w\rangle 10^t \mid t \geq 0 \text{ and } \langle E, w\rangle \in \mathrm{EP} \text{ witnessed by a sequence of} \leq t \text{ edits}\}\ .$$

   Show that PEP is NP-complete. [Hint: The m-reduction from A_{TM} to EP described in lecture is computable in polynomial time (making it a polynomial reduction), and, for any TM M and input w, if M accepts w, then there is an editing sequence of length $3t + 3$ (i.e., polynomial in t alone), where t is the running time of M on input w.]