CSCE 355
2/18/2026

More pumping lemma examples

MT1 review

(1)

Recall: L is not pumpable iff

$\forall p > 0,\ \exists s \in L,\ |s| \geq p$

$\forall x, y, z \in \Sigma^*,\ s = xyz,\ |xy| \leq p,\ y \neq \varepsilon$

$\exists i \geq 0,\ xy^iz \notin L.$

Examples:

$L = \{ w \in \{0,1\}^* :$ $|w|$ is a multiple of 3 and w has a 1 somewhere in its middle 3rd$\}$

$= \{ xyz : x, y, z \in \Sigma^*$ and $|x| = |y| = |z|$ and y has a 1 somewhere (i.e., $y \in L((0+1)^* 1 (0+1)^*)$ $)\}$

Prop: L is not pumpable.

Pf: Given $p > 0$, let $s := 0^p \overbrace{1 0^{p-1}}^{\text{middle 3rd}} 0^p = 0^p 1 0^{2p-1}$

given x, y, z as above {know that $y = 0^k$ for some $k > 0$}

let $i := 0$. Then $xy^0z = xz = 0^{p-k} 1 0^{2p-1}$

~~$|0^{p-k} 1 0^p| = 2p + 1 - k$~~

$|0^{p-k} 1 0^{2p-1}| = 3p - k$

Case 1: ~~Either~~ k is not a multiple of 3: Then $|xz| = 3p - k$ is not a mult. of 3, so $xz \notin L$. done in case 1

Case 2: k is a multiple of 3. Then $|xz|$ is also a multiple of 3. But $k \geq 3$. So $k = 3\ell$ for some $\ell > 0$. Then $|xz| = \cancel{3(k-\ell)} \cancel{= 3k - 3\ell}$. $3(p-\ell) = 3p - 3\ell$

~~Then~~ $\cancel{xz = 0^{p-k} 1 0^{2p-1} = 0^{p-3\ell} 1 0^{2p-1}}$

$\cancel{= 0^{p-3\ell} 1}$

$$xz = 0^{p-3\ell} 1 0^{2p-1} = \underbrace{0^{p-3\ell} 1 0^{2\ell-1}}_{p-\ell} \underbrace{0^{p-\ell}}_{\uparrow} 0^{p-\ell}$$

middle 3rd (no 1's)

$\notin L$.

Pump up: $i > 1$.

$$xy^i z = \underbrace{0^{p+(i-1)k}}_{\uparrow} 1 0^{2p-1}$$

want $\geq 2/3$rds of the whole string

That is, $p + (i-1)k \geq \frac{2}{3}(p + (i-1)k + 1 + 2p - 1)$

$$3(p + (i-1)k) \geq 2(3p + (i-1)k)$$

subtract 3p from both sides

$$3(i-1)k \geq \cancel{6}\,3p + 2(i-1)k$$

$$(i-1)k \geq \cancel{6}\,3p$$

$$\therefore\ i \geq 1 + \cancel{6}\,3\frac{p}{k}$$

so we let $i := \lceil 1 + 3\frac{p}{k} \rceil$

Ex: $L = \{a^m b^n : m,n \geq 0 \text{ and } m \neq n\}$

Prop: L is not regular.

Proof: (by contradiction): Suppose L is regular.

Then $\overline{L}$ is regular (reg. lang's closed under complement).

Then $\overline{L} \cap L(a^*b^*)$ is regular

($L(a^*b^*)$ is regular and reg lang's closed under intersection)

But then

$$\overline{L} \cap L(a^*b^*) = \overline{\{a^m b^n : m \neq n\}} \cup \{a^x b^y : x,y \geq 0\}$$

$$= \{a^x b^y : x,y \geq 0\} \setminus \{a^x b^y : x \neq y\}$$

$$= \{a^x b^y : x = y\} = \{a^n b^n : n \geq 0\}$$

But $\{a^n b^n : n \geq 0\}$ is not pumpable (showed last time)

$\therefore \{a^n b^n : n \geq 0\} = \overline{L} \cap L(a^*b^*)$ is not regular

contradiction.

$\therefore L$ is not regular.

L is not pumpable:

Given $p > 0$, let $s := a^p b^{p+p!}$

Given x, y, z ---- let $k := |y|$ $(y = a^k)$

let $i := 1 + \frac{p!}{k}$ $\{$note: $0 < k \leq p\}$

Review for 1st midterm —

- Given a lang L described in prose or math, find a DFA, NFA, ε-NFA, regex for L

- Conversions:

  NFA → DFA (sets of states constructions)

  ε-NFA → NFA (use the method I covered in class (method 2 in the course notes)

  regex → ε-NFA (recursion on regex syntax)

  DFA, NFA, ε-NFA ⇒ regex (state elimination method)

- Product & complement constructions with DFAs

- String induction/recursion

NO Pumping Lemma

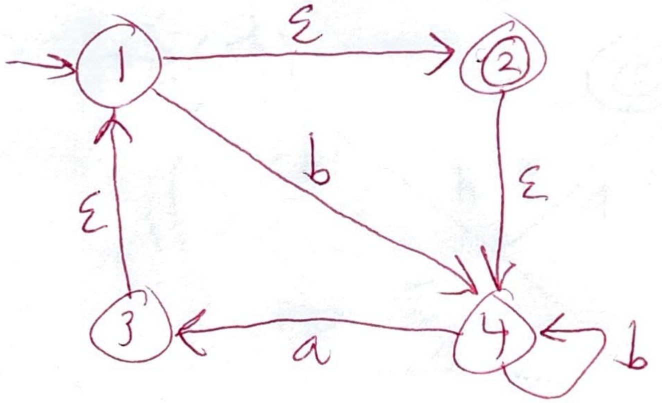
? Closure prop of the reg langs ??

[Confined to HW1,2,3 topics] nonoptional only

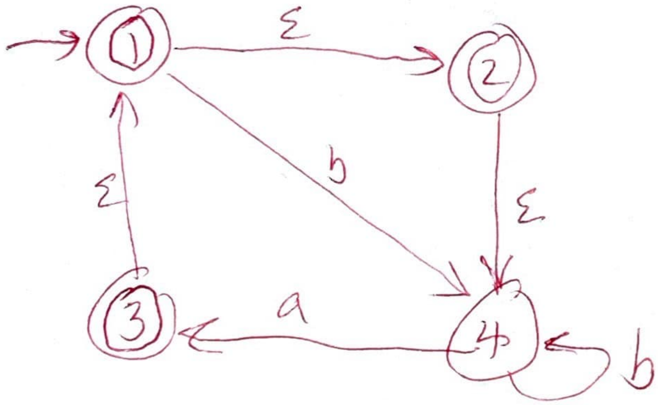
[only string induction from HW1]

ε-NFA → NFA

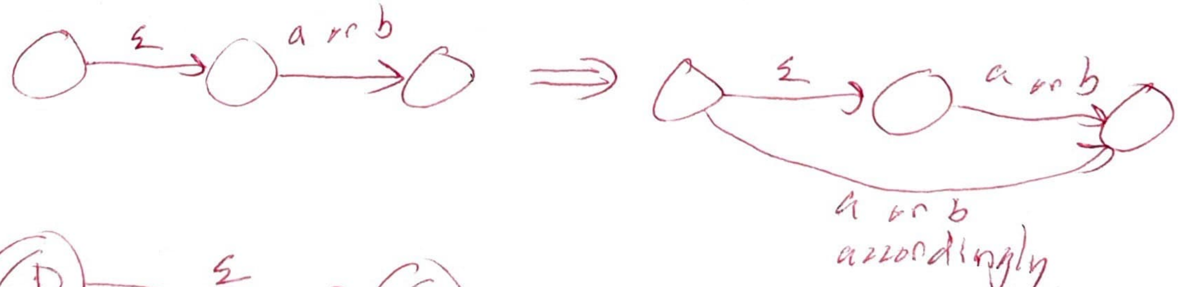
Given

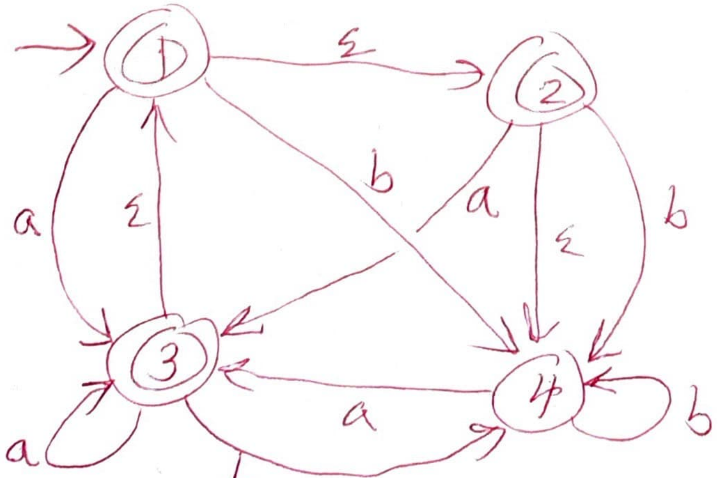


Step 1:

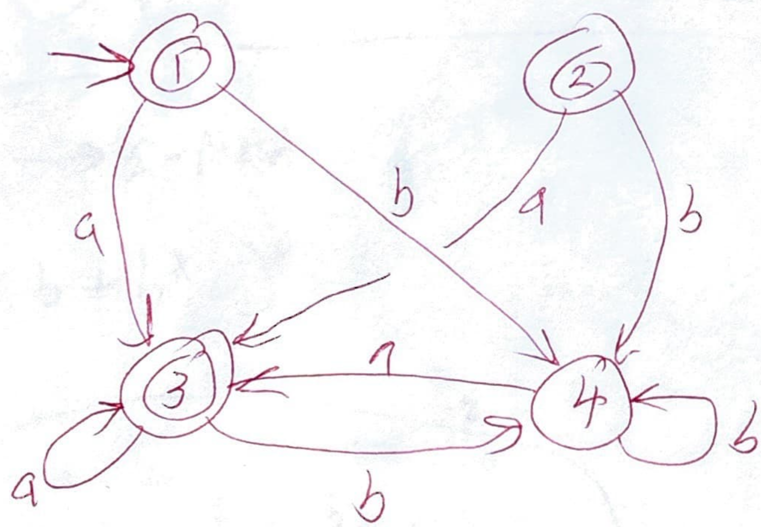


Step 2:



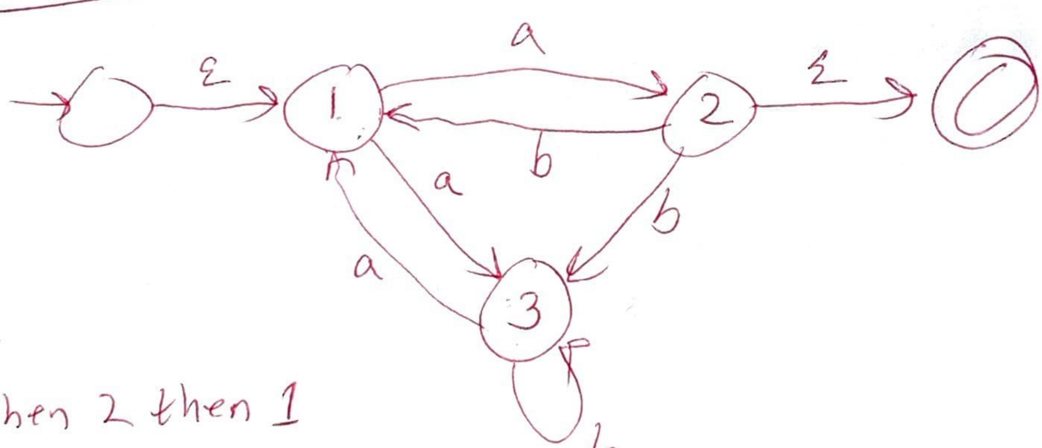


Step 3: Remove all ε-transitions:



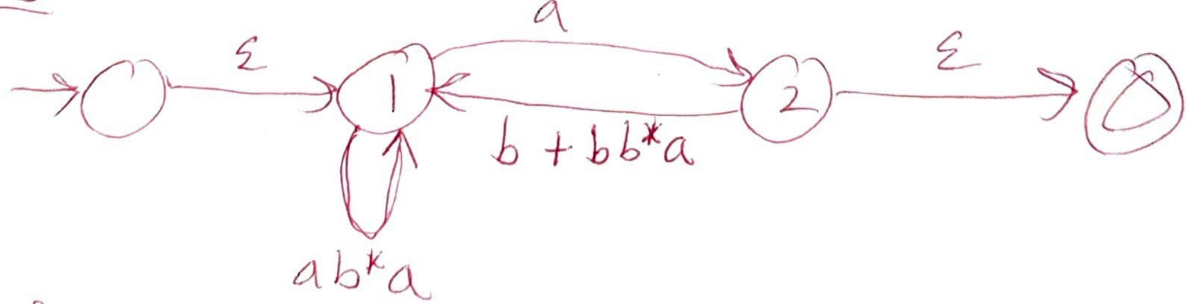
Optional: Remove unreachable states (2 state in this case)
& transitions out thereof

State elim.

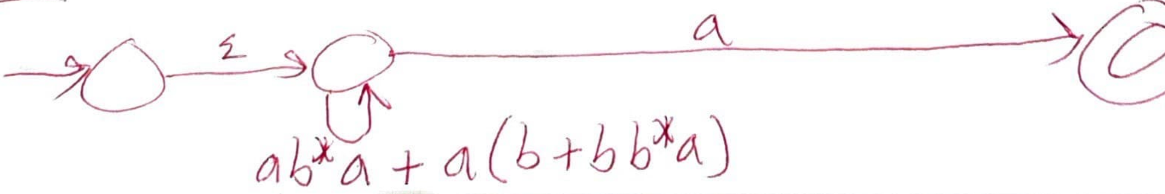


Remove
3 then 2 then 1

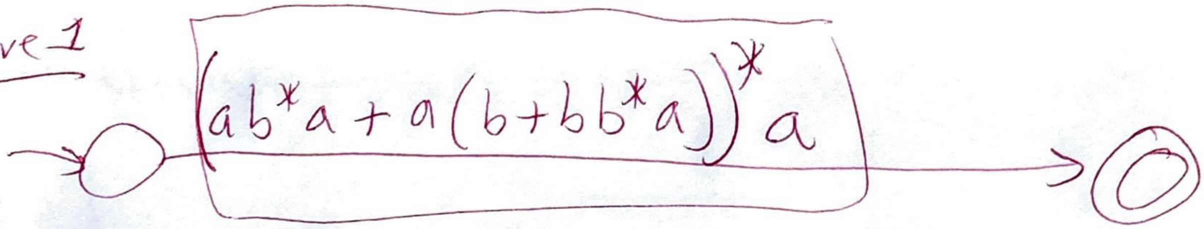
Remove 3



Remove 2



Remove 1



regex $\longrightarrow$ ε-NFA

$(ab + b^*c)^*$